4. a floating body displaces its own weight of water. Therefore sp. gr. of the body

$$= \frac{\text{vol of part under water}}{\text{vol of whole}}$$

in vol water is divided [illegible]
[illegible] weight [illegible]

---

5. sp. gr. is the ratio of the weight of a given volume of a substance to the weight of same vol. of a standard substance (water is standard for solids + liquids)

$$\text{Sp. gr} = \frac{\text{wt of the body in air}}{\text{wt of the same vol of water}}$$

i.e. loss of wt in water.

---

6. [illegible]

7. [illegible]

[illegible]

[illegible]

L. W. Jenkins
285 Newbury St
Boston
Mass.

7. Air Pump Formula

Pressure = vol of Receiver $\left(\frac{\text{vol of Receiver}}{\text{vol of whole app}}\right)^n$

where n = number of strokes.

8. Equilibrant always = resultant but is opposite in direction

9. Moment = force x distance positive when like hands of a clock negative in opposite direction

10. (a) A poundal is a force which acting ... [illegible]

See.

6 a dine gives one gram
same sec. a velocity
1 cm. $\sqrt{\frac{\text{E}}{\text{gr.}}}$ 1 cm against a force of 1 dine

11. Falling bodies.

Velocity = gravity × Time = gt

Space in any Sec = $\frac{g}{2}(2t-1)$

Space for several Sec = $\frac{g}{2}t^2$

12. Momentum = wt. × vel.
momentum before collision = momentum after

13. Kin. Energy = $\frac{\text{wt.} \times \text{vel}^2}{2g}$ ft. lbs

[illegible]

Kin. Energy [illegible]

= [illegible]

[illegible]

[illegible]

A TEXT-BOOK OF PHYSICS

LARGELY EXPERIMENTAL

ON THE BASIS OF THE HARVARD COLLEGE "DESCRIPTIVE LIST OF ELEMENTARY PHYSICAL EXPERIMENTS"

BY

EDWIN H. HALL, PH.D.

ASSISTANT PROFESSOR OF PHYSICS IN HARVARD COLLEGE

AND

JOSEPH Y. BERGEN, JR., A.M.

INSTRUCTOR IN THE HARVARD SUMMER SCHOOL OF PHYSICS, AND JUNIOR MASTER IN THE ENGLISH HIGH SCHOOL, BOSTON

NEW YORK

HENRY HOLT AND COMPANY

1891

15. Sp. Heat = $\frac{\text{heat required to raise 1 gr. of substance 1°}}{\text{heat " " " " 1 " " water " "}}$

16. Latent Heat is heat not shown by the Thermometer but used in melting or vaporizing.

Lat. Heat of melting ice — 80°
" " " Steam — 537

17. Mechanical equivalent of heat = 772 ft. lbs. (= 42670 gr. cm.)

[illegible]

Robert Drummond,
Electrotyper and Printer,
New York.

INTRODUCTION.

TO THE TEACHER.

In the year 1886 Harvard College made a very important change touching the physics in its requirements for admission. For many years candidates for the Freshman class had been examined in physics, but in text-book physics only. It was now decided to establish a requirement of laboratory work to be recommended by the College in place of the text-book work, although the latter, considerably increased, remained as an alternative for those who could not command laboratory facilities.

It soon became evident, in view of the inexperience of teachers and the very different standards and methods likely to be adopted by them, that a special course of experiments, carefully thought out and described with much detail, was needed to make the new plan a success. No one could doubt this who had to do with the first examination held under this plan, when, owing to the delay of specific and full directions, every teacher was left to follow pretty nearly his own plan and devices. The candidates who offered experimental physics came, like the traditional beggars, "some in rags and some in shags and some in velvet gowns."

It appeared from the answers to a circular issued to preparatory schools in 1886 that in many of the best of them the amount of time generally given to the study of physics

was about five school-hours per week in class-room, and probably some hours per week out of class-room, for one year. Upon this basis of time the new course was laid out. As to the money limit, it was evident from the start that the demands of the course must be very modest.

The choice of experiments required careful consideration. There could be no doubt that, if the course was to be kept from degenerating into mere perfunctory trifing with apparatus, there must be a backbone of quantitative work, but of what particular parts should this be made up? The criterion adopted for this selection was that of practical utility. An attempt was made to bring together such experiments as would have the most frequent and important applications in ordinary life, in the conviction that these would be, on the whole, quite as interesting and important in every other way as any that could be chosen under a different principle of selection.

The system of experiments thus originated was described in detail in a pamphlet issued by the College in 1887. It was afterward somewhat revised, the last edition of the pamphlet appearing in 1889, with the title *Descriptive List of Elementary Physical Experiments*. The success of the course has been very gratifying. It is now followed by hundreds of pupils in the schools of New England, and is established in many other places throughout the country. At the examinations for admission to Harvard College, held in 1891, more than half of all the candidates in physics offered this course in place of the alternative text-book course. There has been, however, from the outset a wish on the part of teachers for a text-book to supplement the laboratory course. None of the current text-books, excellent in their way as some of them are, meet fully the special needs of a class using the College pamphlet. Hence the present book, which contains the full text of that pamphlet, with a few unimportant omissions and alterations, inter-

spersed with a considerable number of minor experiments and a large amount of discussion and problem work.

The book is intended for the use of the student, to enable him to derive the full benefit of his experimental work; to guide him in his thinking, but not to relieve him from the necessity of thinking. It is therefore written with certain reservations: for example, the formula for working out the result of Exercise XXVII, on Specific Heat, being given in full, the formulas for Exercises XXVIII and XXIX, on Latent Heat, are not given; in various cases where the subsequent course of the book requires that the conclusion to be drawn from a certain experiment shall be expressly stated, this statement is not given in immediate connection with the experiment which furnishes the basis for it, but is deferred somewhat in order that the student may have an opportunity to frame one for himself; as to the numerical results of the various exercises the book gives little or no hint. Hence the apprehension that some teachers may have, lest the book may give too much assistance to the student, will probably be dissipated upon careful examination.

Exercises taken from the College pamphlet, and the figures illustrating these exercises, are here numbered with Roman numerals. Other experiments and figures are numbered with ordinary numerals. All, or nearly all, experiments are printed in a distinctive type.

Beyond the fact that the text of the College pamphlet is reproduced in these pages, this is in no respect an official publication. The authors have had in mind during its preparation the needs of high schools and academies in general, and believe that the book will prove as useful to those who do not look forward to a College course as to those who do. They believe, too, that it is well suited to the needs of college classes beginning the studyof physics.

It will not be necessary in this place to give, even in outline, directions for the construction of a physical labora-

tory, for its equipment, or for the purchase and manufacture of apparatus. It may be well, however, to say that, if possible, a room should be set aside for the exclusive use of the class or classes in experimental physics, and that the teacher should be absolutely at liberty, not only during the physics hours, but also during several other hours of the week, to arrange for and to direct the experiments unvexed by any care of schoolrooms or of pupils, save those actually engaged in laboratory-work. In too many high schools, of all sizes, this manifest prerequisite to successful experimentation is disregarded.

The size of the divisions for laboratory-practice is a very important matter. Not more than half as many pupils at a time can be directed to advantage as can be heard in recitation: perhaps the number twelve is a fair limit. There must not be so many as to make it inconvenient for the teacher to get to the bench of each pupil at frequent intervals, during his rounds, in order to check at the outset gross error on the part of the experimenter, and thus prevent the whole hour being wasted in following out a misconception of the conditions of an experiment.

Teachers are somewhat divided in opinion as to the feasibility of conducting laboratory-work in physics upon the plan of allowing several different experiments to be carried on at the same time, in order to admit of the use of a single piece of apparatus by many students in turn. It must not be forgotten, however, that the mere performance of experiments in accordance with written directions is not the whole of a successful course in elementary physics. There must be questions, discussions, supplementary experiments, problems, to bring home to the young student the whole lesson that his experiments are capable of teaching. To do all this important work of *enforcement* with economy of the teacher's time requires that the class should move with a nearly even front, so that all of its

members may be interested in the same topics at the same time. It is, moreover, possible to hold the class to more original and independent work in case the experiments are performed upon the simultaneous plan, with each student doing the same work at the same time and using his individual set of apparatus. Only those experiments should be performed by two or more in partnership which cannot be successfully performed without co-operation.

Many other points of practical importance in the conduct of the course are touched upon in the Introduction to the college pamphlet, which Introduction is here given in full.

"The objects to be sought in the course of experimental physics which this pamphlet describes may be stated thus: 1st, to train the young student by means of tangible problems requiring him to observe accurately, to attend strictly, and to think clearly; 2d, to give practice in the methods by which physical facts and laws are discovered; 3d, to give practical acquaintance with a considerable number of these facts and laws, with a view to their utility in the thought and action of educated men.

"The course, limited as it must be in expense of time and money, cannot take up all the topics which in grade would be suited to it and which in a longer course would be desirable.

"It will deal with

MECHANICS.

Certain distinguishing characteristics of solids, liquids, and gases.
Determinations of density and specific gravity.
Simple cases of composition and resolution of forces.
Simple cases in statics: inclined plane, lever, etc.
Friction, the coefficient of.
Dynamical comparison of masses.
Action and reaction.
Work.

HEAT.

Evaporation, boiling, condensation.
Expansion of liquids: study of mercury thermometer.
Expansion of solids: linear expansion of metal bar.
Expansion of gases: study of air-thermometer.
Specific heat of a solid by method of mixture.
Latent heat of melting.
Latent heat of vaporization.

SOUND.

Direct measurement of velocity in open air.
Character of a musical sound: rate of a tuning-fork.
Interference: measurement of wave-length in air in a tube.
Vibrations of strings.

LIGHT.

Photometry.
Images in plane mirrors.
Focal length of a converging lens.
Conjugate foci of a " "
Images formed by a " "

MAGNETISM AND ELECTRIC CURRENTS.

Lines of force due to magnets.
Construction of a galvanic cell.
Lines of force due to an electric current: galvanoscope.
Resistance of wires.
Resistance of a battery.
Electromagnetism: telegraphic sounder and key.

"With very few exceptions the experiments described will require the student to make *measurements* of some kind. To make such experiments intelligible and profitable they must, in many cases, be supplemented by other experiments of a less rigorous character, such as are described in

the text-books of Avery, Gage, and various other authors, and many of which are better fitted for exhibition on the lecture-table than for performance by each student of the class. In one or two cases, where measurement experiments would be difficult to arrange and where the omission of all experiment might leave a serious gap in the student's knowledge, experiments not requiring set measurements are described. Such an experiment is the one on Boiling. The directions given in this pamphlet are in some cases very minute. They are, however, intended to show how the experiments *may* be done, not how they *must* be done. The teacher should decide for himself how closely these directions are to be followed, and should feel at liberty to substitute for the experiments described other experiments covering equally well the same points. In connection with many experiments for which full directions are here given, references are made to somewhat similar experiments in Trowbridge's *New Physics*,[1] or in Worthington's *First Course of Physical Laboratory-practice*.[2]

"An attempt is made to assist teachers by means of references outlining in some measure the theoretical teaching which should be derived from or should accompany the laboratory-work in the first, and in some respects most difficult, half of the course, the Mechanics. Some passages are referred to for the benefit of the teacher which are much too difficult for the pupils to take, directly or indirectly. The book most freely referred to is Lodge's *Mechanics*.[3] Teachers would probably do well to put this book into the hands of their pupils, although some of the passages designated might prove too difficult for them.

"Other books which the teacher should have are Everett's

---

[1] American Book Co., New York. Price $1.20.

[2] Rivingtons, London. 1886. Price 4*s*. 6*d*.

[3] Chambers's Science Manuals. London, 1887. Price 3*s*.

Units and Physical Constants,[1] Goodeve's *Principles of Mechanics,*[2] Anderson's *Strength of Materials,*[3] Niaudet's *Electric Batteries,*[4] Balfour Stewart's *Treatise on Heat,*[5] Everett's last edition of *Deschanel's Natural Philosophy,*[6] Stewart and Gee's *Practical Physics for Schools,*[7] Jones's *Examples in Physics.*[8] [To this list may be added Weinhold's *Physikalische Demonstrationen,* Quant and Händel, Leipsic, from which several illustrations in this book have been taken by permission, and the Third Part of Whiting's *Physical Measurement,*[9] from which most of the data given in the Appendix have been taken.]

"This course in all its aspects is intended to occupy the student about five school-hours a week, with the usual amount of study out of school, for one year. Some subjects will require less, others more, work supplementary to the laboratory-exercises, but it is thought that of the whole time devoted to the course perhaps *not more than half* should be given to these exercises and the calculations connected directly with them. The remainder should be spent on explanations, supplementary experiments, applications, problems, etc.; and for this part of the work a text-book of the ordinary sort should be in the hands of each pupil, unless the teacher is extremely proficient and has at his disposal an amount of time much greater than most teachers in schools can command for physics alone. The exact order in which the laboratory and the text-book work shall be arranged must be decided by the teacher.

---

[1] Macmillan & Co., New York. Price $1.25.

[2] Longmans, Green & Co., New York. Text-books of Science Series.

[3] Macmillan & Co., New York. Price $1.50.

[4] John Wiley & Sons, New York. Price $2.50.

[5] Macmillan & Co., New York. Price $1.00.

[6] D. Appleton & Co., New York. Four vols.; price $1.50 per vol.

[7] Macmillan & Co., New York. Price 60 cts.

[8] Macmillan & Co., New York. Price 90 cts.

[9] D. C. Heath & Co., Boston.

"The number of exercises here given is 46, of which the student may omit any 6. For instance, he may omit the whole subject of Sound, or of Light. A still wider option will be provided in the written examination which the student offering this experimental course of physics is required to take before admission to the College.

"To secure the objects of the course the student during the laboratory-exercises is placed, so far as this is practicable, in the attitude of an investigator seeking for things unforetold. But this attitude, if rigidly maintained, would be likely to keep him for an absurdly long time upon the study of one set of facts, or induce the habit of loose and hasty generalization. He should be required to work carefully, but not with a higher standard of accuracy than the apparatus and time at his disposal will warrant. He should not be told what he is expected to see, but he must usually be told in what direction to look. He should be required to draw inferences from his experiments, but the sources of possible or certain error in his work should be pointed out in order that he may be saved from the danger of coming to think that all so-called physical laws are inferred from demonstrations as loose as his own. In fact, the main value of the student's inferences, *in themselves,* is that they will enable him to understand, and without undue stretch of faith to accept, the established conclusions of physicists, and these conclusions should *in the end* always be made known to him.

"The following extract from a circular issued by the College in September, 1889, contains suggestions likely to be of permanent interest to teachers. The examination to which it refers is that of June, 1889.

"'The great diversity and frequent ambiguity of the teachers' certificates attached to the note-books make it necessary to state more fully than the College Catalogue states them the requirements in regard to the character of these books. The fact is recognized that it is impracti-

cable to have all the notes, including the working out of results and inferences, completed during the laboratory exercises under the teacher's eye. But the record made during these exercises should include, at least, all the numerical data which the student has to obtain for himself, and all other observations of transient phenomena. These first notes must be in the identical book which is to be presented to the examiners. They should be written with a black pencil or in ink, and should not be altered after they are made. Obvious, unquestionable errors occurring in them can be indicated and corrected in pencil or ink of a different color from that originally used without obscuring the original record. Such corrections, however, should occur but rarely. The tendency of the student to regard as unquestionably wrong any observation which is not what he expected it to be, and to make his observation tally with his expectation, is doubtless familiar to most teachers, and it should be one of the important objects of this experimental course to counteract this tendency.

"'It should be made possible for the examiners to tell at a glance what notes were written at the time of the laboratory-exercises and what were written at other times. This can be done by confining one set of notes to the left-hand pages and the other set to the right-hand pages of the book.

"'The following form of certificate will be printed by the College on slips suitable for placing in the note-books, and these slips will be furnished on application to the College Secretary:

"' *This note-book contains the original record of the laboratory-work of*

done under my immediate supervision. The notes which are on the left-hand pages were made in the laboratory at the time when the exercises were performed. The notes which are on the right-hand pages were made

(*Signature*)

"'Most of the note-books presented at the last examination show a satisfactory range of work and a purpose to do the work faithfully; but there are several general criticisms which apply in a greater or less degree to many books. These are:

"'1. Meagreness of experimental data, indicating a hasty performance of the exercises. This fault is most marked in the books from schools which have but lately adopted the experimental course.

"'2. Want of intelligent regard for accuracy. Many details intended to promote accuracy are given in the list of experiments, and more will be given in the next [the present] issue; but much is necessarily left to the watchfulness and ingenuity of the teacher and pupil.

"'3. Failure to work out the numerical results and to state the legitimate inferences.

"'4. Drawing conclusions which, although true, are not warranted by the experiments which the student has performed. This fault is a serious one.

"'5. Lack of proper explanatory and descriptive notes.

"'6. Too great fulness in the verbal notes,—caused by repeating the pamphlet descriptions, or by repeating phrases which would be wholly unnecessary if the numerical observations were arranged in tabular form.

"'7. Failure to keep the results of observations separate from other data and the calculations, in such a way as to make the results conspicuous.

"'8. A general disregard for neatness and good form.

"'In the laboratory-examination the candidates generally showed a very fair acquaintance with the general methods of the various exercises; but here, as in the note-books, their frequent failure to make the most accurate use of the apparatus at their command was noticeable. The increasing skill and experience of teachers will doubtless effect a gradual improvement in this respect.

"'The written examination of candidates who present alternative 2 in physics is a test subordinate in value to the note-books and the laboratory-examinations; but experience has shown that it affords desirable additional evidence of the candidates' acquaintance with the subject.'

"A common error in note-taking is the crowding of notes, the failure to leave generous spaces between exercises or different parts of the same exercise. Others are the failure to make prominent headings, the use of common fractions instead of decimals, etc., etc.

"It is an excellent practice to require the student to put into the hands of the teacher before leaving the laboratory the main data of the experiment which he has just performed, enough to enable the teacher to calculate, if he should desire to do so, the numerical result of the student's work.

"Hints in regard to procuring the more important pieces of apparatus are given in the Appendix.

"The greater part of this pamphlet has been written and revised by Dr. HALL. The experiments have been arranged and a considerable number of them have been devised by him. Some of the exercises were devised wholly or in part by Dr. WHITING. In several cases acknowledgments of suggestions from other sources are made in connection with particular experiments."

The college pamphlet (*Descriptive List of Elementary Physical Experiments*), of which this book is an expansion, is for sale by C. W. Sever, University Book-store, Cambridge, Mass.

TABLE OF CONTENTS.

CHAPTER V.

COMPOSITION AND RESOLUTION OF FORCES.

CHAPTER VI.

GRAVITY AND CENTRE OF GRAVITY.

CHAPTER VII.

MOTION.

CHAPTER VIII.

HEAT : TEMPERATURE, THERMOMETRY, EXPANSION OF SOLIDS, OF LIQUIDS, AND OF GASES.

CHAPTER IX.

NATURE OF HEAT : CALORIMETRY.

CHAPTER X.

CHANGES OF PHYSICAL STATE.

CHAPTER XI.

TRANSFERENCE AND TRANSFORMATION OF HEAT.

CHAPTER XII.

SOUND.

CHAPTER XIII.

LIGHT.

CHAPTER XIV.

OPTICAL INSTRUMENTS.

CHAPTER XV.

MAGNETISM.

CHAPTER XVI.

ELECTRICITY.

sections marked —— important to learn
α means - varies (etc)

PHYSICS.

CHAPTER I.

GENERAL DEFINITIONS; INTRODUCTORY EXPERIMENTS.

1. Definition of the Science of Physics.—It would be well to begin this book with a brief, full, and clear definition of the science of physics; but for the beginner there is no such definition. We can hardly do better than to say that physics is the science of *mechanics, sound, heat, light, electricity,* and *magnetism,* and to leave these words to be defined by the remainder of the book. It may, however, be added just here, that, although this division of the science is a convenient one, rapid progress is being made toward explaining all the phenomena of physics as purely mechanical effects.

2. Definition of Force, Matter, etc.—In the study of mechanics, which will first occupy us, we shall encounter certain words concerning the meaning of which we should have an understanding at the outset.

Force.—A force is a push or a pull. In scientific language, pushes are frequently called pressures, and pulls, tensions.

Matter.—All are familiar with many kinds of material substance or, in physical language, *matter,* such as the metals, marble, glass, wood, oil, water, air, and so on, but it has been found by physicists extremely difficult to con-

struct any definition of matter which really adds anything to our common knowledge of the subject. Perhaps no better definition can at present be given than the following:[1] *matter is that which can be acted upon by, or can exert, force.*

Any aggregation of matter, in whatever state it exists, is called a *body of matter*, or, simply, a *body*. According to the ease or difficulty with which the particles of a body move with respect to each other the body is called a *gas* (or *vapor*), as steam; a *liquid*, as water; or a *solid*, as ice.

Strain.—Any change in the shape or size of a body is called, in scientific language, a *strain.*

Stress.—Any application of force tending to produce a strain is called a *stress.*

It should be noted that the familiar *non*-scientific meaning of the word strain is very similar to the scientific meaning of the word stress.

More precise definitions of these two words, for a particular case, will be given in connection with the study of elasticity.

3. Qualitative and Quantitative Experiments.—An experiment is called *qualitative,* if it shows us *how* something acts. An experiment is called *quantitative,* if it shows *how much* something acts.

4. Units of Matter, Force, and Distance.—Physical measurement is in general the process of determining some unknown quantity in terms of some accepted unit. The units of matter that we shall be most concerned with in this course are the *pound,* the *ounce,* the *kilogram,* and the *gram.* The most common units of force will be those defined as equivalent to the pull of the earth upon the various units of matter. These force-units are in general called by the same names, pound, gram, etc., as the units of matter by means of which they are defined. As units of distance the

---

[1] Thomson and Tait's Nat. Philos., vol. I., art. 207, edition of 1879.

foot and inch and the meter and centimeter will be most used.

For scientific purposes the units of the metric system (See Appendix), some of which have just been mentioned, are more convenient than the common English units, but the latter are so much more familiar to instrument-makers and the public generally in this country, that they can hardly be dispensed with in a book like this.

A much fuller discussion of units will be given later.

5. Experiments in Measuring Lengths and Weighing.—Since practical physics consists in considerable part of measuring lengths and weighing, it is necessary for the student at the very outset to become somewhat familiar with the ordinary processes of performing these operations, and to learn by using them the values of the units employed.

EXPERIMENT 1.

Measure the length, breadth, and height of the physical laboratory, in meters and *decimal parts of a meter*,[1] and calculate its volume in cubic meters.

EXPERIMENT 2.

Measure the length of a page of your physics text-book, and try to read the length on the meter-rod to within 0.5 of a millimeter. The rod is graduated only to millimeters, but the student can with a little effort subdivide each of these finest divisions by the eye into two equal parts. It is important to place the object to be measured against the meter-rod or other rule in just the right way. Do not begin at the end of the rod, but we will say 1 cm. from the end, and lay the thing to be measured against the graduations in such a way as to make the latter *lie against the surface of the object* as closely as possible. In the present experiment, for instance, place the page and rule as block and rule are placed in Fig. VII.

EXPERIMENT 3.

Take a book whose covers do not project over the edges of the leaves, for example, a thick note-book of some kind, place it on the

[1] For instance, do not write 5 m. 3 decim. 7 cm., but 5.37 m.

table just at the edge, put on it some moderately heavy weight, for instance, a 1 or 2 kgm. weight of iron, and measure 5 or 10 mm. vertically on the edges of the leaves. Count the number of leaves and calculate from their number the thickness of an average leaf, in decimal parts of a mm. Repeat once or twice, using different parts of the same book.

EXPERIMENT 4.

Weigh on the most delicate balance in the laboratory 0.5 m. of annealed iron wire, no. 24, or 25,[1] and calculate the weight per linear centimeter of this wire. Then weigh two or more pieces of the same wire, each less than a meter long, which have been carefully measured, cut off, and then rolled up or twisted into spirals by some other person. From the known weight per centimeter of the wire used, calculate the unknown length of the portions given you. Should any of your results be $\frac{1}{2}$ of 1%,[2] or more, larger or smaller than the true value, repeat[3] with new pieces of different (unknown) lengths. In all accurate weighings observe carefully the following rules:

6. Rules for Weighing. — 1. If the balance-pans are provided with a movable support, or can be raised so as to swing freely but otherwise rest on a support, always have the pans supported while putting on or removing weights or other objects.

2. Be sure that the balance is in equilibrium, for example, that the pointer, if there is one, stands at the zero point when the balance is not loaded.[4]

3. Keep the pans clean and dry.

---

[1] See Appendix.

[2] That is, $\frac{1}{200}$ of the *true value of the quantity measured*, whether this be large or small.

[3] If the balance is really a good one.

[4] Many balances are provided with some special mechanical contrivance by means of which the zero reading can be made true. Balances not so provided can be adjusted by putting light objects in one pan or the other before weighing. Many experienced observers instead of adjusting the balance mechanically merely note the position of the pointer without load, and treat this position as the zero from which deflections are to be measured.

4. Keep the bearings free from rust, but not oiled.

5. Always handle the balance gently, and lift the smaller weights with forceps only.

6. Never weigh in a draft of air: if the balance has a case with a sliding window, lower the window to get your final reading.

Do not breathe upon a delicate balance while weighing.

CHAPTER II.

SOME CHARACTERISTICS OF SOLIDS.

7. Tenacity.—Any solid may be broken by a longitudinal pull of sufficient force, and the ratio, *force ÷ area of cross-section of object*, expresses the tenacity of the substance experimented upon. Every one is familiar with the difference in tenacity between cotton twine and twine made of linen or hemp, between paper and cloth, and between weak metals like lead and the stronger ones like brass[1] and steel. A thinnish lead pipe of a centimeter in outside diameter could be pulled in two by a moderately strong man, while a steel wire of 1 millimeter diameter would sustain the weight of a heavy man. Steel pianoforte wire, indeed, is one of the most tenacious of substances, and a wire of 0.44 square millimeters in cross-section will support a weight of 106 kgm., or will sustain 31 kilometers of itself, when hung by one end, before breaking.[2] If tenacity is reckoned, as it may conveniently be, in kgm. per sq. mm. of cross-section, the tenacity of the wire above described would be $\frac{106}{.44} = 240.9$.

MECHANICS.

[Teacher's References for Exs. I–IV. Everett's Units, etc., Chap. V. Lodge, Arts. 144–152. Goodeve, Arts. 175–182. Anderson, pp. 180–182, 130–134.]

---

[1] Brass is not, strictly speaking, a metal, but a mixture of metals, an alloy.

[2] See Encyclopedia Britannica, article *Elasticity*, vol. VII., p. 800 (9th edition).

EXERCISE I.

BREAKING STRENGTH OF A WIRE.

FIRST PART.—Apparatus: A piece of spring-brass wire, No. 27, Brown & Sharpe gauge, about 1 m. long. A Chatillon 30 lbs. straight spring-balance.[1] (This balance has a scale about 12 cm. long and is graduated to ¼ lb.) Some cylindrical body 2 cm. or more in diameter, fixed in position, to which one end of the wire may be made fast. (A water-pipe, large gas-pipe, or table-leg will serve. Two students may pull against each other, one at each end of the wire, thus doing away with the fixed cylin-

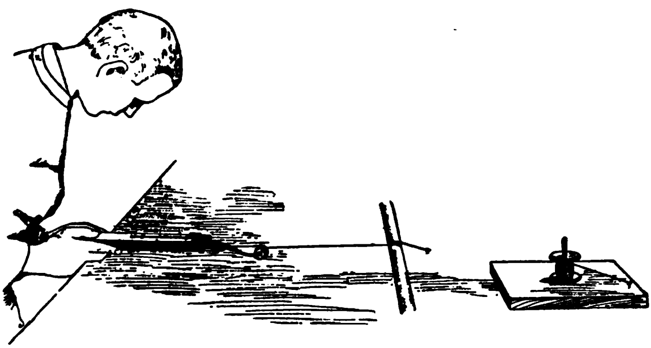


FIG. I.

der.) A short spool or perforated short section of a broom-stick to be slipped on the hook of the balance as a guard. (This guard must not be free to revolve on the hook of the balance. Therefore make it too long to go on the hook

---

[1] When a spring-balance, graduated for use in the vertical position, is placed unstrained in the horizontal position, the index rests behind the zero-mark. The force required to bring the index out to the zero-mark when the balance is in this position can be found with sufficient accuracy by means of another much more sensitive spring-balance, or by attaching to the hook one end of a string reaching over a smoothly-running pulley and bearing a known weight at the other end. This force is to be added to the actual readings of the balance in order to find the true force exerted by it in the horizontal position.

at first, and then cut it away on one side at both ends until it will go on.)

In this experiment care must be taken to have no kinks or sharp bends in the part of the wire which is to be subjected to the full strain. Therefore with one end of the wire make several turns around the fixed cylinder, and then fasten this end to any convenient object, such as a tack driven into the wall, table, or floor near the object around which the wire is wound. Pass the other end of the wire through the eye by which the hook is attached to the balance, fasten it there, and then wind several turns of the wire about the wooden guard which has been slipped upon the hook. Now pull steadily with the spring-balance, gradually increasing the force, watching all the time the index of the balance and taking care to avoid, so far as practicable, friction within the balance. Note the position of the index when the wire breaks.[1] Do this several times, we will say five times, using each time a new piece of wire.[2]

SECOND PART.—Apparatus: A balance weighing to a centigram.[3] Weigh a known length, about 1 m., of the wire and calculate the length of a piece that would just break under its own weight when suspended by one end.[4]

---

[1] Care must be taken to avoid injury to the hands from the recoiling hook of the spring-balance when the wire breaks. Painful accidents have resulted from a neglect of this precaution.

[Attention is called to Fig. 1, as illustrating the proper way of using the spring-balance. The eye should look straight down upon the index. It is convenient to buy the wire wound on spools, and to mount each spool as shown in the figure.]

[2] The danger of tangling the coil of brass wire can be much lessened by laying the coil flat upon a board and driving three or four nails into the board along the inner circumference of the coil.

[3] Such is the scale-pan balance described by Prof. Cooke in his pamphlet "Fundamental Principles of Chemistry," issued by the College. This has a range from 100 gm. to .01 gm. Balances of this latter sort may be obtained at a low price from Eimer & Amend, 205 Third Avenue, New York. Every school should in any case have one or two such balances, or others equally sensitive.

[4] Experiment shows that the breaking strengths, in simple stretching, of different wires of the same kind and quality are proportional to the areas of cross-section of the wires. The weights per unit

Instead of doing this the student can measure carefully the diameter of the wire by means of screw-calipers, and then calculate the breaking strength, in pounds-weight, of a rod of similar material 1 sq. in. in cross-section.

9. Calculation of Tenacity.—Calculate, in the manner explained in § 7, the tenacity of the wire used in kilograms per square millimeter. Remember that the cross-section of the wire is a circle. Its area may be found by the πR^2 formula *area* $= \frac{1}{4}\pi D^2$, in which the Greek letter π stands for the number 3.14, by which the diameter of a circle must be multiplied to obtain its circumference, and D represents the diameter. The diameters of the wires used may be obtained with sufficient accuracy for this calculation by comparing their wire-gauge numbers with the table of diameters given in the Appendix.[1]

10. Elasticity: of Volume, of Figure.—The power which a body has of recovering, more or less perfectly, its original volume after the force which has changed the volume is withdrawn is called *elasticity of volume*. This property exists in perfection in liquids and gases, which recover completely from the effect of any compression, however long it has been continued, when the forces producing it have ceased to be applied.

---

lengths are also, of course, proportional to the cross-sections. Hence a large wire and a small wire of the same kind and quality must be equal in length, if each is just long enough to break under its own weight.

[1] The best wire-gauge to use in physical measurements is the Brown and Sharpe gauge, a circular plate of steel with numbered slits arranged radially about its circumference. The values given in the table refer to the breadths of the slits, not to the diameters of the holes into which the slits are cut. In gauging a wire, much care must be taken to use a straight portion, free from rust or dirt, and to introduce the wire into the slit always with about the same (slight) pressure. Be sure that the wire is neither bruised flat nor roughened at the point where it is gauged.

The power which a body has of recovering, more or less perfectly, its original shape after the force which has changed the shape is withdrawn, is called *elasticity of shape or figure.* Liquids and gases are usually regarded as entirely lacking in this property, although any body of liquid or gas left entirely to its own forces and at rest would finally assume a spherical shape. Elasticity of figure, then, is peculiarly a property of solids. It is called also *rigidity.*

So-called inelastic or plastic solids are those which, like lead, wet clay, or putty, yield readily to a slight distorting force, manifesting little or no ability to resume their original shape when that force ceases to act upon them. Highly elastic solids, like tempered steel, spring-tempered brass, and ivory, offer a constant resistance even to very long-continued distorting forces, and will immediately resume their original shape upon the removal of any such force, provided the strain does not exceed a certain value, called the limit of elasticity, beyond which the substance will act like a plastic solid.

EXPERIMENT 5.

Take some pieces of number 25 or number 27 (B. & S. gauge) copper wire and some spring-brass wire of the same diameter, each about 30 cm. long. Wind a piece of each wire closely about any smooth cylinder—for example, a stout glass tube or student-lamp-chimney; upon letting go the ends notice whether the wires uncoil at all.

Try the same experiment with the brass wire after heating it almost to redness in a gas-flame.

12. Strains in Wires.—If the student repeats Exercise I with softer wires—for example, copper or annealed iron—he will doubtless notice that they stretch considerably before finally breaking. Take the gauge of these wires at various points after breaking and endeavor to discover whether the stretching process affected percepti-

bly their diameters. The stretching which is so evident in copper or soft iron wire is, in scientific language, called a *strain* (in this case a longitudinal one), and any change in the diameter of the wire would constitute a transverse strain. Plainly in this instance the strain is a permanent one, but careful measurements, best made with some more delicate instrument than the wire-gauge, would show a considerable temporary strain, followed by a slight permanent one, in the spring-brass wire as well.

13. Exercise II. Elasticity of Brass Wire. — It has already been stated (in § 10) that the name elasticity is applied to that property which causes any strained portion of matter to resume its original bulk or shape upon the removal of the force which caused the strain. In Exercise II the longitudinal elasticity of brass wire is the phenomenon to be studied.

EXERCISE II.

ELASTICITY: STRETCHING.

[Three persons should work together in this experiment.]

Apparatus: A wire like that used in Ex. I, but 4 m. or more long.[1] A spring-balance like that in Ex. I. Two meter-rods graduated in millimeters. (Shorter rods may be used.)

Drive a nail or screw into the floor or the top of a table and fasten it to one end of the wire. It is not necessary in fastening the wire to take such precautions as in Ex. I; but care should be taken to make a fastening that will not yield much when the wire is pulled. (Some teachers solder the wire to a screw.) Fasten the other end of the wire to the hook of the spring-balance and pull with a force of 1 or 2 lbs., the wire and balance being horizontal and the latter lying upon its back. This will draw the wire tolerably straight. Make the balance fast in this position or have it held while the arrangements are completed as follows: Solder two

[1] Two shorter pieces of wire may be soldered together to make the required length. This soldering, if properly done, will not affect the stretching perceptibly.

pieces of fine wire about ½ cm. long across the long wire at right angles, one near each end, to serve as markers. Put the measuring rods alongside the wire opposite these markers. When now the wire is stretched further, each of these markers will move, and its movement, if sufficiently great, may be measured on the scale opposite. The difference between the movements of the markers

FORM OF RECORD FOR EXERCISE II.

Result of stretching a No. — B. & S. spring-brass wire, four meters long:

Pounds tension.	Total elongation.	Permanent elongation.	Temporary elongation.	Total elongation per lb. tension.	Ratio of total elongation per lb. to length of wire.
1	*mm.*	*mm.*	*mm.*	*mm.*	
2					
3					
4					
5					
6					
7					
8					

is the amount of stretching which occurs in that part of the wire that lies between them.

Great care should be taken in reading the positions of the markers and in handling and reading the spring-balance.

Read the positions of the markers, with a force of 1 lb. applied, and then increase the force 2 lbs. at a time, reading at each addition and coming back to the original 1 lb. for a new reading every time, until the wire becomes strained beyond its limit of recovery and fails to return to its original length.

Record the original length, roughly measured, of that part of the wire in which the stretching is observed.

Put the observations in tabular form and look for some relation between the amounts of stretching and the forces that produce them.

14. Laws of Elasticity.—If Exercise II has been carefully conducted, the tabulated results will enable us to answer such questions as: (1) Is such wire perfectly elastic[1] (that is, does it exactly resume the original length) in case the pull upon it is much less than the breaking-weight? (2) If there is such perfect elasticity, at what value of the stretching force does it fail? (3) Does the temporary strain per kilogram of tension increase or decrease after permanent strain has set in? (4) Does the amount of temporary strain show any simple relation to the amount of tension? (5) Does the amount of permanent strain show any simple relation to the successive increments of tension after permanent strain first appears? The student should answer these questions from the results obtained in *his own experiments*.

The answer to (4) will constitute the first law for linear extension of solids submitted to longitudinal tension. A few additional experiments would suffice to establish the second law:

Elongation is proportional to the length of the wire: and

[1] Of course more delicate tests might show permanent changes which the rough methods of this book cannot detect.

the third law: *Elongation is inversely proportional to the cross-section of the wire.*[1]

15. The graphical method of showing results. The results of this exercise may be rendered apparent to the eye by resorting to what is known as *the graphical method.* This mode of recording and exhibiting the relations of forces to their effects bears the same relation to a numerical statement that geometry bears to arithmetic, and it is in fact an application of geometry. The annexed diagram, Fig. 1, consists of a large number of equal squares, about $\frac{1}{4}$ cm.

[1] The following definitions and formulæ may help to show the relations of the quantities discussed in the preceding paragraphs in a somewhat more definite shape.

$$\textit{Elasticity} = \frac{\textit{stress}}{\textit{strain}}. \quad \textit{Stress} = \frac{\textit{tension}}{\textit{area of cross-section}}.$$

If in the preceding experiment we consider only the change in length of the stretched wire,

$$\textit{strain} = \frac{\textit{change of length}}{\textit{original length}}.$$

Hence

$$\textit{elasticity} = \frac{\textit{tension} \div \textit{area of cross-section}}{\textit{change of length} \div \textit{original length}}.$$

If several bars of various sizes and shapes but made of the same material in the same state of purity, hardness, etc., are subjected to tests, like those of Exercise II., which do not exceed the limit of elasticity, it is found that the quantity, *stress* ÷ *strain;* is the same for all of them. It is called the *modulus of elasticity* for the given material under longitudinal strains. A different material has a different modulus, that is, a different value for *stress* ÷ *strain*. For instance, if force is reckoned in kilograms and area of cross-section in millimeters, the modulus for steel bars of a certain grade is 25,000 and for copper wire of a certain grade about 12,000. Tables of such moduli derived from experiments are printed in many books used by physicists and engineers, but different grades of the same general substance will differ so much in their tenacity and elasticity that materials which are to be put to any severe and important use, steel for making heavy cannon, for instance, are often subjected to special tests to prove their qualities.

on a side,[1] and upon this series of squares the result of Exercise II may be represented as follows: number the larger divisions in a vertical column at the left of the sheet, calling the lowest heavy line 0, the next heavy line above, 1, and so on. These figures may stand for kilograms[2] tension. Next, measure off from the left-hand heavy vertical line, and on a level with the figure standing for each tension

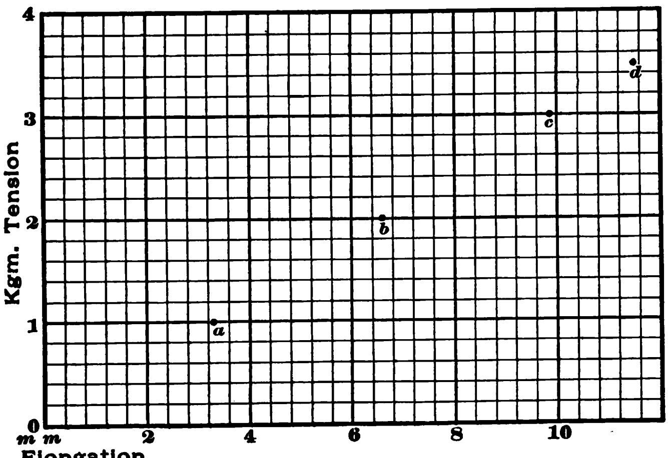


FIG. 1.

in turn, a distance corresponding to the number of mm. stretching which was produced by that tension. In the accompanying diagram the horizontal distances correspond to elongations obtained as the result of an actual experiment. The numbers in a horizontal row at the bottom represent mm. of elongation, and the dots *a, b, c, d,* which extend diagonally across the diagram mark the results obtained by

---

[1] Any ordinary paper, divided into squares, such as is furnished by dealers in drawing materials, will answer.

[2] Kgm. are used here because they go, more naturally than pounds, with mm., in which the extensions are measured.

the successive trials in the experiment. Fig. 2 is a repetition of Fig. 1, but with the dots connected by means of a ruled line. In reducing the students' own results to this form it is probable that no straight line or line of even curvature can be drawn which shall cut all the points located as just described. In that case the line should be drawn between the extreme points, in a position as near the medium position as it can be made to fall.

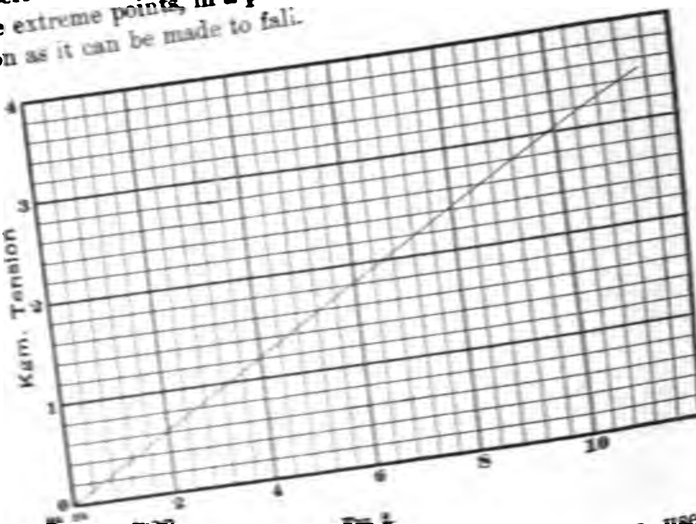


Fig. 2.

In case india-rubber cord or tubing has been used for the experiment, the medium line may be decidedly curved, and in that case it may be constructed with a fairly even curve by taking a strip of hard rubber of uniform thickness or a steel back-saw blade and bending it in various ways until its curve coincides with as many of the points as possible, then drawing a smooth line through them. From the principle in geometry that corresponding sides of similar triangles are proportional, it is evident that if the line joining the squares is straight, it

indicates that, in this experiment, stretching is proportional to tension.

16. Compression.—All that has been said concerning the stretching of bodies by tension is equally true concerning their compression. The same general laws hold for compression as for stretching, and, moreover, the *modulus*[1] for the compression of any substance is numerically the same as the modulus for its stretching, provided the limit of elasticity is not exceeded. The limit of elasticity in compression is, however, not necessarily the same as the limit of elasticity in stretching.

17. Strains produced by Transverse Forces.—The preceding considerations lead naturally to Exercise III, on bending, for it will be found that usually when a body is bent some parts of it are stretched and some parts compressed.

EXPERIMENT 6.

Take a long thick india-rubber "eraser" and measure carefully the length of the two opposite sides. Then bend the eraser sharply, in such a way that one of the measured sides will be on the inner arc and the other on the outer arc, taking care not to stretch or compress the rubber unnecessarily. Measure the length of the two arcs thus formed and determine for each side whether it is longer or shorter than before bending.

Countless instances of flexure or bending produced by some kind of transverse force acting upon more or less elongated objects will occur to every one. A sagging telegraph-wire, the limb of a pear-tree bending under its load of fruit, the bent springs of a loaded carriage, the sagging rafters of an old building, are only a few familiar examples of bodies strained by transverse forces. Exercise III is intended to enable the student to answer for himself the question what general quantitative relation exists between the amount of transverse force applied, and the amount of bending which it will produce.

---

[1] See note to p. 14.

18. EXERCISE III.

ELASTICITY: BENDING.

[Worthington, Exp. 2, p. 129, and Exp. 3, p. 131.]

Apparatus: Two rods of clear white pine of similar grain, each 41 in. long and ⅜ in.[1] thick, one of them ⅜ in. wide, the other 1 in. wide; for supports, three 2-in.-long triangular prisms of wood about 1 in. thick. Weights from 100 gm. to 2 kgms. A scale about 10 cm. long, graduated in millimeters, which may be a short cut from a wooden meter-rod. A very light, straight rod of wood about 32 cm. long.

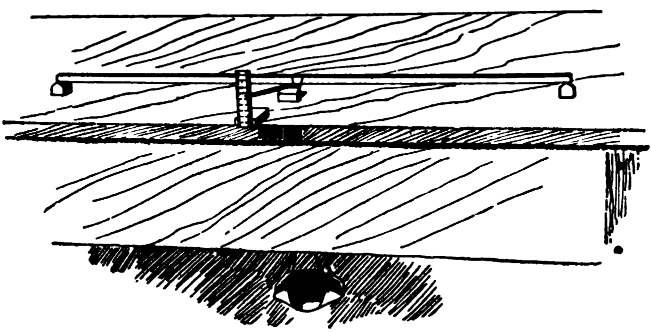


FIG. II.

FIRST CASE.—Lay the narrow rod horizontally upon two of the supports placed parallel to each other with their centres 1 m. apart. Place the other block parallel to the rod and alongside its middle, the centre of the top of the block being 5 cm. distant from the nearer edge of the rod. On this block as a fulcrum support the light piece of wood in the manner of a lever, the short arm terminating under the rod and the end of the other arm lying alongside the 10 cm. scale placed vertical at a distance of 30 cm. from the edge of the meter-rod. This lever is merely an indicator and will show the up and down movements of the middle of the bar magnified five times.

In order that this magnifying ratio may remain constant the lever should bear against the front edge merely of the rod at the

---

[1] Inches are used here because carpenters would find it difficult to follow exact dimensions given in centimeters.

start and throughout. This is easily effected by having its support slightly higher than those of the rod.

Read the position of this pointer upon the 10 cm. scale. Then lay[1] upon the middle of the meter-rod 100 gm. and again read the position of the pointer. Remove this weight and read again. Then put on 200 gm. and do exactly as before; then 300 gm., 400 gm. in turn, measuring every time with weight on and with weight off and stopping at any time if permanent bending is perceived.

If the rod is warped, take care in putting on and taking off weights not to let it rock. Make it touch the supports at the same points all the time.

SECOND CASE.—Use the narrow rod with supports 50 cm. apart. Load from 500 gm. to 2000 gm., adding 500 gm. at a time.

THIRD CASE.—Use the broad rod on broad-side with supports 1 m. apart. Load from 200 gm. to 800 gm., adding 200 gm. at a time.

FOURTH CASE.—Use broad rod on edge with supports 1 m. apart. Load from 500 gm. to 2000 gm., adding 500 gm. at a time.

The following form of recording observations is suggested:

1st case: rod flat; supports 1 m. apart.

Load.	Readings with load.	Readings without load.	Mean.	Rise of pointer on scale.	Deflection of rod.	Deflection per 100 g.
gm.	cm.	cm.	cm.	cm.	cm.	cm.
100	3.40	3.00 3.00	3.00	0.40 ÷ 5 =	0.080	0.080
200	3.75	2.96	2.98	0.77 ..	0.154	0.077
. .	. .		. .	. .	. .	. .
. .	. .		. .	. .	. .	. .
					Mean	. .

As the grain of the wood may differ considerably in different rods, it is well in deducing general laws from this exercise to take the mean of the results obtained with a considerable number of rods, all as much alike in grain as practicable. The effect of

---

[1] If the weights provided are not flat they must be put in a pan suspended from the middle of the bar. For this purpose the middle of the bar can be placed over a hole bored through the table-top.

length, of width, and of thickness upon stiffness can in this way be made out with considerable clearness.[1]

19. Laws of Stiffness.—The results of the exercise should enable the student to answer the questions: (1) What is the relation between the transverse force applied and the flexure of elastic rods? For example, if a force f produces a depression d, how great a depression would be produced by a force $3f$? (2) What between length of rod and amount of flexure, i.e. distance which the middle of the rod is depressed? (3) What between breadth of rod and amount of flexure?

An important law not here shown may be stated: (4) Flexure is inversely proportional to the cube of the thickness of the rod, *depression* $\propto \frac{1}{t^3}$. Show how closely your results agree with the fourth law.[2]

It would be well to try to put the answers to the questions above asked into a single formula (together with the law stated in (4)), placing the several symbols used for pressure, length, breadth, and thickness either in the numerator or denominator, as may be necessary, of the single fraction which shall express in compact form the four laws of stiffness of rods or beams, it being understood that flexure is inversely proportional to stiffness.

20. Breaking-strength.—It is important to notice that the

---

[1] It should be distinctly impressed upon the student that he is here testing the *stiffness*, not the *strength*, of the rod in the various positions. For instance, the *breaking-strength* of a short rod does not bear to the breaking strength of a long rod so large a ratio as the load required to depress the middle of the short rod a certain distance bears to the load required to depress the middle of the long rod the same distance, for the short rod cannot have its middle depressed so far as the long rod can without breaking. It would be well to show the class an experimental proof of this fact.

[2] The symbol $\propto$ is read *varies as*, or *is proportional to*.

formula suggested in § 19 will not express the dependence of *breaking-strength* upon dimensions in the case of rods submitted to the action of transverse forces. Adding to the thickness of a rod, for instance, increases its strength less than it increases its stiffness. A thick rod is hard to bend, but a little bending will break it. We have the following relations: $strength \propto width$; $strength \propto thickness^2$; $strength \propto \frac{1}{length}$. Combining these three relations we get the formula

$$strength \propto \frac{width \times thickness^2}{length}.$$

Experiments on the breaking-strength of rods are too wasteful of material to be available for class-work, but a few tests made by the teacher in the presence of the class will serve to enforce the law just stated.

21. Cross-section of Iron Beams.—It has already been noted (see § 17) that the outer arc of a bent rod is stretched and the inner arc compressed. It has also been stated that in a given substance the limit of elasticity in stretching is not necessarily the same as its limit in compression. Cast iron, for instance, will bear much more compression than stretching before giving way. Accordingly cast-iron beams or rails are made much wider on the side which is to be stretched than on the side which is to be compressed; a cross-section like this (Fig. 3) being sometimes used.[1]

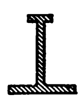


Fig. 3.

22. Elasticity of Torsion.—The elasticity of torsion is most familiarly manifested in the alternate twisting and untwisting to be noticed when a weight is suspended by an ordinary (not a braided) cord or rope. It is much better shown by an elastic wire than by any rope on account of

[1] See Goodeve's Mechanics.

the striking elasticity of the wire, and the consequent promptness with which it springs back to and even beyond its original position after a torsional stress has been removed.

EXPERIMENT 7.

Hang by one end a no. 26 or 28 brass wire, not less than a meter long, from a firm suspension (for example, a small hand-vise), and attach to the lower end a weight of fifty or a hundred grams, to which a pointer of card-board or a straw has been fastened at right angles to the wire. Turn the weight through a considerable angle, then release it and notice the rapid circular movements of the pointer.

23. EXERCISE IV.

ELASTICITY: TWISTING.

Apparatus: Two rods of clear ash of similar grain, each 36 in. long, one of them ⅜ × ⅜ in. in cross-section, the other ¾ × ¾ in., one end of each rod being fitted into the middle of a stout cross-bar 1 ft. long (or better, fitted axially into a circular board 1 ft. in diameter). Some means of fastening the rods rigidly at any section in a vertical position.[1]

A sheet of card-board upon which is traced a circle the diameter of which is somewhat greater than the length of the cross-bars on the rods, one eighth part of the circumference being divided into degrees. Two 8-oz. spring-balances and two of the 30-lb. spring-balances used in Exercises I and II. (It would be well to have two more spring-balances of 4 or 5 lbs. capacity.)

First Case.—Fasten the small rod in a vertical position with the cross-bar downward and close to the table-top, leaving a clear space of 80 cm. between support and cross-bar. (In order that the rod may not be bent to one side or the other it is well to have

---

[1] At the Harvard Physical Laboratory a tall stiff post is placed at each end of a long table and a stout horizontal bar runs from post to post over the table at a height which may be changed at will. To this bar the rods to be twisted may be fastened by means of clamps, care being taken to have the *pinch* extend to the lower edge of the beam. This adjustable horizontal bar is useful in various experiments for suspending pendulums, spring-balances, etc. See Fig. IV.

a stout brad in its lower end, and let this brad reach into a small hole in a thin metal plate set into the table-top, or the rod itself, made cylindrical near the end, may project a little distance through the cross-bar and into a hole bored in the table-top.) Fasten the pasteboard upon the table with the centre of the circle just beneath the centre of the end of the rod, and with the 0° of the circle just beneath a pin driven into the end of the cross-bar for an index.

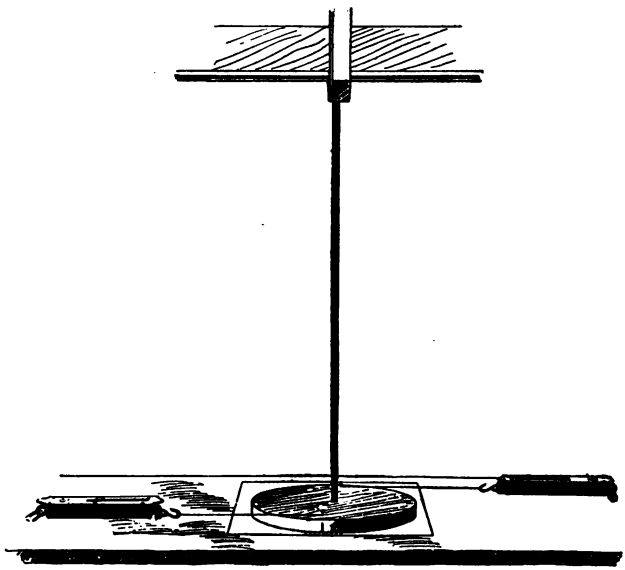


FIG. III.

At each end of the cross-bar, at a point 12 cm. distant from the middle, attach a string. To each string attach an 8-oz. spring-balance, and with these balances pull horizontally in opposite directions at right angles to the cross-bar. Let the force be the same for both balances, and as the bar twists let the direction of the pull be changed in such a way as to keep it always at right angles to the cross-bar. Use in succession forces of 2, 4, 6 and 8 oz., and record in each case the resulting torsion as shown by the readings upon the pasteboard circle.

SECOND CASE.—Fasten the small rods in such a way that only

40 cm. of it will be subject to torsion and then proceed as before, using now, however, forces of 4, 8, 16, and 32 oz.

THIRD CASE.—Use 80 cm. of large rod and forces from 1 lb to 4 lbs.

FOURTH CASE.—Use 40 cm. of large rod and forces from 2 lbs. to 8 lbs.

If in any case the forces here mentioned are not suitable, use others.

The last paragraph and foot-note under Ex. III apply here also, with certain obvious changes.

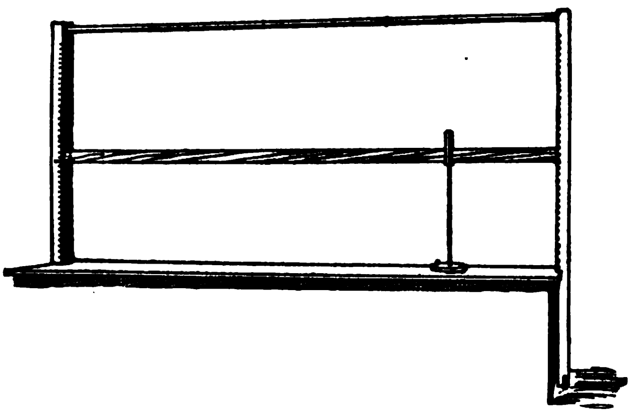


FIG. IV.

24. Laws of Torsion.—Exercise IV affords a means of studying the relation between the torsional forces applied to a rod and the amount of twisting produced, as well as the effect of varying the dimensions of the rod. From his observations the student should try to answer the following questions: (1) What is the relation between the forces applied and the amount of torsion? (For example, if a force f produces a torsion of 5°, how many degrees of torsion will a force $2f$ produce?) (2) What is the relation between the length of the rod and the amount of torsion? (3) How

nearly do the observations agree with the law, *torsion* $\propto \frac{1}{D^4}$, where D is the diameter of the rod? Try, as in discussing the result of Ex. III, to reduce the laws found to a single formula.

25. Are Solids Perfectly Elastic?—We have now seen enough of the phenomena manifested by elastic solids in a state of stress to be able to answer with some degree of assurance the question whether the kinds of wood examined and the spring-brass wire used are perfectly elastic, up to the breaking point, or not. Since each experiment (II, III, or IV), if carried far enough, shows that a permanent strain—that is, bend, twist, or elongation—is obtained before rupture of the wood or metal used, we may conclude that solids in general are not perfectly elastic up to the limit of their strength. We may also infer from the apparently perfect restoration of the object, in each of the three experiments, to its original shape and dimensions, so long as the stress put upon it does not exceed a fixed amount, that solids are practically perfectly elastic within certain limits.

26. Inductive Reasoning: Need of Caution in Drawing Inferences.—The kind of reasoning by which general inferences are drawn from the study of a group of observations upon an object or phenomenon is called inductive reasoning. The value of its conclusions depends upon the accuracy, the number, and the variety of the observations upon which they are based. If a savage who is familiar with no metals except gold and copper should argue, from his acquaintance with these, that metals will sink in water, his opinion would be of little value, because of his very limited knowledge of metals.

Any chemist or physicist of the last century, however, would have drawn from an acquaintance with all the metals then known the same conclusion, namely, that metals will sink in water; and it is only since the discovery, early

in the present century, of sodium, potassium, and other such metals which will float, that it has become possible to dissociate the idea of comparatively great density from the necessary characteristics of metals.

The student must be extremely careful, in the course of his experiments, never to infer more than his observations warrant. If a certain conclusion seems probable from the data obtained, state that it seems probable, not that it has been proved true. If certain quantities are found—as they often may be—to be nearly proportional, it should be stated just how far they are from exact proportionality; for instance, they may be put into the form of a proportion and then the product of the extremes and the means stated in the form of an inequality, by the use of the sign > or < (*greater than* or *less than*).

Questions and Problems on Chapter II.

1. Define elasticity and limit of elasticity. Illustrate your definitions.

2. If no. 27 brass wire breaks with 15 lbs. pull, calculate the breaking-strength of no. 25 brass wire. lbs 23 + ans

3. If no. 27 spring brass wire breaks with 15 lbs. pull, and no. 30 annealed iron wire with 5 lbs., find the relative tenacity of spring brass and annealed iron.[1] Ans as 15 : 10

4. What pull would a wire of each kind 1 sq. mm. in area of cross-section sustain?[2] x = 170 ans

5. What must be the diameter of a spring brass wire that shall just sustain a load of 50 kgms.? Use data obtained in Exercise I. Ans = .97 sq mm

[1] Areas of cross-section are *proportional* to squares of diameters. See table of wire gauge numbers and diameters, Appendix.

[2] See § 8, note. In these calculations it is to be assumed that wires of the same metal are of exactly the same quality and condition. In practice this is not always the case. A thin wire is likely to be stronger, relatively to its size, than a thick wire.

breaking strength of wire varies directly as the square of the diameter of wire.

6. A certain kind of wire breaks with l kgms. tension; it takes m meters of this wire to weigh 1 kgm. Find how many kilometers of such wire will break with its own weight. $l \times m$ or lm

7. If in Exercise II the wire were 8 m. long instead of 4m., how would the change affect the readings in the column headed "Total elongations"? Readings in column "Ratio of elongation per lb. to length of wire"?

8. If a force of 1 kgm. stretches 1 mm. a wire which is 1 m. long and 0.1 sq. mm. in cross-section, how great a force is required to stretch 5 mm. a wire of like material 10 m. long and 1 sq. mm. in cross-section? 5 kgm

9. If a beam 3 m. long, 8 cm. wide, and 9 cm. thick is depressed 0.5 cm. by a certain load, how much would a similar beam 4 m. long be depressed by the same load? 1.1?

10. How much would the same load depress the first beam if its thickness were increased to 12 cm.? .21

11. It is found that an 8-in. floor-joist (i.e., one eight inches thick) is bent 0.5 inch by a certain load. What would be the thickness of one that would be bent only 0.1 inch by the same load?

12. What is the ratio of the stiffness of a rod 50 cm. long to that of another rod 100 cm. long, but similar in all other respects? Tell why the strength (in breaking across) of the shorter rod does not bear an equally large ratio to that of the longer rod.

13. Explain the reason for the form of the cross-section seen in a "T-rail."

14. What pull upon each balance would be necessary to twist the stouter rod of Exercise IV through an arc of ½ degree, the portion of rod used being 2 cm. long?

15. If one end of a rod is firmly fastened and certain forces applied to a cross-piece at the other end twist the rod 10°, how many degrees would it have been twisted by forces twice as great and similarly applied? How many

degrees would these latter forces twist a rod twice as long as this one but similar in other respects?

16. How much twisting would be produced in a rod of the same wood as those used in Exercise IV, and of the same length as the longest used in that exercise, the new rod being one inch square, the pull applied being equal to the greatest pull applied during the performance of that exercise, and the points of application the same?

CHAPTER III.

HYDROSTATICS AND PNEUMATICS, OR MECHANICS OF LIQUIDS AND OF GASES AT REST.—DENSITY AND SPECIFIC GRAVITY.

27. Fluids.—Liquids and gases are both comprehended under the general name fluids, and they resemble each other sufficiently to make it possible to treat them together. Fluids are distinguished from solids by their lack of power to retain any definite form without support. A cube of ice kept at a temperature below the freezing-point will remain a cube, unless submitted to unequally distributed pressure so as to become flattened or otherwise distorted. In other words, the ice, like all solids, possesses rigidity, or the power to resist changes of shape. But let the block of ice be melted into water, or melted and then evaporated into steam, and it will be possible to keep the water or the steam in the form of a cube only by confining it in some kind of cubical vessel.

28. Limpidity and Viscidity (or **Viscosity**).—All are familiar with the great variety of liquids as regards their consistency, some being very *limpid,* as ether, ordinary alcohol, water; others, as glycerine and molasses, *viscid* (or *viscous*); while still other substances usually, at ordinary temperatures, classed as solids,—for instance, sealing-wax, asphalt, coal-tar,—really act somewhat like extremely viscid liquids.

A stick of sealing-wax supported at each end, with its longer axis horizontal and with a pretty heavy weight hung from its middle, will at length become bent without cracking, and a narrow strip of window-glass treated in the same

way will receive a slight permanent bend. A bullet placed on the surface of a barrel of coal-tar (which substance is brittle enough to fly to pieces at a slight blow) will gradually sink to the bottom of the barrel.

A perfect liquid would be a substance which possessed no rigidity of form and no viscosity; no such liquid is known to us.

29. Compressibility of Gases and of Liquids.—Gases differ from liquids mainly in their great compressibility, or capacity to suffer change of volume from change of pressure, and a necessity for treating some phenomena of gases separately from those of liquids arises from this difference.

One of the most important properties of gases is their tendency to expand indefinitely. Not only have they no tenacity, or power to resist forces which tend to pull their particles asunder, but they expand of themselves, unless hindered by external forces.

A block of iron placed on the laboratory-table does not communicate its substance to the rest of the room, nor tend to occupy all parts of the latter at the same time. A cup of any liquid which does not evaporate appreciably at ordinary temperatures, glycerine for instance, behaves in this regard like the solid iron. But a quantity of coal-gas or of hydrogen sulphide gas, brought into the laboratory and there set free, will in a few minutes have distributed itself into every part of the room, as will be shown by the smell. If instead of the room already filled with air a vessel exhausted of air had been used, the gas would still more rapidly have filled it throughout.

It is difficult to measure the exact amount of compressibility of liquids, and expensive apparatus is required for the measurement; but a very simple experiment will suffice to show the great difference in the behavior of gases and liquids under pressure and the comparative incompressibility of the latter.

EXPERIMENT 8.

To the tube of brass, *C*, attached to the end of a stout brass cylinder, like that shown in Fig. 4,[1] well oiled inside, attach a short piece of very stout rubber tubing about 10 cm. long. Wire this tubing tightly on *C*, after slipping it on against the cylinder-head, and then clamp the free end of the tube close up to *C* with a small hand-vise or pinchcock.

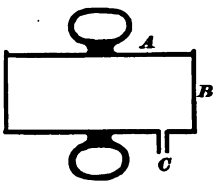


Fig. 4.

Now try to force the piston *A* down to the bottom of the cylinder and notice the amount of compression that can be produced by a moderate pressure brought to bear on *A*; notice too, how exactly the piston returns to its original position after the removal of the pressure, if there has been no leakage of air.

Now remove the vise, push *A* to the bottom of the cylinder, and with *C* under water, open end up, force the piston back and forth a few times, leaving it at last drawn out as far as possible. Clamp the rubber tube once more and again apply 50 or more kgm. pressure to the piston. If a very slight movement of the latter is noticed, it will be found to be due either to leakage of water or to the yielding of the rubber tube.

30. Transmission of Pressure by Liquids.—A very important property of liquids may be shown by means of one of the two-necked bottles known as Woulff-bottles, used in some chemical operations. Procure a stout bottle of the form shown in Fig. 5. Stopper both the necks with very smooth, soft, well-squeezed corks and fill the bottle with water. Perforate one cork, so as to allow a stout glass stirring-rod to slide smoothly through it. Perforate the other cork and fit into it a glass tube drawn out to a very slender point and with the tip then broken off.

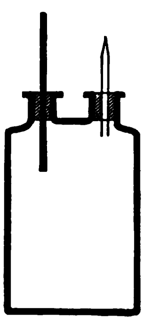


Fig 5

Crowd the cork with the tube down into the bottle until the water rises to the tip of the tube, or runs out of

[1] Manufactured by A. P. Gage, 13 Tremont Place, Boston, and sold under the name of the "Seven-in-one Apparatus."

it. Then push the plunger[1] slowly down into the bottle and notice the proof of transmission of pressure by the water. Suppose the opening in the tube to have become clogged, what would be the result? A cork with a close-fitting plunger may be inserted into an ordinary bottle *full* of water, the cork wired in place, and the plunger then forced down; the result should show both the transmission of pressure by the water, and the comparative incompressibility of water.[2]

31. EXERCISE V.

PRESSURE IN A LIQUID.

[Trowbridge, Exps. 24-25; Lodge, Arts. 158-168, 180-182.]

PRELIMINARY.

The pressure-gauge to be used in this exercise is easy to make and is effective, but the intelligent use of it requires some preliminary knowledge of atmospheric pressure, which the following investigation is intended to give: (a) Does air have weight? See Ex. XI, which, however, need not be performed throughout at this stage. (b) Does the atmosphere press equally in all directions? Take a "thistle-tube," shaped like those used by chemists but with walls thick enough to bear the full pressure of the atmosphere while having a vacuum within, and having a mouth about 2.5 cm. across. Cover this mouth with a piece of sheet rubber about 1 mm. thick, and tie the rubber firmly on with a string passing round the tube just back of the lip. If this does not close the mouth of the tube air-tight, pour melted bees-wax and rosin under the loose edge of the rubber. Connect the stem of the thistle-tube with an air-pump by means of a thick-walled rubber tube, and then exhaust the air. The rubber diaphragm will be forced into a deep cup-shape by the pressure of the atmosphere. Turn the mouth of the tube up, down, and in all directions in order to find an answer to the question proposed above. (c) Does the pressure of the atmos-

[1] A piston that fits only the entrance to the cylinder in which it works, called the "stuffing-box" and represented in this case by the cork, is called a plunger.

[2] Water really is compressed about 0.00005 of its volume by each atmosphere of pressure to which it is subjected.

phere upon a given surface depend upon the shape of the passage by which it reaches the surface? (The answer to this question is really implied in the answer to the preceding one, but the connection is too subtle to be seen at once by young students.) Take a thistle-tube having a mouth about 1.5 cm. across, and cover this mouth with very thin sheet rubber, such as dentists use, thin enough to be translucent and to be easily torn by the fingers. To the stem of the tube attach a small rubber tube about 40 cm. long. [See Fig. V.] Hold the rubber-covered mouth of the thistle-tube stationary in any position and bend the rubber tube into a variety of shapes, watching the diaphragm to see whether it gives evidence of any accompanying change of internal pressure.

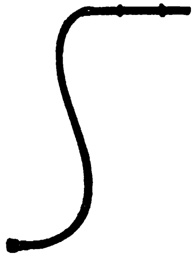


FIG. V.

The student will now be in a position to take up the subject of liquid pressure.

Apparatus: The small pressure-gauge described above, together with an open straight glass tube about 20 cm. long, 1 or 2 mm. in diameter of bore, and of such external size that it will fit closely in the rubber tube of the gauge. One or two rubber rings to fit on this glass tube as reference marks. A one-gallon, or larger, glass battery-jar. Two student-lamp chimneys closed at one end. (One of these chimneys may be replaced by a piece of wood of similar dimensions.)

Nearly fill the battery-jar with water having a temperature differing not more than 1° or 2° from that of the surrounding air. Put the pressure-gauge and the attached rubber tube into the water, keeping the free end of the tube above the surface. Wet the inside of the small glass tube and leave in it near the middle a short column of water. Thrust one end of this tube, which is to be kept always horizontal, into the rubber tube.

Experiment now with a view to answering the following questions:—

A. How is pressure affected by change of level in the liquid?

Place the gauge at various depths in the liquid, keeping its face always horizontal, and watch the index in the horizontal glass tube until an answer to the question proposed is obtained. An answer in general terms only is to be expected.

B. Is the pressure upon the gauge-face, when its centre is kept

at a given point beneath the surface of the liquid, dependent upon the direction in which the face is turned?

Vary the inclination of the face as much as possible, turning it upward, downward, and sidewise, taking care to keep its centre unchanged in position; watch the index and record the result in answer to the question proposed.

C. Is the pressure equally great at all points upon the same level within the liquid?

Points on the same level may not have the same depth, or height, of liquid over them. Thus if the closed end of a lamp-chimney is thrust down into the water and held near the bottom of the jar, the gauge-face when placed beneath this shelter may have only 1 or 2 cm. depth of water above it, while if placed on the same level but not beneath the chimney it may have 15 cm., or more, of water above it. So if the lamp-chimney be first filled with water and then inverted into the jar it may be used to give a depth of 35 or 40 cm. above the gauge-face. The apparatus described, then, furnishes the means for varying greatly the depth of the water immediately over the gauge-face without changing the level at which the latter is placed. The general height of the water surface in the jar should be kept unchanged during the experiments upon any particular level, and as this cannot conveniently be done if the lamp-chimneys are moved up and down, it is better to have them placed in position at the start, one filled with water, closed end up and held high so as to give a tall column, the other pushed deep into the water with closed end down, both remaining unmoved while the gauge-face is carried about from point to point in a given level, sometimes beneath the tall column, sometimes beneath the short column, sometimes under neither. If a difference of pressure is indicated, record the positions of which it is greatest and those at which it is least.

Push the chimney which is filled with water well down into the jar and push the gauge-face up inside the chimney several cm. Compare the pressure at this point with that in the water outside the chimney at the same level.

From these observations and any others that may suggest themselves make up an answer for question C.

It is well to extend this exercise by trying the experiment of forcing the gauge to equal depths in vessels of very different shapes and sizes, such as jars, narrow tubes, and bottles.

32. Discussion of Exercise V.—This Exercise has certain important bearings which the following questions and problems are intended to show.

(1) An upright cylindrical jar 20 cm. deep, the base of which is 100 sq. cm. inside, is filled with water. What is the weight of the water in the jar? What is the weight of water resting upon each sq. cm. of the base? (It is here assumed that the weight of the water is borne entirely by the bottom of the jar, not at all by the vertical wall. *Surface-tension effects*[1] being disregarded, this is true. A distinct experimental proof of the fact is perhaps too difficult for this course, but the experiments which should precede, testing the pressure in vertical cylinders of very different diameters, show, at least, that no considerable part of the weight of the liquid is borne by the vertical sides.) If the pressure of the atmosphere is 1000 gm. upon each sq. cm. of the upper surface of the water, what is the total pressure upon each sq. cm. of the base? 1020

What is the total pressure upon a horizontal sq. cm. at the depth of 3 cm.? At a depth of 10 cm.? At a depth of 15 cm.?

What is the total pressure upon a sq. cm. of the vertical wall, the centre of the square being at a depth of 5 cm. in the water? At a depth of 10 cm.? At a depth of 15 cm.?

(2) Let the jar be closed by a flat cover touching the water, having an open tube 1 sq. cm. in cross-section rising from its centre and extending upward 30 cm. above the top of the jar. Let both jar and tube be full of water. Let the atmospheric pressure be disregarded.

(*a*) What is the total weight of water in the jar and tube? 2030

(*b*) What is the pressure upon that sq. cm. of the base which lies exactly beneath the tube?

(*c*) Is the pressure upon any other sq. cm. of the base greater or less than this? no difference

[1] See any general treatise on physics.

(*d*) What is the pressure upon that sq. cm. of water, which, at the top of the jar, lies exactly beneath the tube? 80

(*e*) Is the pressure of the water against each sq. cm. of the cover greater or less than this? same (no different)

(*f*) What is the total pressure of the water against the whole cover? = 3000 grams.

(*g*) Subtract the total pressure against the cover from the total pressure against the base and compare the result with the weight of all the water in the jar and tube.

(*h*) Suppose now that in any way, by means of a piston, for instance, a pressure equal to the weight of fifty gm. is brought to bear upon the top of the water in the tube. What will now be the pressure upon the sq. cm. which lies at the top of the jar, just beneath the tube?

(*i*) How much will the total pressure against the bottom of the jar be increased by the action of the piston.

(*j*) What is the total pressure upon 1 sq. cm. of the vertical sides of the jar before the piston is made to act, the centre of the square being at a depth of 5 cm. beneath the cover of the jar? At a depth of 10 cm.? At a depth of 15 cm.?

(*k*) What is the pressure upon each of these vertical squares after the piston is made to act?

(3) Suppose a mound of water to be raised temporarily upon the general surface of a lake. Show why such a mound could not be permanent.[1]

33. Pascal's Principle.—Much of what is taught by the preceding experiments and problems is summed up in a

[1] Large masses of liquid at rest are said to have *level* upper surfaces. Any small portion of such a surface appears to be flat or plane. These apparent planes are, however, really portions of slightly flattened spheres, all having their centres at or near the centre of the earth, but the curvature of such surfaces is so slight that it cannot usually be seen at all except at sea or upon great lakes.

The curvature of liquid surfaces in tubes or near the walls of containing vessels is not here considered.

principle announced by the French physicist, Pascal, about the middle of the seventeenth century: "If a vessel full of water, closed in all parts, has two openings of which the one is a hundred times the other, placing in each a piston which fits it, a man pushing the small piston will equal the force of a hundred men who push that which is a hundred times as large, and will surpass that of ninety-nine. Whatever proportion these openings have, and whatever direction the pistons have, if the forces that one applies on the pistons are as the openings, they will be in equilibrium."

The principle is usually stated nearly as follows:

Pressure exerted anywhere upon a mass of liquid is transmitted undiminished in all directions and acts with the same force upon all equal surfaces and in a direction at right angles to those surfaces. This principle will be freely alluded to by name in this book.

34. Mariotte's Bottle.—Admirable practice in applying the principle of hydrostatics is afforded by a study of the phenomena presented by "Mariotte's bottle."

EXPERIMENT 9.

Take a bottle with three lateral openings, each stopped as shown in Fig. 6; fill the bottle full of water and insert in the mouth a perforated cork, through which a glass tube, 30 or 35 cm. long, extends nearly to the bottom of the bottle. Take out each of the stoppers, *a*, *b*, and *c*, in turn, noting in every case whether water runs out freely, and returning the stopper again to its place. Note, after the removal of each stopper, the level at which the water stands in the vertical tube. Now raise the tube until its lower end is level with *a*, open *a* and allow all the water that will run out to do so. Refill the bottle, lower the tube until its lower end is on the level half-way between *a* and *b*, and allow the water to run

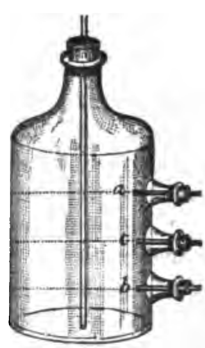


Fig. 6.

from *a* and from *b* (for a short time) in turn. Repeat with the lower end of the tube half-way between *b* and *c*, allowing the water to flow from *a*, from *b*, and from *c*, in turn. Try to explain the observed results by considering the bottle and the tube as communicating vessels. The *free* surface of the water in the *bottle*, as distinguished from the vertical tube, is at the level of any opening, *a*, *b*, or *c*, that may at the time be unstopped. ["Free surface" here means a surface exposed to the direct pressure of the outer air. In unravelling hydrostatic puzzles it is best to begin at a *free* surface, for at this surface we know how great the pressure is. It is simply the pressure indicated by the barometer (§ 42).]

35. Pressure at a Point.—In our experiments with the gauge, we have tested the pressure upon the gauge-face, taken as a whole. It is evident, however, that when this face is not horizontal, different parts of it, being at different depths, are subject to pressures of different intensity. To find the meaning of the phrase *pressure at a point*, imagine the face to shrink continually without change of shape, the centre being kept at a particular point. The total pressure upon the face will decrease continually during this process, but the ratio, *total-pressure ÷ area*, will change very little, if at all. The limit which this ratio approaches, as the circle approximates to a mere point, is called the pressure at the point. It is equal to the total pressure upon a unit surface, at every part of which the pressure is as intense as at the point in question.

36. Average Pressure.—If we find the total pressure upon a given surface and divide this total by the area of the surface, we obtain what is called the *average pressure* over the given surface. If the pressure is equally great at all parts of the surface the average pressure is equal to the actual pressure per unit-area at every part of the surface. If the pressure is not equally great at all parts of the surface the average pressure is greater than the smallest pressure per unit-area, and less than the greatest pressure per unit-area, to be found on any part of the surface.

37. Calculation of Total Pressure on a Non-horizontal Surface.—By definition of average pressure (§ 36) the total pressure upon a surface is equal to the area multiplied by the average pressure. The average pressure on certain surfaces, e.g., a circle or a rectangle placed vertically, is easily seen to be equal to the pressure at the central point, the liquid being supposed of equal density throughout; for pressure in such a liquid increases regularly with the depth, and any given element of the surface below the centre is offset by any perfectly similar element placed at an equal distance above the centre. In Fig. 7, which represents a section of a rectangular box with vertical sides, the depth of the water at A is 0 and at D is $\overline{AD}$ cm. Therefore the average depth is $\frac{1}{2}$ $\overline{AD}$ cm., or $\overline{AE}$ cm., and the average pressure per sq. cm. against the end of the box is equal to the pressure at the middle point, which is $\overline{AE}$ grams. The total pressure, expressed in grams, against the end is equal to its *area* $\times$ $\overline{AE}$, or *area* $\times \frac{AD}{2}$. In any except very simple cases the calculation of the total pressure on a surface is too difficult for the purposes of this book.

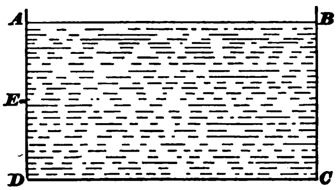


FIG. 7.

38. Pressure at Great Depth.—Since in water each 10 meters depth will give a pressure of 1 kgm. per sq. cm., the pressure encountered in deep soundings, in the lakes or sea, becomes immense. During the days of sailing navigation on the great lakes a curious use was made of this pressure. No ice being carried on board vessels, it was difficult in summer to procure cold water for drinking, but this was often secured by lowering an empty, tightly-stoppered jug, heavily weighted with lead, to a depth of some hundreds of feet. When this was hauled up the

cork would be found forced in and the jug full of water, at a temperature not far from 40° Fahr. (See § 151.)

39. Pressure of Air.—In order to contrast the behavior of gases under pressure with that of liquids under pressure, we may now subject a confined portion of air to various degrees of pressure, as described in Exercise VI, but we shall first need to perform some simple introductory experiments. We have already learned, from the observations in Exercise V, that the air exerts pressure upon objects with which it comes in contact. But the invisibility of air, and the fact that its presence is not so readily perceived by the sense of touch as is that of liquids, makes it harder to realize the importance of the pressure which it exerts, so that some further demonstrations of this are necessary.

EXPERIMENT 8.

Tie firmly a piece of thin sheet-rubber, like that used to make the pressure-gauge in Exercise V, over the smaller end of a "hand-glass," or small air-pump receiver open at both ends. Place the larger opening of the hand-glass on the well-oiled plate of an air-pump and exhaust the air.

EXPERIMENT 9.

Replace the rubber by the palm of the hand pressed tightly down upon *A*, and again exhaust.

EXPERIMENT 10.

Push the piston *A* of Fig. 4 to the bottom of the cylinder, clamp the rubber tube tightly, fasten the ring at one end of the apparatus to a stout hook in the wall or other suitable attachment, and try to pull the piston out to the open end. Two or more of the class may pull at the same time, if necessary.

40. Magnitude of the Atmospheric Pressure.—From the experiments just performed the reality of the atmospheric pressure must have become quite evident. Since we live at the bottom of an aerial ocean of very great depth, it is not remarkable that the pressure of the atmosphere should be capable of producing extraordinary effects, for the

weight of a cubic m. of air, though many times less than that of the same volume of water, is (as may later be calculated from the results of Exercise XI), a very considerable quantity. Under ordinary circumstances this pressure of the atmosphere is equal to about 14.7 lb. to the square inch, or 1033 gm. to the square cm.

41. Principle of the Barometer.—In studying the behavior of gases under pressure it is often necessary to take barometer-readings. A few preliminary experiments will illustrate the mode of action of the barometer.

EXPERIMENT 11.

Into one end of a piece of stout glass tubing of about 1 cm. internal diameter and 1 m. long insert a good cork or a close-fitting rubber stopper. Fill the tube with water, close the open end with the forefinger, place this end under water, after inverting the tube, and remove the finger. Note whether the water in the tube sinks at all. Remove the stopper and note the result.

EXPERIMENT 12.

Fill an "ignition-tube," such as is used in many chemical experiments, or any strong glass tube about 50 cm. long closed at one end, with mercury, close it with the forefinger, and after inverting it, open it under mercury in a small dish, for example a small tumbler. Note whether any fall of the mercury in the tube can be observed. Repeat the experiment with a barometer-tube nearly a meter long, Fig. 8; note the decided fall of the mercury in the tube, and measure as accurately as you can the height of the mercury remaining in the tube above the level of that in the saucer. This last experiment, known from the inventor as Torricelli's experiment, serves to give a tolerably accurate measure of the value of the atmospheric pressure. The fact that the water stands at the very top of an inverted tube a meter long, filled with water, is due to the sufficiency of the atmospheric pressure to support a column of water of that height. The atmosphere sufficed also to support a column

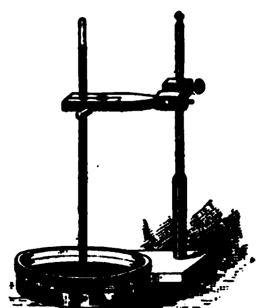


FIG. 8.

of mercury which filled the ignition-tube to the top, but when a much longer tube was taken the mercury sank until its downward pressure just counterbalanced that of the atmosphere upon the surface of the mercury in the saucer. The great weight of mercury as compared with that of an equal bulk of water—mercnry is more than thirteen times as dense as water—accounts for the comparative shortness of the mercury-column which can be supported by the atmosphere. And since mercury is more than 10,000 times as heavy, bulk for bulk, as air at atmospheric pressure and ordinary temperature, it is possible for a column of mercury about 76 cm., or 30 inches, tall, to exert as much pressure as a column of air of equal cross-section reaching to the top of our atmosphere.

42. Construction and Use of Barometer.—Although the principle of the barometer is precisely that of the tube of Torricelli, previously described, it is necessary to give a description of the instrument as ordinarily made, and a few words of explanation in regard to its use. A sufficiently good form of barometer for ordinary purposes is shown in Fig. 9.* It consists essentially of a cistern for mercury, covered to prevent loss of mercury when the instrument is carried about or jostled, and a glass tube, alongside which is placed a scale divided either into inches and tenths of inches, or into centimeter and millimeter.

The scale is marked as if it began at a certain level in the mercury cistern, and the indications of the barometer are correct only when the surface of the mercury in the cistern is maintained at this fixed level. This result is attained in the better class of instruments by raising or lowering the flexible leathern bottom of the mercury-cistern by turning a screw worked by a milled head until the upper surface of the mercury in the cistern just touches the point of a pin reaching down from the top of the cistern. In the use of a barometer of this class the

* If the instrument is provided with a vernier, the teacher will find it well to explain its use by a working model or by blackboard drawings.

level of the mercury in the cistern should first be adjusted, and after this the height of the column read on the scale alongside the tube. The instrument shown in Fig. 9 is provided with a screw for making adjustments of the mercury-level in the cistern, but as this mercury-level is not visible the screw is evidently not for daily use. It is of service in occasional adjustments of the instrument by comparison with a better barometer. The cistern is shallow, but of so great diameter that the ordinary rise and the fall of mercury in the barometer-tube does not seriously affect the mercury-level in the cistern.[1]

FIG. 9.

Unless the tube is of large calibre the mercury column is likely to be held above or below its true position, that is, to stick slightly against the interior of the tube. Errors arising from this source may be obviated by carefully jarring the barometer before taking a reading.

In every case the extreme height of the convex surface of the mercury in the tube should be taken as the height of the column. The divisions of the scale may be subdivided by the eye. If the scale, for example, reads only to tenths of an inch, each tenth may be subdivided into halves so as to give twentieths, or possibly somewhat further.[2]

[1] If one of the cheap siphon barometers is used, its readings are of little value unless the barometer has once for all been calibrated, that is, tested over a wide range of pressures by comparison with a standard instrument in the neighborhood.

[2] Hardly any exercise that can be given affords better training for the eye than this subdivision of scales of equal parts. It will be

43. Variations of Atmospheric Pressure—Measurement of Heights by Barometer.—Two more points remain to be noticed, namely, that the atmosphere varies extremely in its density at different heights, and that the atmospheric pressure, and consequently the barometric readings at any given place on the earth's surface, vary considerably from time to time. The unequal density of the various portions of the atmosphere at various heights above the earth's surface is due to the compressibility of air. As a cubic foot of hay from the bottom of a hay-stack or hay-mow, when it is pressed upon by many feet of hay above, contains more matter than a cubic foot from the upper part of the pile, so a cubic foot of air at the earth's surface at sea-level contains about twice as much matter as a cubic foot taken from a point 3.4 miles above sea-level. An extremely useful application of the barometer depends upon the decrease in atmospheric pressure with ascent from the sea-level. Since, for small changes of elevation, a fall of $\frac{1}{10}$ inch of the barometer corresponds to an elevation of 87 feet, the heights of hills, etc., may be directly ascertained by the observer carrying a barometer up the hill to be measured and noting the fall of the barometer as he ascends, due allowance being made, however, for barometric variations not due to the change in elevation.[1]

The daily and hourly changes in the height of the barometer may be considerable. These barometric changes must be noted when any careful experiments upon press-

---

found a useful means of developing this power of subdividing to issue to the class during written recitations sheets of squared paper (§ 15) on which fine elliptical or other curved lines have been drawn, and require the readings at certain designated intersections to be reported.

[1] Complicated calculations are necessary for considerable heights, as those of mountains. An aneroid barometer, constructed without mercury, is generally used when a portable instrument is required.

ure of gases are being performed, in order that the experimenter may know to just what pressure the gas under examination is really exposed. Since the usual height of the mercurial column in the barometer at sea-level is about 30 inches, or 76 cm., if the barometer at any given time stands at 74 cm., for example, the pressure of the atmosphere indicated is about $\frac{74}{76}$ of its average sea-level value.

44. EXERCISE VI.

COMPRESSIBILITY OF AIR.

[Trowbridge, Exp. 33. Worthington, Exp. 2, p. 79. Lodge, Arts. 189–194.]

Apparatus: A piece of barometer-tubing about 1.5 m. long, diameter of bore about 5 mm., one end closed, bent twice at a right angle 25 or 30 cm. from the closed end, in such a way as to make the two arms parallel and as near together as practicable. (This tube should have a nearly uniform bore in the short arm. It is well to have the open end flared into a funnel-shape to facilitate pouring in mercury; unless this is done a small glass or paper funnel will be required.) Mercury, about 0.5 kgm. A meter-rod graduated in millimeters. A support consisting of a pine rod about 2.5 cm. square in cross-section and 1 m. long, fixed vertically in the middle of a pine base-board 30 cm. square. To prevent warping of the base-board and to prevent loss of spilled mercury the base-board should have a raised ledge about 1 cm. high about it. (This support is not necessary, but it is a very convenient portable means of keeping the glass tube and meter-rod vertical.) A barometer.

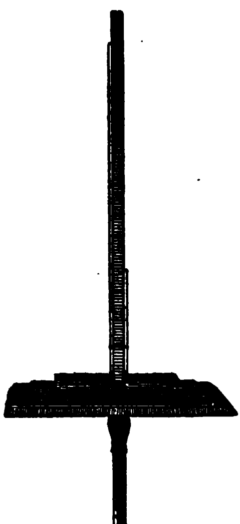


FIG. VI.

Pour into the tube enough mercury to fill the bend and 1 or 2 cm. in each arm. Manipulate the tube until the mercury stands at nearly the same level in the two arms. Then lash the tube, with the meter-rod behind it, to the upright rod of the sup-

port, the bottom of tube and meter-rod resting upon the base-board. [See Fig. VI.] Read and record the distance from base-board to top of bore in short arm and call this A; the distance from base-board to top of mercury-column in short arm and call this S; the distance from base-board to top of mercury-column in long arm and call this T; note the barometer-reading and call this B. Then pour in mercury until the column stands about 20 cm. higher in the long arm than in the short arm and read as before. Then add more mercury and make new readings, proceeding in this way by several stages until the excess of mercury in the long arm is about equal to the barometric column. Care should be taken to prevent changes of temperature in the confined air during this exercise. Therefore the short arm of the tube should not be touched unnecessarily by the hands.

The following form is suggested for recording the observations of this exercise:

A cm.	*S* cm.	*T* cm.	*B* cm.
. . . .	. . . .	. . . .	. . . .
. . . .	. . . .	. . . .	. . . .

The method of working up the observations is indicated by the following form:

L Length of confined air-col. $= A - S$	*h* Excess of mercury in long arm. $= T - S$	*P* Total pressure on confined air-col. $= h + B$	$P \times L$
. . . . .	. . . . .	. . . . .	. . . . .
. . . . .	. . . . .	. . . . .	. . . . .

The other form of this investigation, which uses pressures of less than one atmosphere, may be substituted for that with the bent tube, but it requires more mercury.

45. Discussion of Results: Boyle's Law.—From the result of this experiment, if successfully performed, we may draw the important conclusion that *the volume of a given portion of air is inversely proportional to the pressure to which it is subjected.*

If $V =$ volume and $P =$ pressure,

$$V \propto \frac{1}{P}, \quad \text{or} \quad PV = \textit{a constant quantity.}$$

That is to say, doubling the pressure will halve the volume, quadrupling the pressure will make the volume $\frac{1}{4}$, and so on. This law, known from its discoverers as Boyle's law or Mariotte's law, is extremely important, and we shall have frequent occasions to refer to it. Strictly speaking, Exercise VI establishes this law only for a given amount of air at a particular temperature. A vast number of experiments were needed to enable physicists to state the law, with its slight necessary modifications, for all known gases at all observable temperatures.

46. Graphical Study of Results of Exercise VI.—This exercise may also be recorded graphically. Let each division of the square-ruled paper, Fig. 10, along a vertical line represent a pressure of, let us say, 2 cm. of mercury, and each division along a horizontal line a portion of the confined air-column, 0.5 cm. for example. Let the origin of the line which is to represent the behavior of the air under pressure be taken at a point *a* on some one of the horizontal lines near the bottom of the squared paper. Indicate compression of the air-column by carrying the curve *a i* to the left, and let the rise of the curve from the horizontal zero-line near the bottom of the paper represent increase of pressure.

Each successive length of the confined air-column, as read off at the outset and after each of the successive pourings-in of mercury, is to be located as nearly as possible, first with a sharp-pointed, hard, lead-pencil, and at length, when fully established, by a dot of ink, made with a fine steel pen. Then, by means of the elastic saw-blade, as already described in § 15, draw a fine pencil-line in a smooth curve as nearly as possible through the points *a*, *b*, *c*, *d*, *e*, etc., as shown in Fig. 10.

In studying the diagram thus obtained the student should answer for himself the questions:

(1) Do equal increments of pressure produce equal

amounts of contraction in the compressed air-column? (If so, the line *a i* will be a straight line, as already explained in § 15).

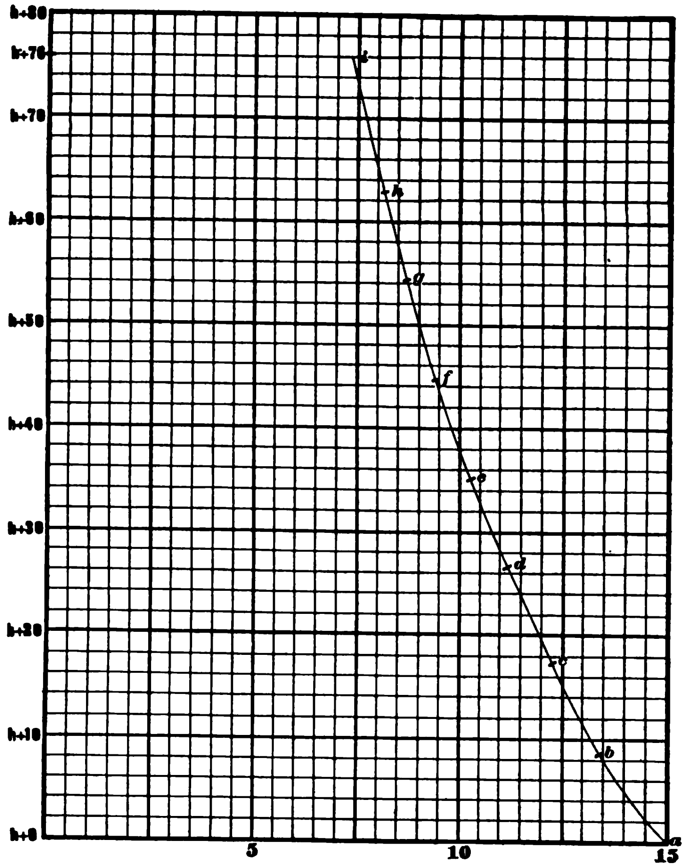


FIG. 10.

(2) If the downward slope of the line *a i* (as seen in its course across successive squares of the ruled paper) is unequal, then compression is most marked where the slope is

steepest. This being borne in mind, what conclusion is to be drawn in regard to the rates of compression due to the first, the second, the third, and the fourth pouring?[1]

47. **Density.**—An important part of our study of the properties of any kind of matter consists in the measurement of *the quantity of such matter contained in a unit of volume.* This is called the *density* of the matter. The quantity of matter is determined by weighing.

48. **Exercise VII.**—The unit quantity of matter and the unit of volume which are in most common use among physicists are the gram and the cubic centimeter; but as the pound and the cubic foot are still largely in use as units of measure in English treatises on physics, as well as in the common business of all English-speaking people, it is necessary in this country to be familiar with these latter units. In Exercise VII we shall, therefore, ascertain the density of some substance with reference to both systems; that is, we shall find how many grams there are in a cubic centimeter, and how many pounds in a cubic foot, of the material experimented upon. This may easily be ascertained by both weighing and measuring a convenient portion of the substance. It is best to have the object to be measured in the shape of a cube or a right parallelopiped.

EXERCISE VII.

DENSITY.

[Trowbridge, Exps. 6 and 9. Worthington, Exp. 12, p. 44, and Exp. 22, p. 56. Lodge, Arts. 32 and 33.

Apparatus: A rectangular-sided block of any moderately dense wood containing 150 or 200 cubic cm. A measuring-stick

---

[1] The teacher can add much to the discussion of this diagram by drawing to scale another one on a much larger sheet of paper, divided off into squares large enough to be seen from all parts of the recitation-room, and with the line *ai* extended both ways so as to indicate what are the geometrical limits of the curve when pressure = zero and when pressure = infinity.

graduated in mms. A spring-balance such as has been used for letter-scales, weighing to 8 oz., and graduated in 0.5 oz., the whole length of scale being about 6 cm. (Such a balance is mentioned because it is in the market. Its readings in ounces can, with sufficient accuracy, be turned into grams by multiplying by 28.3, or a gram scale can be marked with a knife on the balance-face.) Thread to suspend the block.

Considerable care is necessary in order to obtain results of any accuracy with a spring-balance such as is here mentioned.[1] The student should try to read it to tenths of its smallest divisions, that is, to twentieths of an ounce. In every experiment the reading of the balance without load should be observed and noted. When used in making weighings the balance should not be held in the hand, but should be suspended, by means of the ring at its top, from some firm support.

In using the measuring-rod, place it on its edge upon the block, in such a way that the ends of the graduations will come close to the surface the dimensions of which are to be measured. [See Fig. VII.]

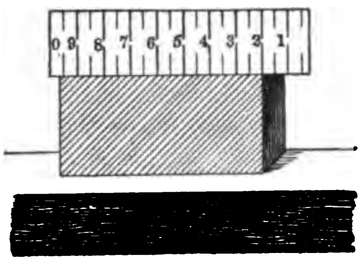


FIG. VII.

Find the density according to both the metric and the English systems, i.e., the number of gm. to the cu. cm., and the number of lbs. to the cu. ft.

It will probably be unwise to insist upon the distinction between mass and weight until after Ex. XVIII shall have been taken.

49. Loss of Weight in Water.—Every one who has lifted a stone from a river-bed, the sea-bottom, or any similar

[1] These balances are cheap, and a weighing can be quickly effected by means of them. The care necessary to attain respectable accuracy with them is by no means unprofitable. In the description of Exs. 7, 8, 9, and 10, care has been taken to select for weighing objects that lie within the range of this balance. Teachers may, however, prefer for their students a more sensitive instrument. See Ex. XI and foot-note to Ex. VIII.

situation, must have noticed how much heavier it appeared to become at the moment when it left the water. Somewhat more exact, though by no means accurate, experiments, like those which immediately follow, may give us a more precise idea of what happens to the submerged stone.

EXPERIMENT 13.

Weigh a thick piece of roll sulphur about 5 cm. long on a balance sensitive to less than ½ gm. (see Exercise XI). Attach a piece of flexible rubber tubing about 10 cm. long to the copper tube on the side of the upright cylinder of "Apparatus A." (§ 140). Stand this cylinder on a table, and fill with water until some overflows through the tube. Now place a small vessel, weighed to ½ gm. or less, under the rubber tube of the cylinder and, by means of a thread tied about the piece of sulphur already weighed, lower the latter carefully into the water. Re-weigh the small vessel with the water which it now contains, and by subtraction ascertain the weight of the water which was made to overflow by immersing the piece of sulphur. Finally attach the sulphur by a fine thread to one pan of the balance, suspend it in a jar of water, as shown in Fig. VIII, and weigh. Its weight is now much less than in air. Compare the loss of weight in water with the weight of the overflow water, as already ascertained.

The record may be made as follows:

(A) Weight of sulphur in air =
(B) " " vessel =
(C) " " vessel and displaced water =
(D) " " displaced water (C − B) =
(E) " " sulphur in water =
(F) Loss of weight in water (A − E) =

Compare C − B with A − E.

The piece of sulphur when immersed evidently displaces its own bulk of water. We are now in a position to decide what relation the loss of weight of an object immersed in water bears to the weight of the displaced water, that is, to the weight of a quantity of water equal in bulk to the object itself. In order in a slightly different way to get at the answer to the question just asked, we may perform one more simple experiment.

EXPERIMENT 14.

Take a rod of light wood, in the shape of a square prism, 1 cm. square and about 30 cm. long, loaded at the lower end with lead to make it float upright. Such a rod can be made (best of white wood, poplar, or willow) by any good carpenter. It should be graduated in cm., and then well varnished with shellac varnish. A little cylinder of stiff paper should be attached to the top, to contain weights, etc. Float this rod upright in water in a tall glass jar, Fig. 11, an ordinary hydrometer-jar for example. Then holding the eye at the water-level and sighting along the under part of the concave surface of the water, as shown in the diagram, Fig. 12, drop, if necessary, very fine shot ("dust-shot") into the little cylinder at the top of the rod, until some one of the graduations comes just to the water-level, taking care not to allow the rod to float against the side of the vessel. Now load the rod with a 1-gm. weight, then with a 2-gm. weight, etc., and at each addition note how far the rod sinks. Draw any conclusions that you can from your results, bearing in mind always that a cu. cm. of water weighs 1 gm.

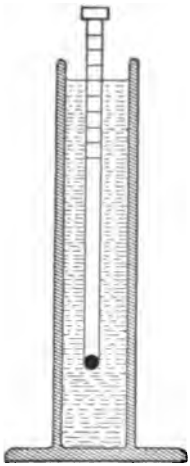


FIG. 11.

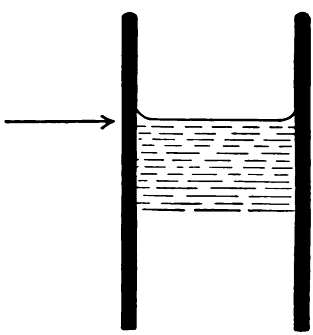


FIG. 12.

50. Discussion of Experiment 14.—The results of the experiments just performed may be more easily understood by reference to a simple diagram. Suppose the jar shown in Fig. 13 to be nearly filled with water and a cubical block, whose volume is 1 cu. cm., to be immersed just to the water-level. The latera pressures, indicated by the horizontal arrows, will neutralize each other. All that remains capable of producing any unneutralized tendency to move the block as a whole is the upward

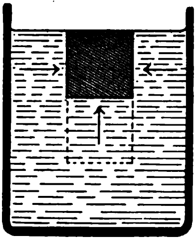


FIG. 13.

pressure, indicated by the vertical arrow. This pressure being equal to that of a column of water 1 sq. cm. in cross-section and 1 cm. deep will amount to 1 gm. (§ 32). Suppose the block now to be submerged 1 cm. farther, to the position indicated by the dotted lines. The upward pressure on the bottom of the block is now equal to that due to a column of water 2 cm. deep, but this is now opposed by a downward pressure due to a column of water 1 cm. deep immediately over the block.[1] The net result of the joint action of these upward and downward pressures is still equal to the pressure of a column 1 cm. deep. In short, to whatever depth the block is immersed, we shall find that the net upward pressure, or buoyant effect, is always equal to that of a column of water of the same volume as the displaced water.

51. Principle of Archimedes.—From results of experiments and the reasoning just given, we may derive what is known from its discoverer as the *principle of Archimedes.* This may be stated, for liquids in general, as follows: *A*

[1]Atmospheric pressure, which affects equally the top and the bottom of the block (Pascal's principle, § 33), is left out of account in estimating the buoyant action of a liquid upon a floating submerged body.

body immersed in a liquid loses an amount of weight equal to the weight of the liquid displaced.

If the body is lighter than water, it may lose more than its own weight, that is, it may exert an upward push or pull on the object by which it is held down in the liquid (see Exercise X).

52. Specific Gravity.—The *specific gravity* of a substance is *the ratio which the weight of a given volume of the substance bears to the weight of the same volume of some standard substance.* The standard of specific gravities for solids and liquids is water at its temperature of maximum density, which is very near 4° centigrade (§ 143). In the experiments of this book water at the ordinary temperature of the laboratory will be taken as the standard substance. Chemists often find it more convenient to refer specific gravities of gases to hydrogen gas as a standard.

To illustrate in the simplest way what is meant by specific gravity, suppose that a cubic centimeter of marble weighs 2.7 gm. Since a cubic centimeter of water weighs 1 gm., the specific gravity of marble is 2.7. But in order to determine the specific gravity of a substance it is not necessary to obtain just a unit volume of the material and compare its weight with that of a unit volume of water. Any convenient portion of the substance to be examined may be compared, by means of the principle of Archimedes, with an equal volume of water, and the specific gravity may be calculated from the results.

The relation of specific gravity to density can perhaps best be shown by means of formulas. By definition, in the case of solids and liquids,

$$\textit{specific gravity} = \frac{\textit{weight of the body}}{\textit{weight of equal volume of water}}$$

$$= \frac{\textit{vol. of body} \times \textit{density of body}}{\textit{vol. of water} \times \textit{density of water}} \,(\S\, 47) = \frac{\textit{density of body}}{\textit{density of water}}.$$

The specific gravity of a body is, therefore, equal to the ratio which its density bears to the density of the standard substance.

53. Numerical Relation of Specific Gravity to Density.— As specific gravity and density are easily confused, it may be well to dwell somewhat further upon the distinction between them.

Water being taken as the standard of specific gravity (§ 52), its own specific gravity is unity in any system of units.

In the C. G. S. system (Appendix I) the unit of volume being the cubic centimeter, and the unit of mass the gram, the density of water (§ 47) must be unity. Would it still be unity if the cubic decimeter were to be assumed as the unit of volume, the unit of mass remaining 1 gm.? Or if, the unit of volume remaining, as now, 1 cu. cm., the kilogram were to be made the unit of mass? What, in each of the cases above supposed, would be the density of water? In the English system, which has the cubic foot as the unit of volume and the pound as the unit of mass, the density of water (weight of a cubic foot) is 62.426.

What would be the density, reckoned by the C. G. S. method, of chestnut wood (sp. gr. 0.6)? What, by the English method? What is the general ratio between density and specific gravity in each system; i.e.,

$$\frac{density}{sp.\ gr.}(C.\ G.\ S.) = what\ number?$$

$$\frac{density}{sp.\ gr.}(English) = what\ number?$$ [1]

[1] One great advantage of the metric system of weights and measures arises from the simplicity with which the relations of the units are expressed. It is easy to remember that a cubic centimeter of water weighs a gram, or a cubic decimeter (or liter) a kilogram, and to calculate what would be the weight of any number of liters, say 150. But it is not so easy to remember that a cubic foot of water weighs 62.426 lb., and few people could mentally reckon easily what, for example, would be the weight in pounds of 32 gallons (a barrel) of water.

54. EXERCISE VIII.

SPECIFIC GRAVITY OF A SOLID THAT WILL SINK IN WATER.

[Worthington, Exp. 6, p. 67.]

Apparatus: A glass bottle, without stopper, weighing a little less than 8 oz. The spring-balance, of 8 oz. capacity, used in Ex. VII. A vessel of water in which to submerge the bottle. Some means of suspending the balance, e.g., a nail in the edge of a table. A thread for suspending the bottle.

Weigh the bottle, empty and dry, in air. Weigh the bottle entirely filled with water in water. From these two weighings find the specific gravity of the glass of the bottle as compared with the water used.

With the spring-balance[1] it is very difficult to get good results

[1] Mr. J. Y. Bergen, who has taught this course in the Boston Latin School, uses in the experiments on specific gravity the platform-

FIG. VIII.

balance mentioned in Ex. XI. He takes a wooden box of suitable size, removes top and bottom and all of one side except a strip about 8 cm.

when a very dense substance, like most of the metals, is used. Hence glass is here used. The bottle is to be regarded merely as a piece of glass.

Tabulate the results obtained in Exercise VIII on the same general plan as that adopted for Experiment 13.

55. Formula for Specific Gravity.—In calculating specific gravities of solids and liquids the formula employed will frequently be the following, or some modification of it:

$$\textit{specific gravity} = \frac{\textit{weight of substance in air}}{\textit{loss of weight in water}},$$

for the loss of weight in water is (by Archimedes' principle) the weight of the displaced water, and therefore of a volume of water equal to the volume of the object which is being examined.

56. Illustration of Differences of Specific Gravity.—To illustrate roughly the relative specific gravities of some familiar substances the following experiment may be performed.

EXPERIMENT 15.

Pour into an ordinary hydrometer-jar mercury enough to fill it to the depth of several cm., and then nearly fill with water the remaining portion of the jar. Drop into the water an iron ball or an iron nut of convenient size and note at what point in the jar it comes to rest. Fill another similar jar half full of water, and most of the remaining half with benzine. Drop into this a small block of heavy hard wood, for example red oak, hickory, or apple-wood. Make out a list of substances experimented upon in the order of their specific gravities.[1]

---

wide. On this strip, lengthwise, the balance is placed, the jar of water in which objects are to be weighed standing beneath the strip. The object to be weighed is suspended by a loop of thread which runs across the platform and embraces, without touching, the supporting strip of wood. [See Fig. VIII.]

[1] The experiment can be performed with all three liquids together in one jar.

57. EXERCISE IX.

SPECIFIC GRAVITY OF A SOLID THAT WILL FLOAT IN WATER.

FIRST METHOD: BY SUBMERSION WITH A SINKER.

[Worthington, Exp. 8, p. 67.]

Apparatus: The wooden block of Ex. VII soaked in melted paraffine so that it will be water-proof. A sinker[1] weighing less than 8 oz. under water, but heavy enough to sink the block. Some means of suspension for the spring-balance.

Weigh the block suspended by a thread alone in air. Place the sinker on the block and weigh both together under water. Weigh the sinker alone under water. From these three weighings find the specific gravity of the block.

SECOND METHOD: BY FLOTATION.

[Worthington, Exp. 12, p. 70.]

Apparatus: The same block. A vessel in which to float the block in water. A short scale divided to mms.

Measure the thickness of the block at each of the four corners and take the average. Float it in the water and measure the depth to which it sinks at each corner.[2] Take average depth. From these measurements find the specific gravity of the block.

58. Discussion of Exercise IX.—The calculation for the first method is to be performed by the aid of the formula given in § 55, but it must be noticed that the loss of weight

---

[1] A flat-bottomed piece of metal with a hook at the top is very convenient for this purpose. Such a sinker can be easily made by pouring melted lead into a small baking mould, smaller at the bottom than at the top, inserting a copper wire while the lead is still hot, and holding it in place until the lead solidifies. The submerged part of the copper wire should have a hook in it. The interior of the mould should be smoked before the lead is poured in. By putting the mould, resting on a piece of wood, on a platform-balance one can arrange to have the balance tip when the desired amount of lead is poured in.

[2] The *meniscus* caused by the water-surface rising where it touches the block makes this measurement a difficult one. It can perhaps best be performed by looking *through* the glass and *through* the water, raising the eye until it just comes on a level with the flat water-surface, previously seen from beneath. [See Fig. 12.]

now suffered by the block represents all of its own weight and all of the weight of the immersed sinker except the weight noted for the immersed block and sinker taken together.

Before discussing the result of the second method in Exercise IX a simple experiment may be performed which will throw some light on the principle involved.

EXPERIMENT 16.

Arrange the cylinder of Apparatus A, as already described in Experiment 13, weigh a wooden ball or block of such size as to fill the cylinder pretty completely but not to fit closely in it. Fill the cylinder with water until some runs off through the spout, then put the block carefully into the water, catch the overflow and weigh it, and from your results ascertain what relation exists between the weight of the floating block and the weight of the displaced water.

Further experiments with a variety of liquids and of floating objects would establish the general law:

A body floating in a liquid displaces its own weight of the liquid.

The calculation of results from the second method of Exercise IX is readily intelligible from this law. Since the displaced water weighed as much as the entire block,

$$\textit{volume immersed} \times \textit{density of water} = \textit{volume of whole block} \times \textit{density of block},$$

whence

$$\frac{\textit{density of block}}{\textit{density of water}} = (\textit{sp. gr. of block}) = \frac{\textit{volume immersed}}{\textit{volume of block}} =$$

(in this case owing to the rectangular shape of the block)

$$\frac{\textit{thickness of immersed portion}}{\textit{thickness of the whole block}}.$$

59. ## EXERCISE X.

SPECIFIC GRAVITY OF A LIQUID.

FIRST METHOD: BY THE "SPECIFIC GRAVITY BOTTLE."

[Trowbridge, Exps. 16 and 17. Worthington, Exp. 20, p. 58.]

Apparatus: A few ounces of water and of kerosene. (Other liquids, for instance alcohol or a saturated solution of sulphate of copper, may be substituted for the kerosene.) A bottle which will hold 2 or 3 oz. of water, with ground glass stopper, the bottle and stopper weighing about 4 oz. The 8-oz. spring-balance already described. A thread to suspend the bottle. A small piece of cloth with which to wipe the bottle.

Weigh the bottle empty; weigh it full of kerosene with stopper in; weigh it full of water with stopper in. Take care to exclude air-bubbles in putting in the stopper.

From the three weighings find the specific gravity of kerosene.

SECOND METHOD: BY MEANS OF BUOYANT ACTION.

[Trowbridge, Exp. 11, p. 9. Worthington, Exp. 10, p. 68.]

Apparatus: The bottle used in the first method, filled with water to make it sink readily in water, and with the stopper firmly in place. A jar of kerosene in which to submerge the bottle for weighing. A jar of water for the same purpose. Other articles as in the first method.

Weigh the bottle and the water it contains in air, in kerosene, and in water.

From these weighings find the specific gravity of kerosene.

THIRD METHOD: BY BALANCING COLUMNS.

[Worthington, Exp. 1, p. 62.]

Apparatus: Kerosene and water. A glass tube 8 or 10 mm. in internal diameter and about 1 m. long, bent into the form of a long and very narrow U. A small funnel. A meter-rod reading to millimeters. Some support to keep the tube vertical.

Fasten the U-tubes upright with the meter-rod behind it. Pour water into the tube until it fills the bend and about 20 cm. of each arm. Then pour kerosene into one arm until it is nearly full. Measure the height of the kerosene column in one arm, and in the other arm the height of the water column above the level of the bottom of the kerosene column. From these measurements find the specific gravity of kerosene.

Alternative Method of Balancing Columns.

[Trowbridge, Exp. 28, or Worthington, Exp. 3, p. 63.]

Apparatus: Two small tumblers, one containing water, the other kerosene. A lead Y-tube, each arm of which is about 5 cm. long and 5 or 6 mm. in internal diameter. Two pieces of glass tubing each 50 cm. long and about equal to the lead tube in outside diameter. Two pieces of rubber tubing about 5 cm. long and one piece about 10 cm. long, all of the proper size to fit closely over the glass or the lead tube. A pinch-cock. A meter-rod.

Connect two arms of the lead Y-tube with the glass tubes by means of the short pieces of rubber tubing. Slip the longer piece of rubber tubing, carrying the pinch-cock, upon the third arm of the lead tube. Place the free ends of the glass tubes, one into the water, the other into the kerosene, and fix them in a vertical position. With the mouth draw out some air through the Y-tube, thus raising the two liquids in their respective tubes. [See Fig. IX.] Close the pinch-cock and watch to see that there is no falling of the columns due to leaking in of air. (To prevent leakage it is well to wet the inside of the rubber tubes.) Measure the height of the columns.

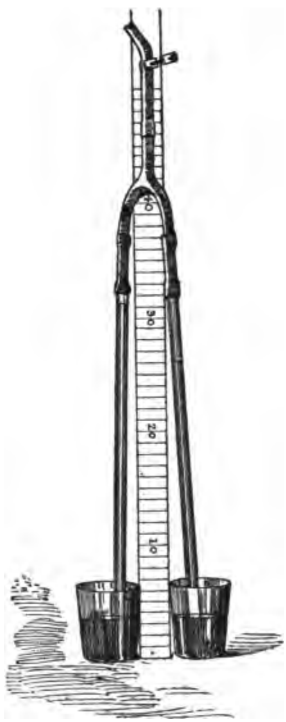


Fig. IX.

Note the height to which "capillary action" alone draws the liquids and make corrections accordingly.

From the measurements find the specific gravity of kerosene.

60. Disscussion of Exercise X.—It is easier to find the specific gravity of liquids than of solids, and many methods may be resorted to for the purpose. The first method of

Exercise X consists in measuring equal volumes of the two liquids and comparing the weights of these equal volumes. The calculation is too simple to require any explanation. In the second method the process really depends on the same principle as in the first method, but now the volumes that are being compared in weight are those which are displaced by the bottle. It is clear that the loss of weight in water represents the weight of a quantity of water equal in bulk to the outside measurement of the immersed bottle. The loss of weight in the kerosene represents the weight of an equal bulk of kerosene. The third method depends upon the principle that when two liquid columns in connected vertical tubes balance each other by the pressure due to their own weight only, the heights of the columns are inversely proportional to the specific gravities of the liquids. The truth of this principle in the case where the two tubes are of equal cross-sections is made sufficiently evident by what has gone before. When these conditions do not hold the question is more difficult, but Exercise V has shown us that the pressure per unit area at the bottom of a column of liquid does not depend upon the size, in horizontal cross-section, of the column. Now where one column meets the other column the cross-section of the two is the same, and the pressure at this meeting surface is just what it would be if both columns, retaining their present heights, were of the same cross-section throughout. We may therefore disregard their difference of cross-section.

In the alternative method of balancing columns in Exercise X the columns do not really balance each other, but the pressure per unit area on the tops of the two columns is the same, and as the pressure per unit area at the bottom of each column (inside the tube at the level of the outer surface) is equal to that caused by the atmosphere, the pressure per unit area produced by the *weight* of either column must equal the pressure per unit area produced by

the weight of the other column. The two columns would therefore balance each other directly if they met at the bottom, and the calculation may be made as if they actually did so.

61. Height of the "Homogeneous Atmosphere." — The student should notice that the barometer-tube as used in Torricelli's experiment represents the principle of balancing columns, and that if we know just the ratio which the density of mercury bears to that of air, we can readily calculate the height of the "homogeneous atmosphere," that is, the height of an air-column of *uniform density from top to bottom* which would just balance the barometric column. In order to make such a homogeneous atmosphere air would need to become an incompressible fluid; nevertheless the calculation is worth making, for it enables us to make very readily rough determinations of the changes in atmospheric pressure produced by small changes of altitude.

62. Specific Gravity of Gases.—The specific gravity of air, or any other gas, depends so much on its temperature and the pressure to which it is subjected that it is customary to give that value which holds when the temperature is 0° C. and the pressure equal to that of 76 cm. of mercury. These are the so-called standard conditions, but in the following exercise we shall determine the specific gravity of the air at the temperature and pressure which hold in the laboratory at the time of the experiment. Since neither the air nor the water can be weighed without some containing vessel, it will be necessary, as in obtaining the specific gravity of liquids by the "specific-gravity bottle," to obtain the weight of the bottle empty, as nearly as may be, and then by subtracting this from the weight of the bottle filled with air, and again from the weight of the bottle filled with water, we shall get the weights of equal volumes of air and water. Strictly, we cannot get all

the air out of the bottle, and we have to make allowance for what remains in it.

63. EXERCISE XI.

SPECIFIC GRAVITY OF AIR: DEGREE OF EXHAUSTION.

Apparatus: A two-liter bottle, with rubber stopper perforated for a short piece of glass tubing. A piece of thick-sided, flexible, rubber tubing about 15 cm. long. A pinch-cock. A balance[1] weighing to 0.1 grm. or less. An air-pump. (A little glycerine helps to make the junctions between glass and rubber air-tight.)

Weigh the bottle with the stopper, glass tube, rubber tube, and pinch-cock to 0.1 grm. or less. Draw out as much air as possible with the air-pump, then let the pinch-cock close the tube, and re-weigh to find how much air has been taken out. Be sure that there is no steady gain of weight due to leakage, then open the bottle with its mouth under water,[2] let it fill with water so far as it will in this position, and then replace the stopper. Dry the outside of the bottle and weigh again to find how much water has entered. This weighing requires only common scales sensitive to 5 or 10 grms.

When the bottle is opened under water the latter replaces bulk for bulk, at the atmospheric pressure, the portion of the air which has been exhausted; hence the ratio of the loss of weight due to removing the air to the gain of weight due to admitting water is the specific gravity of air referred to water.

Fill the bottle with water, and weigh again in the same manner to find how much water it holds. The ratio of the two weights of water to each other shows the proportion of air which was exhausted.

The interior of the bottle must be carefully dried before it can be successfully used a second time. To dry it quickly hot air

[1] See platform-balance described in the Appendix. This balance should be tested without load before and after each weighing.

[2] For strict accuracy, the bottle, while the stopper is being replaced, should be held at such a depth that the water-level inside will be the same as that outside, and the temperature of the water should be the same as that of the surrounding air. The errors arising from neglect of these precautions are likely to be small compared with other errors of the experiment.

must be forced through it. A little suction-pump which can be attached to a faucet and operated by a stream of water is very useful in this connection and in others. Such a pump can be bought for about $2.

As the drying process is somewhat troublesome when a class is large, the following method may by some teachers be preferred for this exercise: Determine the interior volume of the bottle once for all by weighing it empty and again full of water, and let the volume be marked upon the bottle. Weigh the bottle full of air. Connect the bottle with the air-pump by means of one of

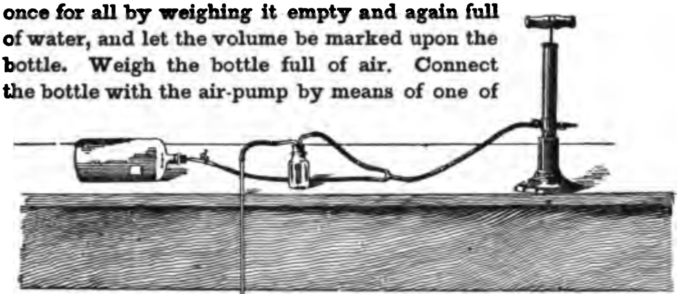


FIG. X.

the lead Y-tubes described above, and connect the third branch of the Y with a straight glass tube 4 or 5 mm. in internal diameter and about 1 m. long, this tube standing vertical in a vessel of mercury (interposing, however, between the Y-tube and the long glass tube a small bottle to catch mercury that might otherwise by accident be drawn into the air-pump). [Fig. X.] The height to which the mercury rises in this tube under the action of **the pump will, when compared with the barometric pressure existing at the same time, give the necessary means for determining the degree of exhaustion. Weigh the bottle again after exhaustion. All the data necessary for determining the specific gravity of air at the existing temperature will now have been obtained.**

64. Record and Calculation of Results.—In recording the result of the experiment, let

$a =$ weight of bottle full of air;
$b =$ " " " after partial exhaustion.

Then $\quad a - b =$ weight of air exhausted.

Let c = weight of bottle with water taking the place of the exhausted air; $c - a$ = weight of water admitted; etc.

The remainder of the calculation may well be left to the student.

Questions and Problems on Chapter III.

1. Describe carefully the experiments which you have made in regard to the pressure of liquids and the conclusions reached. (Omit the preliminary experiments in regard to atmospheric pressure.)

2. At what depth in water will the pressure due to the water alone be 1 kgm. per square centimeter? 1000

3. A cylinder 20 cm. long, closed at each end and completely filled with water, stands upright. A tube enters the side of the cylinder at the middle of its height. This tube is filled with water to a height of 50 cm. above the opening into the cylinder. The pressure of the atmosphere being disregarded, how great is the pressure of water against each square centimeter of the top of the cylinder? 40 Against each square centimeter of the bottom? = 60

4. A tank with rectangular sides and bottom is filled with water. It is 3 m. long, 2 m. wide, and 1 m. deep.
1500 kgm Disregarding atmospheric pressure, (*a*) find the total
1000 " pressure on one side; on one end. (*b*) Find the pressure per square centimeter at a point half-way down one side.

5. Let a Mariotte's bottle have two lateral apertures, and let an open tube extend vertically through the stopper at the top of the bottle to a point between the level of the two lateral apertures. The bottle and the tube being originally full of water, (*a*) state and explain what will take place when the higher lateral aperture only is opened; (*b*) state and explain what will take place when the lower aperture only is opened.

6. A box 20 cm. square and 20 cm. deep has opening

into its top a tube 4 cm. square and 10 cm. tall. If the whole be filled with water, how great is the total pressure upon the bottom? How much does the whole of the water weigh? If the two quantities here required are not equal, explain the difference. (Atmospheric pressure is neglected.) 16 : 160 :: 400 : x Ans. 4800 + 12000

7. A cubical box 10 cm. deep is closed at the top except at one place where an open tube 1 sq. cm. in cross-section projects vertically 20 cm. Both box and tube are full of water. How great is the pressure of the water upon the bottom of the box? How great is the pressure of the box upon its support? Explain the difference. (Neglect air-pressure and weight of box.) 2000 + 3000 + 1020

8. A diving-bell is allowed to sink in water until its mouth is 20 ft. beneath the general surface. The surface of water in the bell is 3 ft. above the mouth. If the pressure of the atmosphere is 15 lbs. per sq. in. and is equal to that of a layer of water 34 ft. deep, how great is the pressure to which the air in the bell is subjected? 15 : x :: 34 : 57 ... 22.5

9. Give a careful account of Exercise VI of this course, describing the apparatus, the experiment, and showing the method of reasoning and the conclusions drawn.

10. When the height of the mercury column of the barometer (specific gravity of mercury 13.6) is 76 cm., how tall a column of water can the pressure of the atmosphere support in a tube, there being a perfect vacuum at the top? 13.6 x 76 = 1033.6

11. If the column of air in the short arm of the tube in Exercise VI is 16 cm. long at the beginning of the experiment, the barometer column being 76 cm. tall and the mercury standing at the same height in both arms of the apparatus, what is its length when the difference of level of the mercury in the two arms is 22 cm.?

12. If a liter-bottle from which the air has been perfectly exhausted is found to gain 1.293 gm. in weight upon the

air being readmitted, the barometer at the time standing at 76 cm., what would be the gain in weight on another day when the barometer stood at 74 cm.?

13. Describe in detail what experiment you would perform and what measurements you would make in order to ascertain the value in grams of the pressure of the atmosphere per square centimeter.

14. Define density; specific gravity. Explain why the density and specific gravity of a body are numerically equal when one particular system of units is used.

Show what numerical relation holds between density and specific gravity in the English system of units.

15. A stone weighs 80 gm. in air and 50 gm. in water. Find its specific gravity.

16. A body whose volume is 1 cu. ft. weighs 100 lbs. in air, 37.6 lbs. in water, and 20 lbs. in another liquid. Find the density (English) and specific gravity of the second liquid.

17. Give exact directions for finding the specific gravity of a blockof wood by submersion in water, stating all the precautions that you have been expected to take in this experiment in this course.

18. If the volume of a brick-shaped body were to be determined by measurement, which of the three dimensions should be measured most carefully? State your reasons, illustrating by a numerical example.

19. If a man's weight is 75 kgm. and his volume, exclusive of his head, 72 cubic decimeters, how many cubic decimeters of cork, of specific gravity 0.25, would be required to keep the man floating with his head above water?

20. A certain body weighs 50 grams in air; another weighs 20 grams under water. The two together weigh 30 grams under water. What is the specific gravity of the first body?

21. Explain why the pressure of the atmosphere does not, and why the pressure of the water in a lake does, increase uniformly with increase of depth.

22. How much would a piece of glass, sp. gr. 2.5, that weighs 100 gm. in air weigh in sulphuric acid whose specific gravity is 1.84? In alcohol whose specific gravity is 0.92?[1] 26.4 63.2

23. A rod of wood of uniform cross-section and 20 cm. long floats, when prevented from falling over, with 12 cm. submerged. What is its specific gravity? .6

24. The weight of a lead sinker when immersed in water $= A$; that of a block of wood in air $= B$; that of block and sinker together under water $= C$. Find the specific gravity of the wood.

25. A cubical block of wood 10 cm. on an edge floats upright in water with 9 cm. of its depth submerged. How many centimeters will be under *water* after kerosene (sp. gr. 0.8) has been poured into the jar in which the block is floating, until the block is entirely submerged by the kerosene?

26. A rectangular aquarium with a bottom 3 m. long and 1 m. wide is filled to a depth of 80 cm. with sea-water of sp. gr. 1.026. What is the weight of the water? 2462.4

27. A certain solid floats in the water with only two-thirds of its volume submerged. What is the specific gravity of this solid? .666

28. Soundings have been made at sea to a depth of 7100 m. (*a*) What would be the pressure per square centimeter at this depth, the density of sea-water being 1.026 in the C. G. S. system? (*b*) If a body of air occupying 400 cu. cm. at the surface were lowered to this depth 728.46

[1] In all the examples of this chapter the buoyant effect of the air is to be neglected unless expressly mentioned.

in a stout tube closed at one end and kept with the open end down, what would its volume there be?[1]

29. Gold-leaf can be made $\frac{1}{10000}$ of a millimeter in thickness. How much surface can be covered by a gram of such gold-leaf? (See table of specific gravity in Appendix.)

30. What must be the volume of a cast-iron kilogram-weight?

---

[1] If the solution of this question were to be given with precise accuracy it would be necessary to allow for increase of density in the sea-water (due to compression from its own weight) at the rate of about $\frac{1}{100}$ of itself for every 2300 meters.

CHAPTER IV.

SOME HYDROSTATIC AND PNEUMATIC MACHINES.

65. The Hydrostatic Bellows.—This apparatus in its simplest form consists of two boards, each some 50 cm. square, connected by a well-oiled piece of leather tacked closely around the sides. A tube, bent at right angles at the bottom, opens into the interior of the bellows at the upper surface of the lower board. When water is poured into the upright portion of the tube until the bellows is filled, the upper board of the bellows is forced up with a pressure proportional to its area and to the height of the water in the tube above the level of the water in the bellows. Each centimeter in height of the column in the tube will give rise to a pressure of 1 gm. per square centimeter in the bellows (see "Pascal's Principle," § 33). If the area of the cross-section of the tube were 1 sq. cm. and that of the top of the bellows were 2500 sq. cm., it is clear that each cubic centimeter (or gram) of water poured into the tube would exert a pressure of 2500 gm. against the bellows-top, if the latter were prevented from rising. Since the leather sides soon become hardened and crack, the metal cylinder already described (Fig. 4) may conveniently be used to illustrate the principle of the bellows. For this purpose it is only necessary to wire a stout rubber tube firmly on to the short brass tube *C*, force the piston to the bottom of the cylinder, and then, holding the free end of the tube 3 m. or more above the cylinder, pour in water through a funnel, while the upward movement of

the piston is resisted by the application of heavy weights or by one of the pupils pushing strongly against it.

66. The Hydrostatic Press.—In the case of the apparatus just described the pressure which moves the piston is due to the weight of the water in the upright tube. The hydrostatic press depends for its action on the pressure imparted to water in one cylinder by a piston or plunger working in another cylinder, often at a considerable distance from the former one. The ram or larger plunger *A* is so arranged

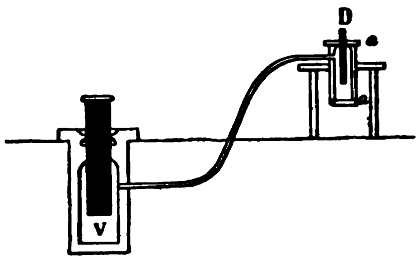


FIG. 14.—Hydrostatic Press.

as to work water-tight in the cylinder *V*, and the plunger *D* works similarly in the small cylinder *a*. Small quantities of water are successively forced from *a* into *V* by the movements of *D*, and *A* is raised very slowly, but with great force, carrying with it whatever weight may have been placed upon it, or compressing strongly any material, for example a bale of cotton, confined in an open-bottomed chamber above the ram, into which the latter rises.

Since the pressure per square centimeter exerted by the small piston is transmitted undiminished to every part of the body of water in both cylinders, the total resulting pressure exerted by the ram *A* will be to the total pressure applied by the plunger *D* as the area of the cross-section of *A* is to the area of the cross-section of *D*. The principle of the hydrostatic press may be readily illustrated by the use

of the brass cylinder and piston, Fig. 4. The cylinder being placed in a stout frame, so arranged that the rise of the piston may break in two a short stick fastened across just above the piston, water may be forced into the cylinder by means of a small force-pump, or even (if the laboratory is supplied with water at considerable pressure) by connecting the cylinder, by stout tubing, with the faucet at the sink.

67. The Siphon.—Let a vessel be filled with water to a certain level. Take a rubber tube about 30 cm. long and fill it completely with water. Closing both ends with the fingers, dip one end beneath the water-surface and then open it. Keeping the other end still closed, hold it somewhat higher than the water-level and consider whether the pressure of the water at this closed end is greater or less than the atmospheric pressure, and whether, therefore, the water would or would not flow out if the tube were opened, guiding yourself in your inquiry by what you have learned in connection with Exercise V. Having made up your mind upon this point, remove your finger and see what course the water takes. Repeat the experiment and the inquiry, holding now the outer end of the tube lower than the water-surface. After making these experiments you will see that you have in your hands a very useful device for the transfer of liquids. Continue the experiments by having the rubber tube, after it is filled, connect the water in one vessel with that in another vessel, the level of the two water-surfaces being different in the two vessels.

PROBLEM: Is there any limit to the height which the upper bend of an acting siphon can have above the level of the water in the vessel which it is emptying? If so, what is this limit at the ordinary pressure of the atmosphere?

68. Pumps for Raising Water.—Most pumps depend upon atmospheric pressure as one necessary condition for their operation.

In the *lifting-pump* (often called suction-pump), Fig. 15, the piston is perforated by two openings, covered at the top by the upward-opening valves *V*, *V* (shown closed in the figure). The piston slides closely, but with little friction, in the barrel, or *cylinder*, *A*. Water is supplied from below by the "suction-pipe" which leads from a reservoir. At the top of this pipe is an upward-opening valve. As constructed for actual use the valves often have a form very different from that of the simple "clack-valve" shown in the figure. As here represented the piston is in the act of rising, hence the lower valve is opened by the upward-flowing water and the upper valves are closed by the pressure of the water and atmosphere above them. The height to which water can be lifted, above the lower valve, depends only upon the strength of the pump and the power applied to work it. The lower valve, however, cannot, for the reason already noted in connection with the siphon, be successfully placed at a height of more than 10.33 m. (about 34 ft.) above the level of the water in the reservoir, when ordinary atmospheric pressure prevails. In practice the valve is not set so high as this.

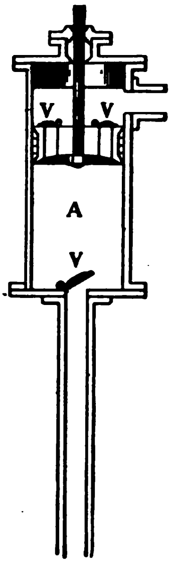


FIG. 15.

In *force-pumps* the water is not merely lifted but it is forcibly driven out of the cylinder. The piston is solid, and the upper valve opens outward, or away from the cylinder, as shown in Fig. 16, into the pipe through which the pump discharges.

With the simple form of pump shown in Fig. 16 the water is thrown out in successive jets. This defect may be remedied by delivering the water through an air-cham-

ber, as shown in Fig. 17. The water forced in through the tube *A* at each impulse compresses the air above the water-level *B* in the chamber, and this compressed air gradually expanding continues the stream of water through the tube *P* while the piston is ascending.

69. Air-pumps.—The principles of construction of the air-pump are essentially the same as those of the water-

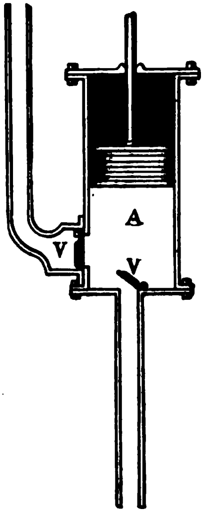


FIG. 16.

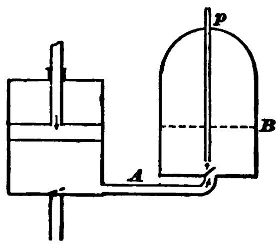


FIG. 17.

pumps just described; and, indeed, the great blowing-engines that supply the intense blast of air to many kinds of furnaces for smelting metals work force-pumps which drive air through tubes, much as an ordinary force-pump drives water. The ordinary air-pump, however, is more nearly like a lifting-pump (§ 68), and air-pumps differ from the latter mainly in the shapes and proportions of their parts and the construction of the valves. A simple and efficient kind of air-pump is shown in Fig. 18.

Used in the ordinary way the air-pump cannot give a perfect vacuum. Even if it is so constructed that the valves will be opened and shut alternately, whether there is enough air-pressure in the cylinder to move them or not, the continued action of the pump can only reduce the fraction of an atmosphere remaining to smaller and smaller

values without ever diminishing it to zero. For each upward stroke of the piston can at most remove nearly all of the air in the cylinder, leaving the air in the receiver and connecting-pipe to expand and fill the entire space contained by the cylinder (below the piston), the connecting-pipe, and the receiver. To make the case as simple as possible, suppose the cylinder and the receiver to be of equal capacity, and the pipe to be so small that its contents

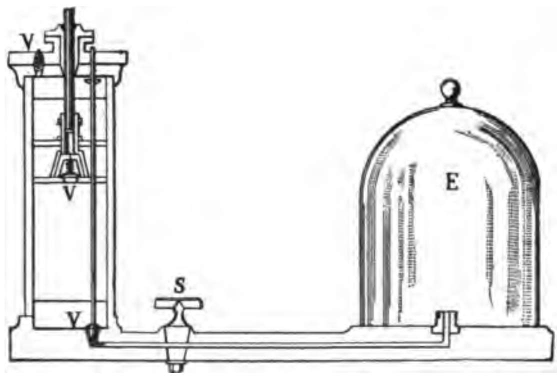


FIG. 18.

may be neglected. If the volume of air before the first up-stroke were 2 liters, the volume after that stroke, in receiver and cylinder, would evidently be 4 liters, and the tension would have fallen to $\frac{1}{2}$ what it was at first. The down-stroke would make no change in the pressure within the receiver. After the second up-stroke the tension would become $\frac{1}{2}$ of $\frac{1}{2}$, or $\frac{1}{4}$, and so on. Let the capacity of any receiver be r, and that of the cylinder be c; let the tension of the air before exhaustion be t, that after one upward stroke be t_1, and that after the nth upward stroke t_n. Then, by Boyle's law,

$$t : t_1 :: c + r : r;$$

therefore, $$t_1 = t\frac{r}{c+r}.$$

After the second upward stroke we shall have

$$t_1 : t_2 :: c + r : r;$$

therefore, $$t_2 = t_1\frac{r}{c+r}.$$

Substituting for t_1 its value derived from the preceding equation, we have

$$t_2 = t\left(\frac{r}{c+r}\right)^2,$$

and after n up-strokes,

$$t_n = t\left(\frac{r}{c+r}\right)^n \text{ [1]}$$

Questions and Problems on Chapter IV.

1. The inside diameter of the cylinder in Fig. 4 being taken at 14.5 cm., what weight can be supported by the piston when the rubber tube attached contains a column of water 2 m. above the water-level in the cylinder?

2. Suppose the same cylinder to be used as a hydrostatic press, the rubber tube attached being laid horizontally, and let a plunger of 1 sq. cm. area of cross-section be forced into the open end of the tube by a pressure of 0.5 kgm.

---

[1] These formulas imply that any desired degree of exhaustion can be attained by continued action of the pump, but with ordinary pumps this is not the case. Leakage and imperfect fitting of the piston at the bottom of the cylinder make it impracticable in most cases to reduce the pressure of the air in the receiver much below that of 1 mm. of mercury. When the exhaustion must be carried farther than this, mercury-pumps are usually employed.

in excess of friction. What weight could be sustained by the piston?

3. What difference would it make in problem (1) and in problem (2) if oil of sp. gr. 0.9 were substituted for water?

4. How high could sulphuric acid of sp. gr. 1.84 be raised above its level in the containing vessel by means of a siphon? How high could the lower valve of a lift-pump, used in pumping up this acid, be placed above the level of the acid to be raised?

5. If the water has all run down to its natural level out of the cylinder and suction-pipe of a lift-pump, it may usually be pumped up again by filling the cylinder with water and working the pump rapidly. State just what occurs when this is done, and explain.

6. In an air-pump the capacity of the cylinder is 0.5 liter and that of the receiver 4 liters. The capacity of the connecting tube may be neglected. How much of the air originally present will remain in the receiver after five double (complete) strokes?

CHAPTER V.

COMPOSITION AND RESOLUTION OF FORCES.

70. Equilibrium.—A force has already been defined in this book (§ 2) as a push or a pull. The natural action of a single force applied to any body is to set the body in motion, or to change in some way the motion that the body already has. But if two or more forces are applied to the body at once, they may neutralize each other in such a way that the body will act as if no force were applied to it, provided we may neglect any change of shape or size that the body undergoes under the action of the forces. Forces so neutralizing each other are said to be in *equilibrium* with each other. But how *does* a body act, as a whole, under the influence of no force, or of a set of forces equivalent, taken together, to no force? For a very long time it was supposed that a body not acted upon by any force must come to rest and remain at rest. It was supposed that motion could be maintained only by a continual application of force. This was an error.

It is true that the moving bodies with which we are most familiar do tend to come to rest. The error lay in supposing that no force is acting upon them while they are coming to rest. Experiment shows that when better and better means are taken for lessening friction, resistance of air, etc., moving bodies show less and less tendency to come to rest, and as a result of all experience and reasoning it is now believed that no body ever comes to rest except because of some obstruction, some application of force which stops it. In fact *the natural behavior of any body,*

the behavior of a body not acted upon by forces outside itself, is to remain at rest if at rest, to move with unchanging motion if in motion. This is essentially Newton's *First Law of Motion.*

The subject of setting bodies in motion or changing their motion will be taken up in connection with Exercise XVIII. We shall in Exercises XII to XVII be dealing with cases of *equilibrium;* that is, *cases in which bodies are at rest or, as nearly as may be, in uniform, unchanging motion.* We shall in studying these cases have in view several objects, one of which is to determine what relations must exist in a set of forces in order that they may neutralize each other in their action upon the body to which they are applied. These relations are called the *conditions necessary for equilibrium,* or, simply, the *conditions of equilibrium.*

71. Illustrations of Equilibrium.—Suppose a lead bullet to be suspended by a slender thread a meter or more in length, so as to hang at rest a few centimeters above the top of one of the laboratory tables. If the air is still and the building is not jarred, by footsteps or otherwise, the bullet will remain motionless with reference to any point on the table-top; for instance, the head of a pin stuck into the table immediately beneath it. There are, however, forces at work upon the bullet. Burn off the thread and the bullet will fall, impelled towards the earth by what we call the force of gravity. As long as the bullet remained suspended and at rest the various forces acting upon it must have antagonized each other so as mutually to destroy each other's effect and thus produce a balanced condition in the bullet. Many forces contribute in greater or less degree to this result. The bullet, as we know, is acted on by the pressure of the air from all sides, and is also slightly attracted by surrounding objects in the room, by more distant objects on the earth's surface, and even by every one of the

heavenly bodies, especially the moon and the sun. Other illustrations of equilibrium of forces will readily suggest themselves to the student. Floating bodies owe their position in the liquid in which they float to the joint action of gravity, and of the upward thrust or buoyancy of the liquid. A picture hung against the wall is supported in position by the simultaneous action of gravity, of the two segments of the picture-cord on either side of the nail or hook from which it is suspended, and of the push of the wall against the lower part of the picture-frame. A balloon, held fast to the ground by a rope, owes its position to the force of the rope and that of gravity, on the one hand, and to the buoyant effect of the air, on the other hand. The spherical form of the balloon is maintained by the outward pressure of the contained gas, acting against the silken envelope which contains it, and the resistance of the silk would not be sufficient to prevent the balloon from bursting, if it were not for the pressure of the surrounding air upon its outer surface.

72. Specification and Representation of Forces.—In order to calculate what effect will be produced by a given force we must know the *amount*, the *direction*, and the *position*, or *point of application*, of the force. The directions of forces with reference to each other may be represented by lines, and the lengths of the lines may represent the relative magnitudes of the forces. One end of a line will represent the point of application of the force, and an arrowhead upon the line will indicate whether the force is directed towards the right or towards the left along it. Any scale may be chosen, as for instance one centimeter to the kilogram.

73. Bodies to be Acted Upon.—It would be most desirable in experimenting upon the effect of forces to have as the object to be dealt with some body of matter entirely free from all external forces except those to be applied by

the experimenter. This, however, is impossible. The best that we can do is to arrange some body in such a way that it will move with as much freedom as possible in all directions, or, at least, in the directions which the forces to be applied would tend to make it follow. It is comparatively easy to secure considerable freedom of motion in directions nearly or quite horizontal. We can do this by hanging up the body by a long suspension, by floating it in some liquid, or by supporting it on easy-running rollers. For some purposes we may use a body so light that its weight and the resistance it encounters from friction may be disregarded in comparison with the forces to be applied, as in Exercise XII.

74. Equilibrium of Two Forces.—One force applied to a body not acted upon by other forces cannot be a case of equilibrium. The body would suffer some change of motion. The simplest case of equilibrium is that of two forces.

EXPERIMENT 17.

Take the "checker-board" described in Exercise XIV and support it upon marbles. Apply to this board two horizontal forces, each equal to several pounds, by means of two spring-balances, and find the conditions of equilibrium. These conditions may, in accordance with § 72, be put in the form of: 1st, a relation between the magnitudes of the forces; 2d, a relation between the directions of the forces; 3d, a relation between the points of application of the forces.

75. Definitions.—Several definitions will now be needed.

Resultant. It is a familiar fact that two or more forces acting upon a body may, in many cases, be replaced by a single force; that is, in many cases, a single force can be found whose unaided action upon the body would produce upon it the same effect that is produced by the joint action of the given forces. This one force is called the *resultant* of the combination to which it is equivalent.

Components. The several forces of the combination are called the *components* of the resultant.

Composition. The process of finding the resultant of a given combination of forces is called the *composition* of forces.

Resolution. The process of finding a set of components to equal a given resultant is called the *resolution* of forces.

Equilibrant. The force which will exactly neutralize a combination of other forces is called the *equilibrant* of that combination. It is evident from what precedes that the resultant and the equilibrant of a combination are exactly equal and opposite to each other.

These definitions may be illustrated by a diagram. Let the lines *A, B,* and *C,* in Fig. 19, represent three forces which are in equilibrium with each other. Then either one of the three is the equilibrant of the other two, since it neutralizes their joint effect.

A is the equilibrant of *B* and *C.*
B " " " *A* " "
C " " " " " *B.*

The dotted lines *a, c, b,* represent forces equal and opposite to *A, B, C,* respectively. It is evident that *A,* which exactly neutralizes *B* and *C* together, would exactly neutralize *a* alone, which is therefore seen to be the exact equivalent, or resultant, of *B* and *C.* So *b* is the resultant of *A* and *C,* and *c* is the resultant of *A* and *B.*

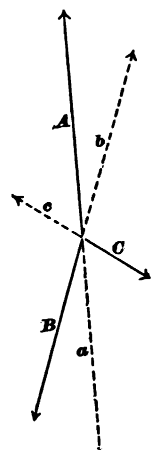


FIG. 19.

76. Exercise XII. Equilibrium of Three Forces.—We shall consider in Exercise XII the case of three forces all all of which are applied at the same point, but this Exercise is to be considered as covering not merely cases of three forces applied at one point, but also cases of three forces whose lines of action, when continued, meet at one point. Such forces will be spoken of as meeting at one point. The experiment already suggested (§ 74) with two forces ap-

plied to the checker-board can readily be so directed as to show that the point of application may, without change of effect, be changed to any new point in the line of the force.

EXERCISE XII.

COMPOSITION OF FORCES.

[Trowbridge, Exp. 61. Worthington, Exp. 10, p. 95. Lodge, Arts, 96-103.]

Apparatus: Three spring-balances of 30 lbs. capacity. Two pieces of slender, strong, hard-twisted string 70 or 80 cm. long. The block used in Ex. VII. A short scale in mms.

Connect the hooks of two of these spring-balances by means of one of the strings. Attach the other string to the hook of the third balance and let a loop in the other end of this string pass around the first string so as to slide freely upon it. Put the three balances so connected on a horizontal surface. Fasten balances one and two by their rings to pegs, nails, or other fixed objects that will serve as pivots around which the balances may swing so as to point in any horizontal directions. Pull now the third balance by its ring in any desired direction, and with such force as to draw the strings tight and so make each balance point in the same direction as the string attached to its hook. Make a careful reading of each balance. Then put a note-book beneath the junction of the strings, and in this, using the block as a guide, draw a line a few cms. in length parallel to and beneath each string. From the point where these three lines meet lay off along each line an arrow proportional in length to the reading of the corresponding balance, the arrow in each case pointing away from the junction. From the tip of any arrow lay off a line parallel and equal to one of the other arrows, and from the end of this line lay off another, parallel and equal to the third arrow. Consider whether the resulting figure appears to have any significance.

On the other hand, from the point whence the three original arrows spring draw a line equal in length to one of the arrows, but exactly opposite in direction, and consider whether this line appears to have any significant geometrical relation to the other two arrows in place.

Four students can work together on this experiment, thus avoiding the necessity of fastening the balances by means of nails or pegs. Each one holding a balance should place it on its back on the table and hold it firmly with both hands, looking directly down upon the index. Each student in turn should draw the

lines in his book and make the diagrams, using a system of forces somewhat different from those used by the others.

This experiment shows how to find the magnitude and direction of a single force which shall be equivalent to two other forces the magnitudes and directions of which are known. It shows also how to find the magnitudes of two forces, in prescribed directions, which shall be equivalent to a single force the magnitude and direction of which are known. An interesting and important application of its teachings is found in the inclined plane. See Exs. XIII and XXI.

77. Statement of Results.—All the conditions of equilibrium in this case may be included in a single statement concerning the geometrical relations of the lines representing the forces, and this statement may be put in either of two forms, which should be suggested in following out the directions already given. The first form is called the *principle of the triangle of forces.* The second is called the *principle of the parallelogram of forces.*

78. Calculation of Resultant of Two Forces at Right Angles.—A very important case of the composition of forces is that in which two forces act at right angles to one another. In order to obtain the numerical value of the resultant for such forces we need only to make a simple calculation. Suppose a force *ON*, Fig. 20, of 3 kgm., to act due north, and a force *OE*, of 4 kgm., to act due east, both forces being applied at the point *O*. Completing the rectangle *ONRE*, and drawing the diagonal *OR*, we have in *OR*, by construction, the required resultant. (Notice that, as is always the case, *the resultant is nearer, i.e., makes the lesser angle with, the greater force.*)

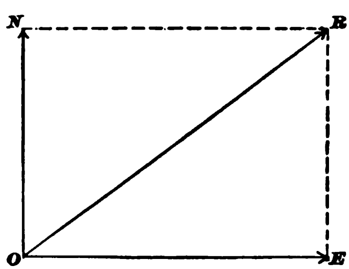


FIG. 20.

Now the line *OR* is the hypothenuse of the right-angled

triangle ONR, and, by a familiar theorem in geometry, $OR = \sqrt{\overline{ON}^2 + \overline{OE}^2}$.[1] From thiswe may derive a convenient rule for finding the resultant of two forces at right angles to each other: *If the two forces are at right angles to each other, their resultant will be numerically equal to the square root of the sum of the squares of the two forces.* In the triangle just constructed the value of OR would be equal to $\sqrt{3^2 + 4^2}$, or 5. From this calculation we get the numerical value of the resultant force, but gain no knowledge as to its direction, so that the geometrical construction gives a more complete knowledge of the resultant than can be gained from the arithmetical method alone.

79. Resolution of Forces.—A problem the converse of that just discussed, namely, how to find two or more forces whose effect shall equal that of a single given force, often presents itself for solution.

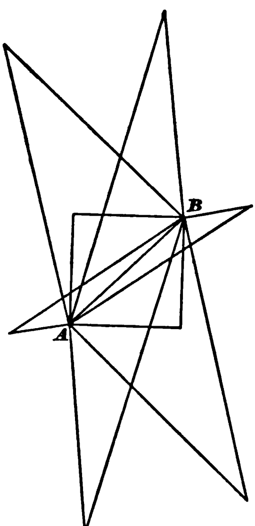


FIG. 21.

Since the force to be resolved is to be considered as the diagonal of a parallelogram, of which the required forces form the sides, the number of pairs of forces that can be found, each pair of which is equivalent to a given force, is equal to the number of parallelograms that can be drawn with the given force for a diagonal. The number of such parallelograms is unlimited. The annexed diagram, Fig. 21, shows a few of the possible parallelograms formed upon the diagonal AB.

It is often important to resolve a force into two others

---

[1] Since $OE = NR$.

one of which shall have a certain definite direction, or to resolve it in such away that the two forces found shall form a certain definite angle with each other, usually a right angle.

Suppose, for example, that a horse is attached to a point O at the front of a car and pulls in the direction OC, Fig. 22, while the rails extend in the direction EF. In order to find what useful result the pull of the horse produces we

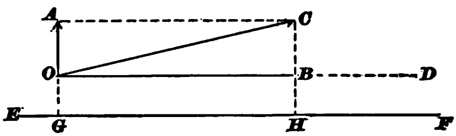


FIG. 22.

must resolve it into two forces, one of which is parallel to the line of the rails and the other perpendicular to this line. Complete a parallelogram with diagonal OC and with sides OB and OA, respectively parallel and perpendicular to the line EF. If OC represents the whole pull of the horse, OB is the useful component, and OA is useless, or, because of the friction it causes against the rails, worse than useless. OB is called the *projection* of OC upon the line OD.

80. Resultant of Forces in Several Planes.—We have in Exercise XII been dealing with three forces only, three forces lying in one plane; but the power we have thereby gained of finding the resultant of any two forces meeting at a point enables us to find the resultant of any number of forces meeting at a point, forces not necessarily confined to one plane but pointing in any direction. We first find the resultant of any two of the forces and replace the two forces by this resultant, thus reducing by one the number of forces. The same process continued leaves at last a single force, which is the resultant of all those originally given. For example, find the resultant

of the five forces, A, B, C, D, E (Fig. 23), all applied at the point O.

The resultant of	A	and	B	is	R_1;
"	"	"	R_1	"	C	"	R_2;
"	"	"	R_2	"	D	"	R_3;
"	"	"	R_3	"	E	"	R_4;

which last is the resultant of all the original forces, A, B, C, D, E. Observe that the outline of the resulting figure, from O around to P, the extremity of R, in the direction

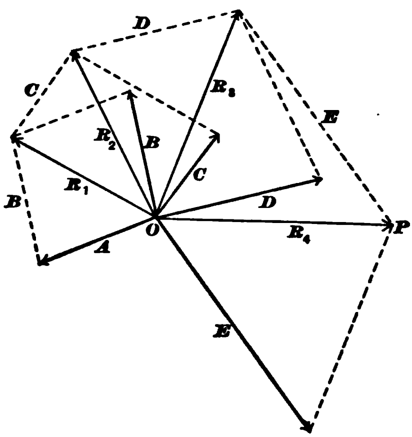


FIG. 23.

of its construction, is made up of lines equal and parallel to the arrows A, B, C, D, E. This, in fact, leads to a general and very convenient rule for finding the resultant of any number of forces meeting at a point. Put together the arrows representing the forces, tail to tip, without change of direction, each arrow being used but once. Then draw a straight line from the tail of the first arrow to the tip of the last. This line represents in magnitude and direction the resultant required. When this resultant is zero, that is, when the set of forces is in equilibrium, the

arrows when put together in the way just described form the outline of a closed polygon. This is called the *principle of the polygon of forces.*

For instance, suppose the six equal forces A, B, C, D, E, F, Fig. 24, to be applied at the common point O, so as

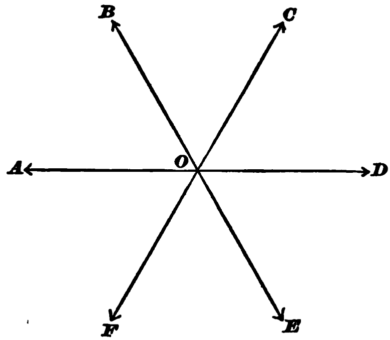


FIG. 24.

to make the angles at the centre equal to each other, and then, using the same forces, apply them tail to tip, as above described, so as to form a polygon, Fig. 25. The

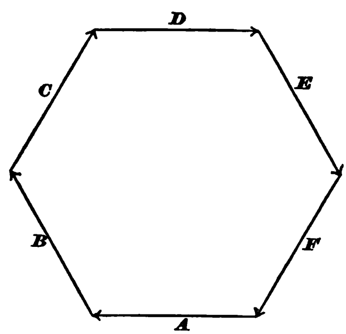


FIG. 25.

polygon is seen to be a regular hexagon. Mere inspection of the six forces in Fig. 24 will suffice to show that they pro-

duce equilibrium, and the fact that the polygon in Fig. 25 is a closed one shows the same thing in another way.

81. Forces all in the same Straight Line.—Cases of any number of forces lying all in the same straight line are so simple that they can be treated without the continued use of diagrams. They can be dealt with by mere addition and subtraction.

EXAMPLE. Let a force of 20 kgms. act due north, and two forces, one of 10 kgms. and one of 8 kgms., act due south along the same line with the 20 kgm. force. Find the magnitude and direction of the resultant and of the equilibrant of this set of forces.

82. Forces not meeting at One Point.—Often a set of forces whose lines of action do not meet at one point can, by finding resultants for some of the forces, be reduced to a set of two forces whose lines of action do meet at one point, and the resultant of the whole original set can thus be found. Thus in Fig. 26 the lines of the forces *A, B, C,*

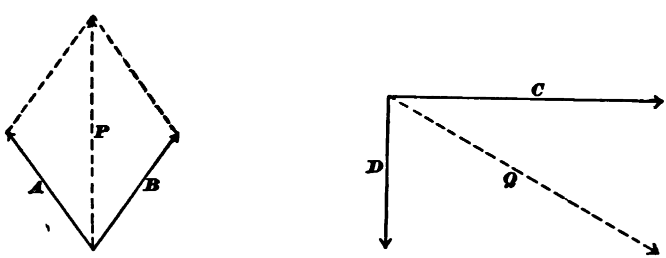


FIG. 26.

and *D* would not meet at a common point if they were continued. But the line of action of *P*, the resultant of *A* and *B*, crosses the line of action of *Q*, the resultant of *C* and *D*, and the resultant of these two forces *P* and *Q* is therefore easily found. If a set lying in one plane cannot be reduced to two forces crossing at one point, it can, as practice with diagrams will show, be reduced to two

or more forces whose lines would not meet however far continued, that is, parallel lines. In Exercises XIV and XV we shall study sets of parallel forces.

(Exercise XIII of the College pamphlet, which deals with the coefficient of friction, makes application of principles established in Exercise XII. It will, in this book, be placed just before Exercise XXI, which also makes an application of Exercise XII.)

83. Parallel Forces: Exercise XIV.—We will begin by finding, in Exercise XIV, the conditions of equilibrium of three parallel forces confined to one plane, the points of application lying all in a straight line which is at right angles with the lines of the forces. In seeking to discover and frame these conditions the student must consider three particulars to which his attention has already been called, viz., the magnitudes of the forces, the directions of the forces, the points of application of the forces. The set of conditions may be expressed in various forms. A good form, although not the briefest, consists of

1) A statement concerning the magnitude of the largest force as compared with the other two taken together;

2) A statement concerning the direction of the largest force in relation to the direction of the other two;

3) A statement concerning the point of application of the largest force with reference to the points of application of the other two. The every-day experience of students may have made them familiar with some of these conditions, but a careful performance of Exercise XIV will serve to make their ideas definite and systematic, and therefore applicable to such questions and problems as those which are to follow.

EXERCISE XIV.

PARALLEL FORCES IN ONE PLANE.

[Trowbridge, Exps. 55, 56, 57, and 58. Worthington, pp. 100, 101, and Exp. 7, p. 90. Lodge, Arts. 113–117.]

Apparatus: Three or more spring-balances of several lbs. capacity. A smooth flat board, protected from warping by cross-pieces at the ends, upon which a square 1 ft. on the side is laid off and divided into squares each 2 in. on the side, the lines being made with a pencil or a knife, a hole about $\frac{1}{8}$ in. in diameter being punched, not quite through the board, at every crossing. [Fig. XI.] Several iron pegs fitting rather closely in the holes in the board. Three smooth round marbles equal in size.

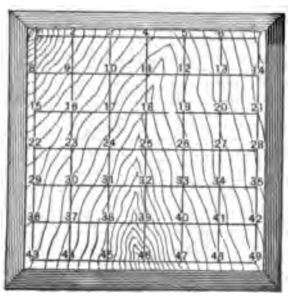


FIG. XI.

Put the marbles on a level table and lay the board, marked side up, on the marbles. Put three pegs into holes along any straight line on the board and attach a spring-balance by means of a string one or two feet long to each peg. [Fig. XII.] Pull upon the three spring-balances in lines at right-angles to the line of the pegs with any forces that will just balance each other, so that the board shall not move after the forces are fully

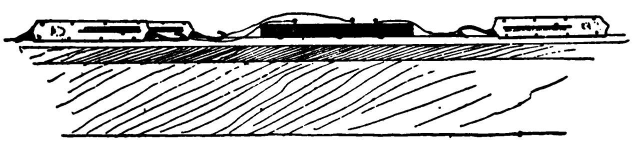


FIG. XII.

applied. (To insure good results in Exs. XIV, XV, and XVI, the lines and holes on the boards should be very accurately placed and the spring-balances should be used with great care as in Exs. II and XII. Each student should hold his balance firmly in both hands, resting his knuckles upon the table. He should, until equilibrium is secured, devote his main attention to keeping his balance exactly in line with the string to which it is attached and

keeping this string exactly over the line traced upon the board, taking care that it does not bear down on the edge of the board. His head meanwhile should be in such a position that when the word is given to read he may look directly down upon the index without moving anything except his eyes. It should be remembered, too, that errors due to the friction of the board upon its supports and inaccuracies of reading are less important when the forces exerted by the balances are considerable than when they are small.) A record should be made in the note-book of the conditions which produce equilibrium in this case. For this purpose a diagram of the board should be made in the note-book and the points indicating the holes should be numbered from 1 to 49. The record can then be made briefly thus:—

Hole.	Force.	Direction.	
. . .	. . .	$\uparrow$, $\downarrow$, $\rightarrow$, or $\leftarrow$	Equilibrium.
. . .	. . .	. . . .	
. . .	. . .	. . . .	

This being done, change the distances between the pegs, find and record the conditions of equilibrium for the new arrangement. Then move the pegs to some other straight line on the board, some line differing greatly in position from the one just used, and do as before.

Study the record of all these experiments with the purpose of obtaining a mastery of the principles which apply in them.

84. Cases in Composition and Resolution of Parallel Forces.—1) To a rod extending east and west are applied two forces, one of 10 lbs., the other of 5 lbs., due south, the points of application being 6 ft. apart. State the magnitude, direction, and point of application of their equilibrant; of their resultant. (See § 75.)

2) If the two forces applied to this rod were 6 lbs. due north and 15 lbs. due south, the points of application being 7 ft. apart, state what would be the magnitude, direction, and point of application of the equilibrant; of the resultant.

3) Let two forces, one of 10 lbs. due north, the other of 10 lbs. due south, be applied to the rod, the points of application being 6 ft. apart. Find, if possible, the mag-

nitude, direction, and point of application of the equilibrant; of the resultant. [A set of two equal forces like those just described, applied to the same body in exactly opposite directions, but not in the same line, constitute what is called in mechanics a *couple*. The student should bear carefully in mind what he here learns concerning the equilibrant and the resultant of a couple.]

Having now learned how to calculate the resultant of two parallel forces, we can easily find the resultant of any number of parallel forces, even when they do not lie in one plane. We have merely to find the resultant of any two of a given set and replace the two by this resultant, thus reducing the number engaged by one. This process continued gives at last a single resultant for the original set, unless it happens that the set is equivalent to a *couple* such as we have already discovered.

4) To the rod already mentioned let three forces be applied, one of 5 lbs. due north, one of 10 lbs. due south, one of 10 lbs. due north, the points of application being respectively 2, 4, and 6 ft. from the western end of the rod. State the magnitude, direction, and point of application of the equilibrant of these three forces; of their resultant.

5) Let the forces be 6 lbs. north, 8 lbs. south, 11 lbs. north, and 15 lbs. south, the points of application being respectively 3, 4, 5, and 6 ft. distant from the western end of the rod. State the magnitude, direction, and point of application of their equilibrant; of their resultant.

6) A board one foot square is placed horizontal and is loaded at the north-east corner with 10 lbs., at the south-east corner with 10 lbs., at the south-west corner with 20 lbs. Find the position where a single supporting point must be placed and the load which this point must bear, the weight of the board being disregarded.

The six preceding problems are cases in the composition of parallel forces. It is easy to apply the principles of Exercise XIV to the resolution of forces also.

7) A force of 20 lbs. acts due north from a point 8 ft. from the western end of the rod. State the magnitudes, directions, and points of application of three pairs of forces, each one of which pairs would just neutralize this force.

85. Parallel Forces Continued: Exercise XV.—We have now considered at some length the cases of parallel forces whose points of application lie all in one straight line at right-angles with the line of the forces. We now take up in Exercise XV cases of three parallel forces whose points of application lie in a broken line, or in a straight line not at right angles with the lines of the forces. After performing this exercise the student is advised to write out the conditions of equilibrium for such cases in a form as nearly as possible like that used in Exercise XIV, making use now of *perpendicular distances between lines of action* instead of distances between points of application. Having written out this set of conditions, let him find whether it will serve also for the cases of Exercise XIV.

EXERCISE XV.

Apparatus as in Ex. XIV.

FIRST PART.—Starting with one of the cases of equilibrium already recorded in Ex. XIV, experiment with this in view: to find, all the forces remaining unchanged in magnitude and direction and two of them retaining their points of application, what new points of application, if any, there are for the third force which will leave the system in equilibrium as before. Take each force in turn as the roving one, and record the trials in the notebook with any general conclusions that may be obtained.

SECOND PART.—Starting again with any one of the cases of equilibrium of Ex. XIV, try whether the equilibrium will continue when the three forces, unchanged in magnitude and points of application, and still remaining parallel to each other, are veered about into new directions. Record briefly the trials and the

result. Try the same with any case of equilibrium that may have been found in the First Part of this XVth Exercise. Record briefly the trials and the result.

(The ordinary lever, when the power acting and the resistance overcome are parallel, is simply a case of three parallel forces, one of which is exerted by a fixed support called the fulcrum.)

Before Ex. XVI is taken up the idea and definition of the moment of a force should be given with an application to Exs. XIV and XV, and the student should work out in his note-book the moments for at least two cases of equilibrium in Ex. XIV and two in Ex. XV. Let him in discussing each case take for the fulcrum each of the three pegs used in that case, and at least one other peg, in turn. This will make at least twelve moments calculated for each case of equilibrium. The calculations, however, are extremely simple. Call a moment positive if it tends to produce rotation around the given fulcrum in the direction of motion of the hands of a watch as seen from above; call a moment negative if it tends to produce rotation in the opposite direction around the fulcrum.

86. Moment of a Force.—Even if one set of three conditions is found which will apply to all the cases of Exercises XIV and XV, a still briefer form of these conditions can be written, if we become familiar with the idea of what is called the *moment* of a force, and make use of algebraic terms to a very slight extent. The moment of a force may be defined, in general terms, as *the tendency of that force to turn or rotate the body to which it is applied about some selected axis.* We will therefore consider at this point the difference between those forms of motion which are called respectively *translation* and *rotation.*

87. Definitions of Translation and Rotation; Examples.—Motion of translation is motion from one place to another; rotation is spinning or whirling motion. An ice-boat, sailing straight forward on the ice, or an ordinary hotel-elevator moving vertically upward or downward on its guides, illustrates motion of translation in a straight line.* The

* In either of these cases if we take into account the earth's mo-

revolution of a fly-wheel or of a circular saw illustrates well the properties of rotation uncombined with motion of translation, while a top which is spinning and meanwhile travels about, or a moving car-wheel, furnishes an example of combined translation and rotation.

The characteristic feature of pure motion of translation is that every point of the moving object travels in a straight line. If we suppose the body not to change its shape during translation, all straight lines rigidly connected with the body will remain parallel to their original direction during translation. Suppose a right cylinder to have a number of parallel straight lines ruled on its convex surface, at right angles to the bases, and suppose a number of other lines to be ruled on the bases. If the cylinder is rolled along on a perfectly level surface, the ruled lines on its convex surface will at all times remain parallel to their original direction, but those on the ends will successively form all kinds of angles with their original directions. The only way in which the parallelism of every line with its original direction could be maintained, with the cylinder in motion, would be to slide the cylinder without allowing it to roll or its axis to change its direction.

If a body rotates in such a way that some straight line drawn through it does not change its position, this line is called the *axis of rotation*. Frequently the axis of rotation is outside the body itself, being a line so placed that if the body were rigidly connected with it the line would not have to change position in consequence of the motion of the body. Thus, if we regard our earth as being at rest and the moon as revolving in a circular path about it, turning, as the moon does, always the same face toward the earth, the axis of the moon's rotation is a straight line passing

---

tions, the motion of the object in question through space would describe a complex curve.

through the centre of the earth and at right angles always with the straight line connecting the centre of the earth with that of the moon. Previous every-day experience will perhaps have taught every student that the effectiveness of a force in producing rotation about a given axis depends not merely upon the magnitude of the force, but also upon the distance of the line of the force from the axis of rotation, increasing as this distance increases. In fact, as we shall presently see, the importance, or *moment*, of a force with reference to rotation about a given axis, or the measure of the value of the force for the production or prevention of rotation about the given axis, is obtained by multiplying the number representing the force by that representing the perpendicular distance between its line of direction and the axis of rotation. Let the line P, Fig. 27, represent a certain force, and let O be the point where the required axis of rotation, perpendicular to the paper, is cut by a perpendicular drawn from the line of P, extended if necessary. Then $P \times OL$ is called the *moment* of P with reference to the given axis through O. Moments whose tendency is to produce rotation in the same direction with the hands of a clock, *clock-wise rotation*, we shall call *positive*, and those of opposite tendency *negative*. In Fig. 27 the moment of P is negative.

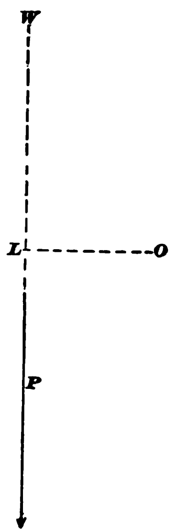


FIG. 27.

88. Illustrations of Moments.—In order to become familiar with moments and to test the truth of the assertion that the moment of a force, calculated according to the directions just given, measures the tendency of the force to produce rotation, we will consider one or two simple cases. The student will recognize these as being

cases of equilibrium of parallel forces (§ 83), and will see whether the idea of moments can be successfully applied in them. The first case is defined in Fig. 28, in which

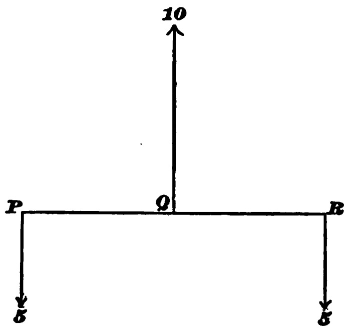


FIG. 28.

$PQ = QR = 2$. This being a case of equilibrium, the given forces produce *no translation and no rotation.* If an axis perpendicular to the plane of the forces were put through the line PR at any point, there would still be no rotation and yet in this case any one of the given forces, *taken alone,* would in general tend to turn the body to which the forces are applied about the axis. Now since the forces, taken together, do not produce this rotation, it must be that their several rotative influences neutralize each other, and if what has just been asserted (§§ 86 and 87) concerning moments is true, the algebraic sum of the three moments, calculated with respect to the given axis, must be zero.

Imagine, for instance, the axis to pierce the point Q. The moments, calculated with respect to this axis, are

$$\left.\begin{array}{r} -(5 \times 2) = -10 \\ 10 \times 0 = \quad 0 \\ +(5 \times 2) = +10 \end{array}\right\} = 0.$$

Let the axis pass through the point P. Then the moments are

$$\left.\begin{array}{r} 5 \times 0 = 0 \\ -(10 \times 2) = -20 \\ +(5 \times 4) = +20 \end{array}\right\} = 0.$$

Take the point R. The moments become

$$\left.\begin{array}{r} -(5 \times 4) = -20 \\ +(10 \times 2) = +20 \\ 5 \times 0 = 0 \end{array}\right\} = 0.$$

Take a somewhat less simple case of equilibrium, repre-

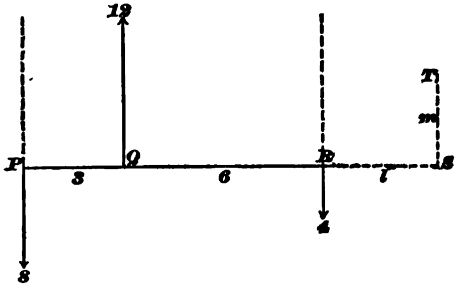


FIG. 29.

sented in Fig. 29. With respect to P we have the moments

$$\left.\begin{array}{r} 8 \times 0 = 0 \\ -(12 \times 3) = -36 \\ +(4 \times 9) = +36 \end{array}\right\} = 0.$$

With respect to Q,

$$\left.\begin{array}{r} -(8 \times 3) = -24 \\ 12 \times 0 = 0 \\ +(4 \times 6) = +24 \end{array}\right\} = 0.$$

With respect to R,

$$\left.\begin{array}{r} -(8 \times 9) = -72 \\ +(12 \times 6) = +72 \\ 4 \times 0 = 0 \end{array}\right\} = 0.$$

But we need not confine the axis to positions indicated by P, Q, or R. If we should put an axis anywhere else through the line PR, perpendicular to the plane of the forces, there would be no rotation about it, and the algebraic sum of the moments calculated with respect to it must be zero, if moments have the significance that is claimed for them. Let us therefore, in the case shown in the last figure, imagine an axis perpendicular to the plane of the forces to pass through this plane at a point lying in the extension of PR at a point S at a distance l to the right from R. The moments with respect to this axis are

$$\left.\begin{array}{lcl} -[\ 8 \times (9+l)] & = & -72 - \ 8l \\ +[12 \times (6+l)] & = & +72 + 12l \\ -\ \ (4 \times l) & = & \qquad -\ 4l \end{array}\right\} = 0.$$

Now as l can have any magnitude, and can be either negative or positive, without destroying this equation, it is shown that with the given system of forces the algebraic sum of the moments is zero for all axes, perpendicular to the plane of the forces, that pierce the line PR or its extension.

But imagine an axis to pass in the same direction through a point T at a distance m from S on a line drawn from S parallel to the forces. The moments with respect to this axis are, as a little consideration of the definition of moments will show, precisely the same as the moments with respect to the axis through S. Now as m, like l, may have any magnitude and be either positive or negative, we see that the algebraic sum of the moments of the given set of forces is zero with respect to any axis whatever perpendicular to the plane of the forces. If we were to try other sets of three parallel forces in equilibrium we should find the same statement to be true of all of them. This statement, therefore, expresses a necessary condition of equilibrium for such cases.

89. Restatement of Conditions of Equilibrium.—We can now replace the set of three conditions suggested in § 83 by a set of two conditions only. The first one of the three can be put into the following form:

1) *The algebraic sum of the forces, those in one direction being called positive and those in the opposite direction being called negative, must be zero.*

The second and third conditions of the first set are covered by the following condition concerning moments:

2) *The algebraic sum of the moments of the forces, taken with respect to any axis whatever, perpendicular to the plane of the forces, is zero.* This second condition really comprises the first, and is alone sufficient to cover the whole ground, but it is convenient to have the first one expressly written out.

The argument thus far has merely gone to show that every set of three parallel forces that is in equilibrium fulfils these two conditions. If the student will take set after set of three parallel forces *not in equilibrium,* he can soon convince himself of the additional fact that no such set of forces can fulfil these same conditions.

90. Application to Cases of More than Three Parallel Forces.—This set of conditions, like the one first suggested, has been written for cases of equilibrium of three parallel forces in one plane, but, unlike the first set, it will be found equally applicable to cases of equilibrium of any number of parallel forces in one plane. To illustrate its use let us find the equilibrant, and so the resultant, of the set of forces described in Fig. 30.

We will call forces acting downward on the page positive and those acting upward negative. Then we must, in order to give the proper signs to the moments, call distance towards the right positive, towards the left negative.

Applying condition 1), we find, x being the magnitude of the equilibrant,

$x + 4 - 6 + 8 - 10 = 0$, or $x = +4$, a downward force.

In applying condition 2) let us first calculate the moments with respect to an axis through the point P. We shall have

$$\left.\begin{array}{r} 4 \times 0 = \quad 0 \\ (-\ \ 6) \times (+2) = -12 \\ (+\ \ 8) \times (+4) = +32 \\ (-10) \times (+6) = -60 \\ (+\ \ 4) \times y \quad = \quad 4y \end{array}\right\} = 0;$$

the last being the moment of the equilibrant at an unknown distance, y, from P. We thus find $4y = 0 + 12 - 32 + 60 = 40$ and $y = 10$, to the right from P.

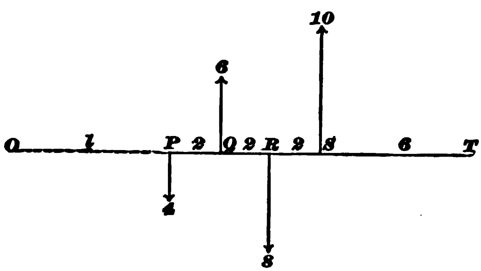


FIG. 30.

Suppose we were to choose R as the point for the imagined axis to pierce. We should have

$$\left.\begin{array}{r} (+\ \ 4) \times (-4) = -16 \\ (-\ \ 6) \times (-2) = +12 \\ (+\ \ 8) \times \quad 0 = \quad 0 \\ (-10) \times (+2) = -20 \\ (+\ \ 4) \times (y') \quad = \quad 4y' \end{array}\right\} = 0.$$

Then $4y' = +16 - 12 + 0 + 20 = 24$, or $y' = 6$.

But 6 to the right from R is equal to 10 to the right from P, so that the result reached is the same as before.

Let us now take the point T as the intersection of the plane by the imagined axis.

The moments are

$$\left.\begin{array}{l}(+\ 4) \times (-12) = -48 \\ (-\ 6) \times (-10) = +60 \\ (+\ 8) \times (-\ 8) = -64 \\ (-10) \times (-\ 6) = +60 \\ (+\ 4) \times y'' \quad = +4y''\end{array}\right\} = 0.$$

Whence $4y'' = 48 - 60 + 64 - 60 = -8$, and $y'' = -2$, which gives the same point of application for the resultant that has been found before. In fact, the result will be precisely the same whatever point in the plane is chosen to mark the position of the axis. With the "checker-board" rolling on marbles the class should test this result, determining whether the equilibrant, as found by calculation, does really balance the original set of forces. The resultant is at once known as soon as the equilibrant is found, being simply an equal but opposite force applied at the same point.

91. Calculation of Moments in Exercises XIV and XV.—The student should at this point calculate the moments required in the last paragraph of Exercise XV. The balance-readings, properly corrected, stand for the forces; the spaces between the holes on the board may be taken as units of measure in reckoning the distance from the line of application of the force to the *centre of moments*, that is, *the point with respect to which the moments are reckoned.* The word *fulcrum* as used in Exercise XV may be taken as equivalent to *centre of moments*, or *axis of rotation.*

92. Cases of Straight Levers.—In many practical cases of parallel forces in one plane there is a certain fixed axis about which alone rotation will occur, if it occurs at all. Such a case may be that of a crow-bar used to pry up a

weight, a pair of nut-crackers, a pair of tweezers, etc. Such instruments are classed in mechanics under the head of *straight levers,* or combinations of such levers. The axis about which rotation may take place is called the *fulcrum,* or *pivot.* In studying the laws of levers it is most convenient to consider cases of equilibrium, and the fulcrum, or pivot, is to be regarded as applying one of the forces necessary for equilibrium. We will consider a case as an example. Let a rod be balanced in a horizontal position upon a supporting pivot. If a weight of 8 lbs. be suspended from the rod at a point 2 feet from the pivot, find the magnitude and point of suspension of a second weight that would balance the first. (The weight of the rod itself is not to be considered.)

The moment of the first force with respect to the given pivot is $(8 \times 2) = 16$. We will suppose this to be a positive moment. The force, whatever its magnitude, which is applied to the rod by the pivot, has no tendency to turn the rod upon the pivot. The second weight must be such, and must be so applied, as to have a moment of -16; and any weight so applied as to have this moment will make equilibrium, for the equality of the moments, positive and negative, insures that the forces shall produce no *rotation* upon the pivot, and the upward push of the pivot will just balance the downward force of the weights, thus preventing the force from causing *translation* (§ 87). The second weight may be, for instance, 4 at a distance of 4 to the left from the pivot, or 2 at a distance of 8 to the left etc. In the first case the force which the pivot must exert is $(8 + 4)$ lbs., in the second case $(8 + 2)$ lbs. The variations of the force at the pivot in accordance with the variation of the other forces is a feature of the case which students are apt to overlook. It explains various puzzles, such as we shall find in the next article.

93. Moments of Couples.—Let one force of 8 lbs. down-

ward be applied to the horizontal rod at a point 4 ft. to the right from the pivot, and another force of 8 lbs. upward be applied at a point 4 ft. to the left from the pivot. These two forces constitute a couple, and we have learned (see § 84) that a couple cannot be neutralized by a single force. Suspend from the rod, however, at a point 2 ft. to the left from the pivot a weight of 32 lbs. Now calculate the moments with respect to the pivot. We have

$$\left.\begin{matrix}(+\ 8) \times (+\ 4) = +\ 32 \\ (-\ 8) \times (-\ 4) = +\ 32 \\ (+\ 32) \times (-\ 2) = -\ 64\end{matrix}\right\} = 0.$$

There will therefore be no rotation and, if the pivot can bear the load, no translation. We seem to have balanced a couple by means of a single force. A moment's thought, however, shows that the pivot is now bearing a load of 32 lbs. In other words, the pivot *pushes up* against the rod with a force of 32 lbs. We really have, therefore, when equilibrium is secured, one positive couple consisting of two 8 lb. forces 8 ft. apart, and one negative couple consisting of two 32 lb. forces 2 ft. apart. We have one couple balanced by another couple. If instead of the 32 lbs. 2 ft. from the pivot we had suspended 16 lbs. 4 ft. to the left from the pivot, the pivot itself would have borne a load of 16 lbs., and we should thus have a negative couple consisting of two 16 lb. forces 4 ft. apart, making equilibrium with the positive couple of two 8 lb. forces 8 ft. apart. In fact, whatever single weight we add making equilibrium with the first two forces of 8 lbs. each, we shall find that the pivot in consequence of the addition exerts a force equal to this weight, and that the equilibrium is really due to the conflict of two equal but opposite couples. The cases just detailed can be tested by means of spring-balances and the checker-board, Fig. XII.

In order to examine further the characteristics of couples,

let us imagine the two 8-lb. forces to be moved any unknown distance l towards the right without change of direction, the distance between them, which is called the *arm of the couple,* remaining unchanged. The moment of the couple with respect to the pivot will now be

$$\left.\begin{array}{l}(l+4)\times(+8)=+8l+32\\(l-4)\times(-8)=-8l+32\end{array}\right\}=+64.$$

Had we moved them any unknown distance m to the left, we should have

$$\left.\begin{array}{l}(-m+4)\times(+8)=-8m+32\\(-m-4)\times(-8)=+8m+32\end{array}\right\}=+64.$$

Taking any new points of application in the lines of the forces would, as we know (§ 88), make no difference in the moment of either force. We conclude, therefore, that the moment of a couple with regard to any axis perpendicular to the plane of the couple is equal to the product of the number representing one of the forces by the number representing the arm of the couple, and is not at all dependent upon the location of the axis, provided this is perpendicular to the plane of the forces. This statement may well be experimentally tested to a certain extent by means of the torsion apparatus of Exercise IV.

94. Parallel Forces not lying in One Plane.—We have now discussed pretty fully the laws of parallel forces *in one plane,*[1] but in very many cases we have to do with parallel forces not lying in one plane. For instance, the earth's

[1] Strictly there are no real cases in which a number of forces lie in one plane. In fact no one force exerted upon the strings that are used in the preceding exercises lies in one plane, for every string has dimensions in all directions. In using the phrase, *forces lying in one plane,* we have meant forces lying between closely adjacent parallel planes, each force being symmetrically placed with respect to the two planes. We shall continue to use the phrase with the same meaning as before.

attraction for any body of ordinary size consists practically of such forces. The matter is, therefore, important.

It is easy to see how the resultant of any set of parallel forces may be found, if they have a resultant. Any two of the forces lie in the same plane with each other. Find the resultant, then, of any two of the forces and replace the two by this resultant, thus diminishing by one the number of forces to be considered. This process continued will yield at last a single force, the resultant, or a *couple* (§ 84).

For studying the *moments* of parallel forces not in one plane one may use a common *rolling-pin*, mounted by its handles, in a horizontal position, in suitable well-greased bearings, a number of wooden rods six or eight inches long for attaching weights being set into the body of the roller at right-angles with the axis.

95. Exercise XVI.—In connection with Exercise XII we considered the case of several forces in one plane all meeting at the same point. In exercises XIV and XV we considered the case of several forces in one plane all parallel to each other. In the exercise which follows we shall consider experimentally the case of several forces all in one plane but not all parallel, being grouped in two sets which make right angles with each other. We might work out the laws for such a case from the experiments and discussions that precede, but it is better to reach them by more direct methods. The case (see Problem, p. 125) is of great practical importance, as will presently be seen.

EXERCISE XVI.

FORCES IN ONE PLANE BUT NOT ALL PARALLEL.

[Lodge, Arts. 126 and 127.]

Apparatus: The same as in Exercises XIV and XV, but with one more spring-balance.

Make at least three cases of equilibrium, with four spring-balances in use at once, pulling in four different directions, varying

the conditions as widely as practicable, but in all cases applying the forces along the lines marked on the board.

Record as in Exercise XIV.

Compare the magnitudes of the forces in each case of equilibrium. Is there any fixed relation between the magnitude of the force north and that of the simultaneous force south? between that of the force east and that of the force west? between that of the force north and that of the force east? etc.

Calculate in at least one of the cases of equilibrium the moments of the various forces with respect to some one of the pegs used and compare the sum of the positive moments with the sum of the negative moments. Do the same with respect to each of the other pegs used in this case of equilibrium and with respect to one or two other points on the board. State carefully any general laws inferred from the exercise.

96. Inferences from Exercise XVI.—The "general laws" to be inferred from this exercise include a statement of the conditions of equilibrium. Probably the most useful form of such a statement is that which resembles as closely as may be the set of conditions for the equilibrium of parallel forces that is given in § 89.

Questions and Problems.

1. Make a tabular view of the conditions of equilibrium for all the cases of equilibrium discussed in Chapter V, classifying them as follows:

   I. Two forces.

   II. Three or more forces.

   (*a*) Passing through a point.

   (*b*) Parallel.

   (*c*) Acting at different points and in different directions.

2. Find the resultant of a force *A* of 5 kgms. and a force *B* of 12 kgms., acting at right angles to each other.

3. Construct a diagram and thus find the resultant of

two forces, A of 5 kgms. and B of 8 kgms., acting at an angle of 60° with each other.[1]

4. Four forces act on the same point O, and with the following directions: A north, B east, C south, D west. $A > C$; $B > D$. Express in letters the value of the resultant. What is its general direction?

5. Resolve a force of 100 kgms. into two components acting at right angles to each other, one being twice as large as the other.

6. A loaded freight-car weighs 25 tons. It is held at rest on a track whose grade rises 1 ft. for every 200 ft. of horizontal distance. What force parallel to the track is needed to hold the car in place, the effect of friction being neglected?

7. Show by constructing the triangle of forces whether the three forces

$$A\ (= 5\text{ kgms.}),\ B\ (= 6\text{ kgms.}),\ C\ (= 12\text{ kgms.}),$$

can balance each other.

8. State the points of similarity and of difference between an equilibrant and a resultant.

9. If the angle COD, in Fig. 22, is of 30°, find the useful component of a pull of 400 kgms. in the direction OC.

10. Define the "moment of a force." A force of 5 lbs. and a force of 10 lbs. are applied in parallel but

[1] The student should use a protractor and a scale of equal parts, or find the angle of 60° by constructing an equilateral triangle. The instructor can, in case of any construction similar to the foregoing one, test the accuracy of the student's work by calculating the value of the required resultant by use of the following formula:

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha.$$

R is the resultant, P is one force, Q the other, and α the angle between the forces. It is important to notice that for obtuse angles the sign of the cosine will be negative.

opposite directions to a straight rigid bar, the distance between the points of application being 8 feet. What is the magnitude, direction, and point of application of a third force that would neutralize the effect of these two?

11. When any number of forces are acting upon a pivoted body, what is the necessary condition for their causing no rotation?

12. When a body is subjected to forces in one plane only, what conditions must be fulfilled in order that the body may neither slide nor turn?

13. If a carriage-wheel is resting upright upon the ground and is prevented from slipping at the bottom, how great a force applied horizontally at the top will just neutralize a force of 50 lbs. applied horizontally in the opposite direction at the centre of the wheel?

14. Let the board used in Exercises XIV, XV, XVI be placed with the lines 1–7, 8–14, etc., east and west.

(*a*) Let a force of 10 lbs. north be applied at point 28, and a force of 15 lbs. south be applied at point 26. Tell the direction, magnitude, and point of application of a third force that would just balance the first two.

(*b*) Let a force of 10 lbs. north be applied at point 28, and a force of 10 lbs. south be applied at point 26. Tell the direction, magnitude, and point of application of a third force that would just balance the two.

15. Where is the axis of rotation (§ 87) in the wheel of a moving carriage?

16. The forces P and Q, extending in the same direction, balance each other when applied at perpendicular distances M and N from the axis of rotation. Express the ratio of P and Q in terms of M and N.

17. Suppose in Fig. 30 all the forces to be in the same direction. Find (*a*) the amount of the resultant; (*b*) its point of application.

18. Suppose in the same figure everything to remain as

there shown, save that the force applied at S must have such a magnitude as to make the resultant 20. (*a*) What must be the magnitude of the force applied at S? (*b*) Where must the resultant be applied?

19. Construct a diagram for four forces, acting in four different directions, not producing equilibrium, and prove that equilibrium is impossible with the forces as given.

20. In driving a wood-screw with an ordinary screw-driver, (*a*) Show that the forces by which it is driven constitute a couple. (*b*) By what other couple or couples is this one opposed? (*c*) What happens when the opposing couples are equal? (*d*) Which couples have a positive and which a negative moment? (*e*) Why is a screw-driver with a very broad handle best for driving large screws?

21. Show how the principle of resolution of forces and the principle of the couple need to be applied to explain the action of a windmill.[1]

22. Suppose a bullet to be suspended by a very long and perfectly flexible thread.

(*a*) What two forces hold the bullet in place?

(*b*) Do these forces help or hinder, at the start, the action of a horizontal force applied to the bullet?

(*c*) Suppose it possible to remove a body from the influence of all forces but one; would that one produce more effect upon it than if the body were all the time subjected to a set of additional forces making equilibrium with each other?

23. Construct a diagram to show how equilibrium is secured in the case of a fishing-line, with a heavy sinker, dropped into a swift-flowing stream and allowed to move down-stream until the line stops it. (Neglect the action of the water on the line but consider the action of the water on the sinker.)

---

[1] Toy windmills illustrate the principles perfectly well.

24. A child is seated in a swing which is drawn forward from its position of rest and held motionless by a cord. Make a diagram to show the number and direction of the forces engaged, then assume that the child's weight is 50 lbs., and find by means of your diagram the magnitude of the other forces.

CHAPTER VI.

GRAVITY AND THE CENTRE OF GRAVITY. STABILITY.

97. Direction in which Gravity acts.—Frequent allusion has been made in the preceding pages to the force of gravity, and its familiar tendency to draw all objects towards the earth has been assumed to be known by the student. The direction of gravity may be found by means of a plumb-line, Fig. 31, and such a line will always be found to point nearly towards the centre of the earth. The line *OP*, or *OC*, passes through the centre of the plumb-bob *P*, and shows the *line of direction* in which gravity acts upon the bob taken as a whole.

Since the earth is nearly spherical, no two plumb-lines on the same hemisphere could be precisely parallel, except in cases where local disturbances of the direction of gravity are caused by the neighborhood of mountains, etc. Plumb-lines far apart are in general very different in direction, as is shown in Fig. 32. On account of the magnitude of the earth, however, the lines of direction of gravity acting upon the different parts of all ordinary objects form

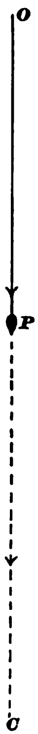


FIG. 31.

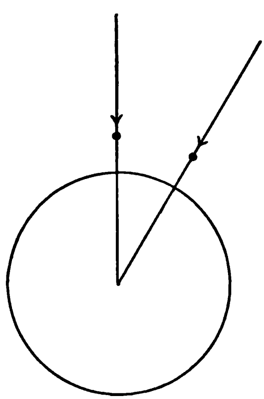


FIG. 32.

with each other angles too small to be measured; that is, they are practically parallel. The action of gravity upon bodies, therefore, gives rise to a large and important class of problems upon parallel forces.

98. Centre of Gravity.—Since gravity must be supposed to act upon every particle of a body, it is clear that the total effect of gravity upon any object is the resultant of a very great number of parallel components. The point of application for this resultant, or the centre about which, as a point of support, a body will balance in any and all positions, is called the *centre of gravity.*

It is an easy matter to find approximately by experiment the location of the centre of gravity. If a slender cylinder, such as a knitting-needle, is, after a series of trials, made to balance across a knife-edge, it is plain that the centre of gravity must lie in the centre of that cross-section of the needle which is vertically above the knife-edge. If a flat piece of card-board can be balanced on the point of a needle or a pin, in this case the centre of gravity lies half-way between the two surfaces of the card-board, vertically above the point of the needle. In a body which is of uniform density throughout the centre of gravity and centre of figure coincide; that is, the centre of gravity of a sphere is at its centre, that of a right cylinder is at the centre of its axis, that of a cube is at the intersection of its diagonals, and so on.

99. Determination of Centre of Gravity by Plumb-line.—Such an object as a sheet of writing-paper, of card-board, or of ordinary rolled zinc may safely be assumed to have a nearly uniform thickness throughout. If the shape of such an object is a regular figure, as a regular polygon, it is a simple matter to find its centre by geometry. If the shape is irregular, the centre of gravity may still be easily found by means of the plumb-line. Suppose the irregular piece of card-board *ABC*, Fig. 33, is to have its centre

of gravity ascertained. At any point, *O*, not near the centre of *ABC*, thrust through the card-board a fine needle, and hang from the needle a plumb-bob *P* by means of a slender thread. Work the needle about at *O* in the card-board until the latter swings with perfect freedom about the needle as an axis. Hold the needle so as to allow the card-board and the plumb-line to swing freely, then grasp the thread where it crosses the margin of the card-board at *D*, and trace with a pencil the course of the line *OD*. This line is the *line of direction* of the resultant for the forces due to gravity, acting on the several particles of *ABC* in their present position. Now repeat the experiment, choosing a new position for *O*, and trace the line of direction. The point where the two lines of direction intersect will mark the position of the centre of gravity, or, more exactly, the latter is half-way between the two surfaces of the card-board, on a line drawn through the point of intersection and perpendicular to the surfaces of *ABC*. The expressions "*centre of gravity of a line*," "*centre of gravity of a surface*," etc., when they are met with in books on mechanics, may be interpreted to mean centre of gravity of an extremely slender cylinder, of an extremely short parallelopiped, etc.

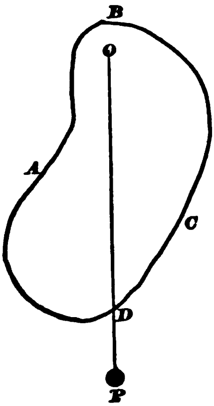


Fig. 33.

100. Stability.—When we speak of the stability of an object resting upon a supporting surface, we usually have in mind the angle through which it must be tilted to overturn it. Stability depends upon two factors, the area of the base upon which the body rests and the height of the centre of gravity above the base. In illustration of the influence of the area of the base, compare the angle trav-

ersed by the centre of gravity of a square prism, *A*, Fig. 34, 1 cm. square and 10 cm. long, stood on end and then overturned, with the angle traversed under similar conditions by the centre of gravity of a decimeter cube, *B*. The block

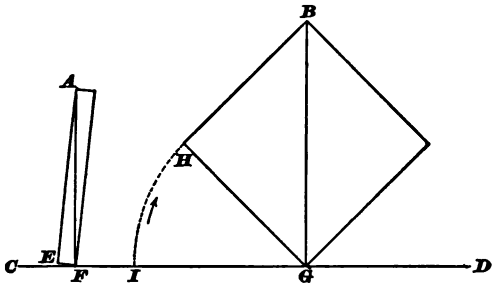


FIG. 34.

A will not of itself overturn until after its diagonal *AF* has passed the vertical position which it has in the figure. In the same way *B* will of itself overturn only after *BG* has passed the vertical position. For an overturn the

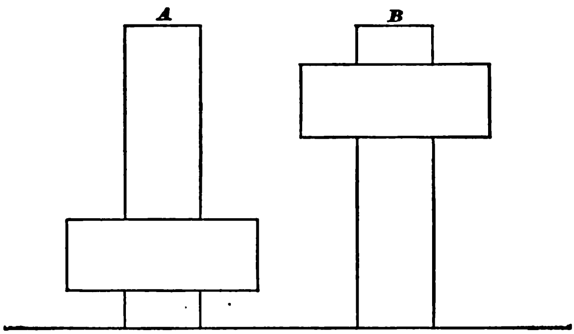


FIG. 35.

block *A* has to be tilted only through the very small angle *EFC*, while the block *B* has to be tilted through an angle *HGC* of 45°. In order to show the influence of the height of the centre of gravity on stability, suppose a heavy iron

nut, with a hole 1 cm. or more in diameter, to have a cylindrical piece of wood inserted in it. When the nut is near the supporting surface, as shown in *A*, Fig. 35, the stability will be considerable; when the nut is near the upper end of the stick, as in *B*, the stability will be very slight.

The common term *top-heavy*, applied to objects like *B*, well expresses the actual reason of their instability.

101. Kinds of Equilibrium.—Bodies are said to be in *stable*, in *unstable*, or in *neutral* equilibrium according to their behavior upon being tilted or displaced through an extremely small angle. If they tend to right themselves, that is, to return to their first position, their equilibrium is stable. If they tend to become overturned, their equilibrium is unstable. If they remain indifferently wherever placed, their equilibrium is neutral. A single solid, the cone, in different positions illustrates all three kinds of equilibrium: resting on its base (on a level surface) it is stable; on its apex, unstable; on its convex surface, neutral.

A simple diagram (which the student can construct for himself) will serve to show that the displacement, or tilting through a small angle, alluded to in the preceding section, leaves, in the case of stable equilibrium, the line of direction of the earth-pull from the centre of gravity downward, well, inside the boundary of the base; in the case of the unstable equilibrium throws the line of direction outside the boundary of the base; and in the case of neutral equilibrium leaves the line of direction unchanged in its relation to the base. In estimating the size of the base of an object it must be noticed that the base consists of the whole area included by straight lines which connect the adjacent points of the boundary of the support, two by two. For instance, the base of a three-legged stool, or a tripod-support for a photographer's camera, is a triangle, of which the foot of each leg forms a vertex.

If the plane surface on which a body rests is inclined, the condition of the object as regards stability may be changed. A sphere, for instance, on an inclined plane is no longer in equilibrium, because the line of direction falls outside of the base.

102. Practical Applications.—Regard should be had to the principles of stability in very many practical operations, as, for example, in the work of the engineer, the architect, the carriage-maker, furniture-manufacturer, and the car-builder, and in such operations as loading vessels, freight-cars, or wagons. In very many constructions the effort is made to secure stability by the use of a large base and, if practicable, by using the heaviest of the materials employed near the bottom of the structure. Perhaps the most remarkable instance of stability and comparative lightness in a tall structure is to be found in the case of the celebrated Eiffel tower at Paris. In this instance additional power of resistance to transverse forces was attained by planting the base of the structure in the ground.

103. Stability of Suspended Bodies.—Since the centre of gravity is subject to a constant tendency towards the earth's centre, it is evident that an object *W* hung from a point *O*, Fig. 36, about which it may swing freely, will be in stable equilibrium when the centre of gravity of *W* falls in the line *OG* which represents the direction of one of the earth's radii. *W'*, then, is the position of rest for the weight when left free to settle. The plumb-line and the pendulum illustrate this principle.

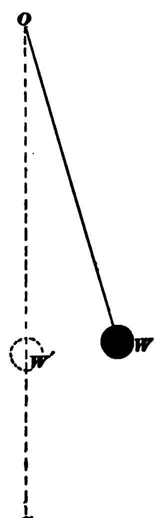


Fig. 36.

104. Centre of Buoyancy: Stability of Floating Bodies.—In a body of liquid at rest any portion not at the bottom may be regarded as *floating* in the remainder.

The liquid pressure against any body wholly or partly submerged depends upon the shape and position of the submerged part, but does not depend upon its kind of material. The resultant of the liquid pressure against the submerged part, whatever its density, is a force upward through what would be the centre of gravity of this submerged part if it were of the same density as the liquid. This point is called the *centre of gravity of the displaced liquid*, or the *centre of buoyancy*.

When a floating body, for instance the hull of a vessel, is in equilibrium, the centre of buoyancy lies in the same vertical line with the centre of gravity of the body. When the vessel rolls, the upward push of the water and the pull of gravity constitute a *couple* which tends to right the

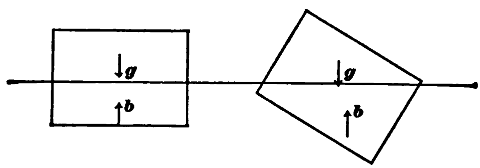


FIG. 37.

vessel. It is an interesting fact, which the student can verify for himself by floating a light board, that a floating body may be in very stable equilibrium with the centre of gravity above the centre of buoyancy, and even above the surface of the liquid. The fact is in such cases that when the body is tipped, the centre of buoyancy, owing to the change of figure of the displaced body of water, moves in the direction of the tipping more rapidly than the centre of gravity, as Fig. 37 will show, g being the centre of gravity and b the centre of buoyancy.

105. EXERCISE XVII.

CENTRE OF GRAVITY: INFLUENCE OF THE WEIGHT OF A LEVER.[1]

[Worthington, Exp. 5, p. 89. Lodge, Arts. 118, 119, 132–135.]

Apparatus: The two boards described in Exs. XIV and XVIII. Two iron weights, about 4 and 8 lbs. respectively. (Safety-valve weights, which cost four or five cents a lb., are most convenient for this purpose, as they have a handle in which is a slot, which makes it easy to suspend them.) A lead-pencil. A meter-rod.

Drive a nail into each end of the long board. Weigh the two boards together and each of the iron weights. Lay the long board flat on a table with one end projecting slightly beyond the edge of the table. Hang the smallest weight upon the nail in this end of the board. Lay the square board on the other end of the long board, keep it always in the same place, and regard the two henceforth as one body. Put now the lead-pencil, reaching straight across the board, between the board and the table to serve as a fulcrum, in such a position that the board as now weighted will balance, as nearly as may be, upon it. Considering the weight of the iron ball as one downward force and the weight of the lever (the two boards together) as the other downward force, find, by applying the teachings of Ex. XIV, at what distance from the fulcrum the weight of the lever, if applied at a single point, would have to act in order to produce the state of equilibrium that is observed. Mark on the board the point thus designated. Record in the note-book the distance of this point from the fulcrum and the distance of the fulcrum from the point of suspension of the iron weight. Then use the other iron ball and proceed as in the case just described. If time permits, try a case in which both the iron balls are used at once, the nails at both ends of the board being brought into service. Finally remove the balls and balance

---

[1] The student is expected to obtain the idea of the centre of gravity by means of familiar experiments with suspended card-board, etc. (Trowbridge, Exps. 53 and 54. Worthington, Exp. 1, p. 101.) The case of the lever is, however, so different in *appearance* from that of the suspended card-board that it may well be treated by itself. By performing this experiment with the lever the student will acquire a confidence in using the idea of centre of gravity which he might otherwise fail to get.

the lever alone, thus finding in what cross-section of the lever its centre of gravity lies. Record in the note-book. Consider now from the whole course of this experiment whether the weight of the lever has or has not in all cases acted as if applied at the centre of gravity of the lever.

(Instead of the two boards mentioned above, the wooden support-stand described in Ex. VI may be used with much advantage, the rod being placed horizontal and the weights being suspended from it at any point by means of a loop of wire or string.) [See Fig. XIII.]

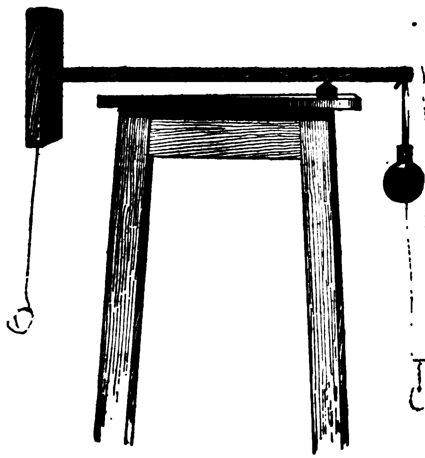


FIG. XIII

In this exercise the student is advised to make all his measurements from one end of the lever instead of from the fulcrum, the position of which varies from one case of equilibrium to another. These distances may be made and recorded for several cases of equilibrium before any calculations are made. Then in the calculations the student should determine for each case of equilibrium how great a pressure is exerted upon the lever *by the support*, and should consider the moment of this force with the moments of the others, taking for the *axis of moments* a line through

that end of the lever from which the distances have been measured.

Questions and Problems.

1. Show by diagrams that a brick laid on a table-top may have three positions, one of greatest, one of least, and one of medium stability. (Disregard the scarcely possible case of balancing it upon an edge or corner.)

2. Show whether the apparatus of Exercise XVII, when supported on its fulcrum, was in stable, unstable, or neutral equilibrium. How would the equilibrium have been affected by fastening a weight to each end of the lever by means of a rigid rod leading downward?

3. A telegraph-pole is made of three hollow iron cylinders joined end to end. Each cylinder is 3 m. long. The lowest weighs 200 kgm., the middle one 100 kgm., and the uppermost 50 kgm. Find the centre of gravity of the pole. (See last paragraph in § 105.)

4. A spherical ball is put into a V-shaped trough, slanted so that the ball rolls down. Supposing the effect of the earth's attraction to be applied to the centre of the ball, show why the conditions for equilibrium are not fulfilled.

5. A cube of wood 10 cm. on each edge and of specific gravity 0.5 is covered on one side by a plate of metal 10 cm. square and 1 cm. thick, of specific gravity 5. How far from the outer surface of the metal plate is the centre of gravity of the whole?

6. Show what forces are at work to maintain equilibrium in the case of a vessel heeling (tilting) over somewhat under the action of a wind which blows at right-angles to her course.

7. A board *AB*, 3 m. long and 30 cm. wide, weighs 5 kgm. On the end *A*, just flush with the end and the sides

of AB, is placed another board 30 cm. sq., whose weight is 1 kgm.

(*a*) Find the centre of gravity of the combination.

(*b*) Find what weight must be hung from the end B to bring the centre of gravity to the middle of the pine board.

8. *ABCD*, Fig. 38, is a beam projecting horizontally

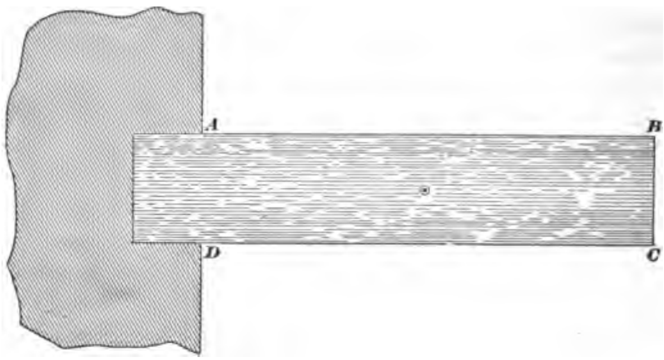


FIG. 38.

from a wall. What is the state of stress of the fibres of the beam *at the section AD?* We may, as Ex. XVII has shown, regard the whole weight of the projecting part of the beam as applied at O, the centre of gravity of this part. We will call this weight W. We know that in order for this part of the beam to be supported an upward force equal to W must be applied to it. Where can this force be applied? It must be applied at the section AD, for this is the only place where anything, the air excepted, touches the outer beam. We have, then, a downward force W applied at O and an equal upward force applied at the section AD. This constitutes a couple having a positive moment. It must be neutralized by a couple having an equal negative moment. Where is this couple to be found? Only at the section AD. A tension of the fibres in the upper part of

this section and a pressure of those in the lower part will produce such a couple, and this is the state of things that we find (§§ 17 to 21). The fibres tend to pull apart at the top of the beam and tend to be crushed and squeezed out at the lower part. Moreover, it is easy to see that a thick beam has an advantage over a thin one, not merely because it has more fibres to be stretched or compressed, but also because the top and bottom fibres, being relatively far apart, have a long *arm* for their couple and therefore work to better advantage. A common house-door supported by hinges presents a case very similar to this.

PROBLEM. A door weighing 40 lbs. has its centre of gravity $1\frac{1}{2}$ ft. distant from a vertical line passing through the pivots of the two supporting hinges. The hinges are of the simplest character and 6 ft. apart from centre to centre. The load is divided equally between the two hinges, but the upper one is supposed to take all the horizontal pull, and the lower one all the horizontal push, caused by the weight of the door. How great is the horizontal pull upon the upper hinge? How great is the horizontal push upon the lower hinge? Find by the graphical method of Exercise XII the direction and magnitude of the total force which the upper hinge *applies to the door.* Find by the same method the total force which the lower hinge applies to the door. Find last by the same method the magnitude, direction, and point of application of the resultant of all the forces applied to the door by both hinges.

CHAPTER VII.

MOTION.

106. Association of Matter, Force, and Motion.—In the preceding chapters it has been necessary to speak of various forms of matter as things familiar to every one, and to consider some of the effects which forces produce on matter. It is well known that the production or change of motion is a very common consequence of the application of any force to any portion of matter. We shall now begin a more careful study of the relation between matter, force, and motion than the student has yet made. This study will require us to look sharply to the meaning of the words *motion* and *velocity*.

107. All Motion Relative.—It may seem at first sight that it would be a simple affair to define the *absolute motion* of a body, that is, to state just how it travels with reference to some fixed point. But there is no point which we know to be fixed. The earth and all the other members of the solar system, not excepting the sun itself, have very complicated motions of their own; and since even the stars and other most distant heavenly bodies appear to have a general drift through space, it is quite impossible to determine the absolute motion of any object. The most that can be done is to observe the departure along a given line from some chosen starting-point which is, for the purpose we have in view, to be regarded as fixed. Thus we might describe the motion of the conductor down the aisle of a car in motion, neglecting in our account the rocking movement of the car from side to side, its pitching, and its gen-

eral motion of translation along the track; neglecting, in fact, everything except the *difference* between the motion of the conductor and the motion of the point from which he started. This difference of motion is called *relative motion*, and all motion that we can describe is relative motion. When nothing is said to the contrary it will be understood that motions spoken of in this book will be relative motions with respect to the earth regarded as fixed.

108. Composition of Motions.—The student may have gathered from what precedes the inference that motions may be compounded or resolved as forces can be (§ 75). This is true; and nearly all that was stated in Chapter V in regard to the triangle of forces, the parallelogram of forces, and the polygon of forces may be re-stated, using the word *motion* instead of force, and the words *rest* or *zero motion* instead of *equilibrium*. An interesting and important class of recording instruments make use of this fact. Such are the self-recording barometer, the sphygmograph, the plethysmograph, used by physiologists for studying the circulation of the blood, and a number of instruments used in the investigation of sounding bodies. In all these pieces of apparatus a moving point is, sometimes by the aid of photography, made to record its motion upon a surface which is itself in motion. The manner in which the motion of the recording surface affects the trace left by the moving point is well shown by a simple phenomenon of frequent occurrence in every-day life. When

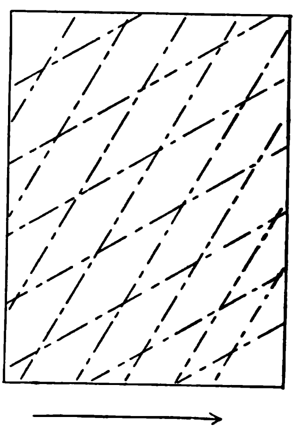


FIG. 39.

rain falls on a comparatively windless day against the window of a railroad-car at rest, the drops trickle vertically down the surface of the pane. But when the car is in motion the line traversed becomes more and more oblique as the speed of the car increases. Fig. 39 contains a graphic record of the course of the drops for two different velocities of the car. The arrow shows the direction of the car's motion.

109. Velocity.—*Velocity is rate of motion.*

When velocity is unchanging it is measured by the number of units of space traversed in a unit of time. When velocity is changing it may be defined as equal, at any instant, to the distance the body would move in a unit of time if the rate of motion which it has at that instant were maintained unchanged. The choice of units depends somewhat upon the kind of motion and the purpose for which it is to be estimated or measured. The speed of rail-road trains, of steamers, of pedestrians, and so on, is usually reckoned, in this country, in miles per hour, the speed of bullets or cannon-balls in feet per second, and the rate of transmission of electrical impulses through wires or of light through air in miles per second. For strictly scientific purposes, however, centimeters per second is a form of statement frequently employed.

110. Condition required for Change of Motion; Inertia.—In the next three exercises we shall be dealing with objects which are neither at rest nor in uniform motion, but are changing from rest to motion, or from one state of motion to a different state. In these cases we shall encounter a very important property of matter which we have almost totally disregarded thus far. Before we give a name to this property let us assure ourselves that it exists.

Suspend an iron ball weighing not less than 10 lbs. from a firm support by a long stout string, so that it may be free to swing at a slight push. Attach a thread to the ball and

with a gentle horizontal pull set it gradually in motion. Stop the ball and again with the thread set it in motion, this time more suddenly. Repeat, starting more and more suddenly each time, until the thread breaks. Then take a string that will bear the weight of several pounds, attach it to the ball and break it as the thread was broken. Take finally a string that will bear considerably more than the weight of the ball and attach it to the latter. Wind the free end of the string several times about some heavy object held in the hand and then, the string being long enough to give the arm free play, attempt to set the ball in horizontal motion with the greatest possible suddenness. The string will probably be broken, while the ball will move but little. (Make sure that the suspension of the ball is very strong.) The hanging back of the ball, which is so extremely obvious in these experiments, cannot be accounted for by its weight, for weight, in its strict sense, is merely the earth's attraction for the ball, and this downward force cannot, save in some indirect way, oppose horizontal motion. (It is true that the ball begins to rise a little when it swings from its position of rest, but this rise is very slight at first and that component of the earth's attraction which is parallel to the path of the ball is therefore very small.) Nor is the behavior of the ball to be accounted for by the resistance of the air, or by the action of any other opposing force applied to the ball by known outside agencies. We must conclude from the experiments described, and other similar ones in which the motion of bodies is arrested or changed in direction (see § 114), that IT BELONGS TO THE NATURE OF MATTER TO REQUIRE FORCE TO SET IT IN MOTION OR TO CHANGE ITS MOTION IN MAGNITUDE OR DIRECTION,[1] *little force if the change of mo-*

---

[1] The words in small capitals are substantially equivalent to what is called Newton's *First Law of Motion*.

tion is very gradual, greater and greater force as the suddenness of the change is increased. This is one of the most important facts that we know about matter, and it is convenient to have a single word which will call it to our minds. We therefore say that this behavior of matter manifests a certain property which we will agree to call *inertia.* To say this, to give the name inertia to a supposed property, does not explain the facts. It is merely a convenience, for it enables us to use the one chosen word, *inertia,* in place of phrases or sentences which would otherwise be required. The word inertia, then, will call up to our minds the tendency of matter at rest to remain at rest, of matter in motion to move on with uniform speed in a straight line. It is encountered when one kicks a cannon-ball at rest upon a smooth and level floor. It is encountered when the cannon-ball pierces the armor of a ship. In either phase it is important.

If we now suspend a much larger or much smaller iron weight than the one just used and repeat some of the experiments with it, we shall find that to start the larger weight with equal suddenness requires a greater force. This being the case, we naturally inquire whether we cannot use the property of inertia in comparing quantities of matter, in determining *how much* is the quantity of matter in one body compared with that of a second body, which process we call *measuring* the quantity of matter in the first body.

111. Quantity of Matter; Mass: Exercise XVIII.— We *commonly* compare bodies as to quantity either by volume or by weight, and we are familiar with the fact that the results usually differ in accordance with the method of comparison, things which are equal in volume being unequal in weight. Each method has its advantages and will always be used, although there is a general tendency to use the weight method more and more. For

instance, grain, which was formerly sold by the bushel, or other volume-measure, is now generally sold by weight. But if we try a method of comparison, or measurement, based on inertia, will the results obtained by this method agree with those obtained by the volume method or those obtained by the weight method, or will they differ from both? Experiment only can answer this question, and to experiment we shall appeal in the following exercise.[1] The word *mass*, which is used in the description of the exercise, is equivalent to the phrase *quantity of matter as determined by the inertia-test.* But if it proves to be the case that the inertia method of comparison gives the same results that are given by some more convenient method, this more convenient method will practically be used instead of the one appealing directly to the inertia-test, even when the object is to compare *masses*.

EXERCISE XVIII.

INERTIA: COMPARISON OF MASSES.

References for Exs. XVIII–XXII: Lodge, Arts. 29–35, 1–12, 18–27, 41–49. 58–62, 74–94. Hall's *Elementary Ideas, Definitions and Laws in Dynamics.*[2]

Apparatus: Two chalk-boxes nearly filled with nails (in one of which boxes is placed a piece of iron or lead weighing about 1 kgrm., in order that the eye may not be able to decide which is the greater mass). Two roller-skates[3] of the simplest construction, well oiled so as to run as freely as practicable. Two smooth, straight, white-pine boards, each about 2 m. long, about

---

[1] For making the test here indicated satisfactory the two objects to be set in motion should be very different in density, which is not necessarily the case when the directions given under Exercise XVIII are followed. It would be well to use in one box iron weights and in the other box stones.

[2] C. W. Sever, Cambridge.

[3] The skates should be so adjusted as to run straight when set in motion along a table. This may require a resetting of the trucks, as they are frequently so placed as to give a tendency to motion in a circle.

15 cm. wide, and about 3 cm. thick. Blocks, boxes, or other means of elevating one end of each board 5 or 10 cm. A meter-rod or measuring-tape. Two strips of india rubber equal in length, about 75 cm., about 1 cm. wide, and about 1 mm. thick. (Pieces of small black rubber tubing serve well.)[1]

Put these strips in line, the end of one slightly overlapping the end of the other. Clamp these joining ends firmly together and then stretch the whole line until it is about twice as long as when unstretched, and fasten it in this condition. Probably one strip will have stretched less than the other. If so, trim its edges until the clamp which holds the joining ends of the strips is drawn to the middle point of the whole line. The strips may now be separated. Hereafter they will, until their properties suffer some change, exert equal pulls when equally stretched. Take now the roller-skates. Place these side by side on a level surface and to each skate attach one end of one of the rubber strips under a block of wood fastened with screws to the heel of the skate, making the strips parallel to each other and to the running direction of the skates. Fasten the free ends of the strips to some fixed object in such a way that the strips, when stretched by moving the skates, may be parallel to the surface upon which the skates rest. Upon each skate fasten one of the chalk-boxes[2] containing nails.

If it were not for the disturbing action of friction we could now apply the test with the greatest ease and decide at once with considerable accuracy which carriage, with its load, has the greater mass. It would be necessary merely to draw the skates along side by side so as to stretch the attached strips equally and to a considerable extent, then release them at the same time and observe which one was drawn back the more slowly. By transferring nails from one box to the other we could make the two loads equal in mass, which condition of things would be indicated by the skates keeping together in their motion.

But friction complicates the case. To meet this difficulty make each skate to run on one of the boards, so inclined that the skate with its load will, once started, the rubber strip being detached,

---

[1] Elastic tape sold at dry-goods stores can be used.

[2] A better carriage is made by taking the trucks from the skate and fastening them to a strong box in the manner shown in Fig. XIV.

roll with uniform velocity[1] down the incline. If one skate naturally runs less freely than the other, this skate will require the steeper incline. The influence of friction is now neutralized by the slight pull of gravity along the inclines and the elastic strips have to deal with the inertia alone, so that the test with the rubber strips may proceed as if with frictionless skates rolling upon a level surface.

When apparent equality has been obtained, compare by means of a balance the weights of the two loaded skates.

112. Absolute Units of Force.—We see in Exercise XVIII that we can compare masses by testing their acceleration with forces known to be equal. We can also compare forces by testing their effects upon masses known to be equal. Indeed the mathematical physicist takes as his favorite units of force those which come under the following definition: *The unit force is that which, acting upon the unit mass for the unit time, imparts to it the unit velocity.*

A particular unit falling under this definition is called the *poundal.* Its particular definition is as follows: *A poundal is that force which, acting alone upon a one-pound mass for one second, would impart to it a velocity of one foot per second,* that is, would, if ceasing at the end of one second, leave the one-pound mass moving at the rate of one foot per second. This does not imply that the one-pound mass would move one foot during the first second of the action of the force.

(The poundal is not the same in magnitude as the pound-force, that is, the force which gravity exerts upon a one-pound mass and which is itself generally called a pound (§ 4). The numerical relation between the two may be inferred from what follows concerning falling bodies (§ 113).

The dyne is that unit force which acting for one second

---

[1] This cannot be perfectly accomplished, but the incline can be made such that friction will neutralize only a small part of the pull exerted by the rubber strip.

upon a mass of one gram would impart to it a velocity of one centimeter per second.

Units of force which, like the poundal and the dyne, are defined with reference to inertia and not with reference to gravitation are called generally *absolute units of force. Inertia-units of force* would be a more expressive name.

Experiment shows that f units of force will impart to a given mass in a given time f times as great a velocity as one of the same units of force, and that a given force will in a given time impart to m units of mass $\frac{1}{m}$ times as great a velocity as the same force would impart in the same time to one of the same units of mass.

113. Laws of Falling Bodies.—It is customary in elementary text-books of physics to give a great deal of attention to the so-called *laws of falling bodies* and to problems illustrating these laws. The most important reason for doing this is that these laws are a study of a very simple case showing the behavior of a body under the combined influence of an applied force and its own inertia. The case is simple because, under the conditions usually assumed, namely, free motion in a vacuum, we have only one applied force and that one practically uniform, a force practically unchanging in magnitude and direction. (This is not self-evident. It will doubtless be admitted without proof that the direction of the earth's attraction for a body remains unchanged during the body's fall, but it is natural to doubt whether this attraction is just as great when the body is in motion as when it is at rest. One can test this matter in a rough way and within a narrow range of velocity by ascending and descending stairs or going up and down in an elevator, at a uniform speed, holding in the hand a loaded spring-balance and noting whether the position of the index is the same as when the observer is standing still.) The so-called laws of falling bodies under these conditions are really

comprised within a single law which is as follows: *The increase of velocity is uniform;* that is, if the body acquires in falling one second a velocity g, which unchanged would carry it g units of distance in the next second, the body in falling t seconds would acquire a velocity $g \times t$, that is, a velocity which unchanged would carry the body $g \times t$ units of distance in the next second.

If, then, v represents the velocity *acquired* in t seconds by a body falling freely, we have

$$v = g \times t. \tag{A}$$

The value expressed by the letter g is called *the acceleration of gravity,* that is, the increase which gravity produces each second in the velocity of the falling body. It is about $32\frac{1}{6}$ when the unit of distance is a foot, and about 980 when the unit of distance is a centimeter.

If at the beginning of the time t the body has already a vertical velocity v_1, the change of velocity will be just as great as if it started from rest, so that the velocity at the end of t seconds will be $v = v_1 + g \times t$.

The case in which the body is started with an upward velocity requires the formula $v = -v_1 + g \times t$, velocities upward being called negative velocities.

To find the vertical distance travelled in a given time by a falling body we shall make use of a graphical method. Let us first consider how we can represent graphically the distance traversed in t seconds by a body moving with a uniform velocity v. Draw a horizontal line OX, Fig. 40, the length of which shall be t units. Draw a vertical line OY, the length of which shall be v units. The area of the parallelogram $OYPX$ is equal to the distance $v \times t$ travelled by the body in the time t.

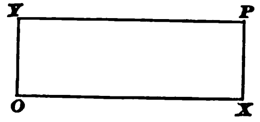


FIG. 40.

If a body travels t_1 seconds with a velocity v_1, t_2 seconds

with a velocity v_2, etc., the total distance travelled is represented by the area of the figure made by joining all the rectangles v_1t_1, v_2t_2, v_3t_3, etc., Fig. 41. Now if the

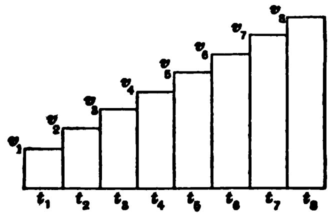


FIG. 41.

periods t_1, t_2, etc., are made equal and very short, and if the increments of velocity are all equal, the figure made by joining the rectangles will approach as a limit a shape like the boundary of Fig. 42. But this figure evidently repre-

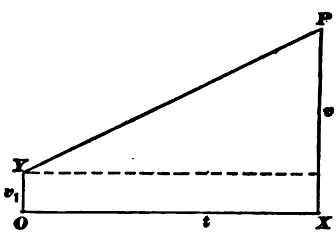


FIG. 42.

sents the case of an original velocity v_1 increased uniformly to a velocity v in t seconds. The distance traversed is represented by the area of the trapezoid, which we know to be $\frac{t \times (v_1 + v)}{2}$. It is numerically equal to the product of t by the *average,* or *mean,* velocity.

If the case is that of a falling body, $v = v_1 + gt$, and we have the distance traversed:

$$s = t \times \frac{(v_1 + v_1 + gt)}{2} = v_1t + \tfrac{1}{2}gt^2.$$

The term v_1t represents the distance the body would go in t seconds if the velocity remained v_1 throughout the time.

When v_1 is negative, that is, an upward velocity, the distance traversed in t seconds is

$$s = -v_1t + \tfrac{1}{2}gt^2.$$

When v_1 is 0, that is, when the body starts from rest under the influence of gravity, we have

$$s = \tfrac{1}{2}gt^2. \qquad \text{(B)}$$

A third very useful equation, obtained from (A) and (B) by eliminating t, is

$$v^2 = 2gs. \qquad \text{(C)}$$

This equation shows the relation between distance traversed and velocity acquired in the case of a body falling from rest. From another point of view it shows the relation between distance traversed and velocity lost in the case of a body starting upward with a velocity v and rising until its velocity is 0. One process is the exact reverse of the other and the two require exactly equal times, for a body loses velocity in going up exactly as fast as it gains velocity going down.

Direct experiments upon the laws of freely falling bodies are rather difficult to make with simple apparatus on account of the very short time that is required for falling ordinary experimental distances, but certain other experiments to illustrate the action of a uniform force in causing change of motion can be made with the familiar Attwood's machine, the construction of which is described in most of the larger treatises upon physics. This apparatus consists essentially of two nearly equal weights fastened to the ends of a thread passing over a pulley with horizontal axis, together with an attachment for beating seconds

and for determining accurately the time of the beginning and the end of motion. The rate of increase of velocity with this machine is uniform, but is much less than in the case of a freely falling body. Another device consists of an inclined plane down which some body is permitted to roll or slide. If the body rolls, however, the case is seriously complicated, in appearance at least, by the very fact of the rotational motion acquired, and if the body slides friction is likely to be troublesome. In view of all the difficulties in the way of successful experiments upon the effects of a uniform force and successful interpretation of these experiments, we shall not attempt in this book an experimental treatment of the matter.

Problems in Uniform Acceleration.

1. How far will a freely falling body travel in the seventh second? What will be its velocity at the end of that second? What will be the total distance traversed during the seven seconds? (The body is supposed to start from rest. Take $g = 32$ (see § 113).)

2. Repeat the calculations of the preceding problem, supposing the body to have had an initial velocity of 10 ft. per second vertically downward.

3. Repeat once more, supposing the body to have had an initial velocity of 50 ft. per second vertically upward; of 400 ft. per second vertically upward. Be careful to show in your answers whether the final direction is upward or downward.

4. If a body falls for 20 seconds and is then instantly released from the influence of gravity, how far will it fall in the twenty-first second?

5. A bullet is shot vertically upward with an initial velocity of 300 m. per second; how long will it continue to rise? How far will it rise? (Take $g = 9{\cdot}80$ (see § 113).)

6. How far would the bullet of (5) rise during the tenth second?

7. A freely falling body falls at the rate of a kilometer per minute; how long has it been falling?

8. A block on an incline slides down without friction. At the end of 4 seconds it is travelling at the rate of 3 m. per second. How long after starting will it take to reach a point 20 m. from the starting-point?

9. How far would the block of (8) slide in the third second?

10. If a body slides down a frictionless incline the length of which is 100 ft., the base 80 ft., and the height 60 ft., what velocity would it acquire in one second?

11. What was the value in feet per second of the uniform accelerating force under the continued influence of which a body travelled a mile in three minutes, starting from rest?

12. If a negative accelerating force were at the end of the three minutes substituted for the positive force of (11), and allowed to act for two minutes, what would be the rate and direction of the resulting motion, the value of the negative force being one and a half times that of the positive?

114. Force required to change the Direction of Motion.—In many cases change of motion affects, not its magnitude, but its direction. Even this change requires the application of force. If the student swings a weight fastened to the end of a string with uniform velocity in a horizontal circle about his head, he is conscious of exerting a continual pull in order to prevent the ball from pulling away. The tendency of the ball to escape from its circular path, which tendency makes necessary the retaining pull, is usually called *centrifugal force.* It must not be supposed that centrifugal force means a tendency to fly straight away from the centre. Experiment and observation will

show that if the ball is at any time released it starts off in a straight line which is a tangent to the circular path at the point where the release occurs. It is this tendency of bodies in motion to move on in a straight line that prevents all the planets of our solar system from falling into the sun. The sun's attraction for them is like the pull between the hand and the revolving weight.[1]

115. Combination of Acceleration with Pre-existing Motion.—It has been stated, or at least implied, in § 113, that the effect of gravity in changing the velocity of a free body is independent of the already existing upward or downward velocity of the body. The student may, however, be in doubt as to what influence a horizontal motion

FIG. 43.

would have upon the effect of gravity. The apparatus shown in Fig. 43 will help to settle this doubt.

---

[1] We say this now with great confidence, although two hundred and fifty years ago very different theories in regard to the motions of the planets were held. Sir Isaac Newton proved about 1680 that the earth's attraction, supposed to decrease in proportion as the square of the distance between its centre and that of the attracted body increases, is just sufficient to hold the moon in its orbit, if the moon acts like ordinary matter with which we are familiar. He showed, too, that the same natural supposition, a mutual attraction universally proportional to the square of the distance between the attracting bodies, would account for the retention of the various planets in their orbits about the sun, provided these planets act like ordinary bodies upon our earth. We accept Newton's theories because of their simplicity—they do not call into action among the stars properties of matter unknown to us upon the earth,—and because of their completeness—they account for the magnitude of the effects observed as well as for their general nature.

Cut six inches from a strip of wood resembling a common lath in shape and size. Fasten the short piece across the long one, as shown in the figure. Cut two equal notches in the two upper corners of the cross-piece to hold marbles or bullets. Fix the apparatus several feet above a level floor, put the marbles in place, and then strike the long bar a sharp horizontal blow near the cross, so as to bend it to one side. One of the marbles will be shot out horizontally with considerable velocity. The other will merely be deprived of its support, and will fall nearly straight down. Which will strike the floor first?

The experiment should be tried several times. The conclusion derived from it may be applied to the action of other forces than gravity.

116. EXERCISE XIX.

THE SIMPLE PENDULUM.

[Worthington, Exp. 11, p. 140; Exp. 19, p. 151; Exp. 20, p. 153.]

[The following exercise on pendulums is given partly for the instruction it gives in regard to the practical applications of swinging periodic motion, and partly for its use in connection with exercises that follow. An ideal "*simple pendulum*" is a mass concentrated at a point and suspended by a string without weight or inertia. Such an ideal is not to be attained in practice, but a small dense object suspended by a slender thread approaches it.]

Use bobs of various weights and sizes, arcs of various lengths, and suspensions of various lengths,[1] for the purpose of finding the effect of these various conditions upon the time of vibration.

Apparatus: A marble, a block of wood, and a lead bullet, all of about the same size. A second block, bullet, or marble, considerably larger than the first. A small piece of sealing-wax

[1] To provide for easy adjustment of the length of the suspension, a teacher recommends pinching the string in a split or pierced cork and fastening the cork suitably as a point of support.

with which to fasten thread to the bullet, etc. Some support from which to suspend the pendulums. (Tacks driven horizontally into the end of a board having several feet of clear space beneath will serve well.) A watch or clock. A meter-rod.

In taking the length of a pendulum it will be sufficiently accurate, unless the bob is more than 3 cm. in diameter, to measure from the point where the thread leaves the support to the centre of the ball.

ALTERNATIVE.

[Trowbridge, Exp. 76, p. 111.]

Apparatus: One ball and suspension. A watch or clock. A meter-rod.

117. Relation of Pendulum Motion to g.—The motion of the pendulum is of course due primarily to the earth's attraction. If the earth's attraction for a given mass were greater than it is, the time of vibration of a given pendulum would be less than it now is. In fact, the time of vibration of a given pendulum is different at different parts of our earth, the earth's attraction for the matter of the pendulum being different in different regions, while the mass (§ 111) of the pendulum as determined in the strict way, by the magnitude of the force required to give it a certain velocity in a certain time, is everywhere the same. The relation between the intensity of gravity,[1] the time of vibration of a pendulum, and the length of the pendulum, is so well known that if any two of these quantities are known the third can be calculated, and from these the intensity of gravity, which is usually expressed by stating the value of g (§ 113), can be found much more accurately than by any other method in use. The determination of g at many different parts of the earth's surface is of much interest on account of the information which the

[1] By this phrase, *intensity of gravity*, is meant the force exerted by the earth upon unit mass. The *force of gravity* upon any body is equal to the number of units of mass in the body multiplied by the number expressing the intensity of gravity.

variation of g from place to place gives in regard to the shape and interior condition of the earth. Accordingly various governments have employed skilled observers to make pendulum observations, and these observers have travelled far and wide over the earth in order that the same pendulum might be studied by the same person in widely different regions. The value of g at the poles is about one part in two hundred greater than its value at the equator, a fact which is accounted for partly by the shortness of the polar diameter of the earth as compared with the equatorial diameter, and partly by the slight tendency of bodies to fly off at a tangent, parting company with the rotating earth (see § 114), which tendency is greater at the equator.

118. All Force a Mutual Action.—We have discussed at some length the effect of a force applied to a body in changing the direction or magnitude of the body's motion. But a force is always the interaction of two bodies, and the two bodies are always affected in opposite directions by this interaction. Illustrations of this mutual action are common in the student's experience. Two sticks struck together will indent each other. A bat stops a ball and turns it back, but the ball stops, or at least checks, the bat. A weight pushes down upon a hand which pushes it up. It is true that in many cases the effect is visible in one direction only. A pebble thrown against a massive stone wall may appear to make no impression upon it. But careful observations may detect in the wall a slight jar at the instant of collision; and even if this is not the case, we are so familiar with the fact that mutual shocks are caused by such collisions, and with the further fact that many effects too small to be easily perceived really exist, that we have no difficulty in believing that the case is only an apparent exception to a universal rule.

119. Object of Exercise XX. Momentum.—In case of a

collision, or any mechanical interaction, between two free bodies of very unequal mass, it is usually evident that the velocity of the larger body is less affected than that of the smaller body, and it is natural to inquire whether the respective changes of velocity bear any simple relation to the respective masses. The most simple supposition not at once shown to be false by common observation is that in the case of free interacting bodies each body suffers a change of velocity inversely proportional to its mass. Exercise XX is intended to test the truth of this supposition.

In carrying out Exercise XX we shall find it convenient to have a name for the quantity $m \times v$, the *number of units of mass in a body multiplied by the number expressing its velocity in units of distance per second.*[1] The name given to this quantity is *momentum.* Now if m and v represent the mass and velocity of a body A, and if m' and v' represent the mass and velocity of a second body B, and if $m = 10m'$, it is evident that increasing the velocity of A from v to $v + 1$ would increase its momentum as much as the momentum of B would be increased by an increase of its velocity from v' to $v' + 10$. The increase of momentum in the case of A would be $m \times 1$, and in the case of B, $m' \times 10$, equal quantities. In this case the supposed changes of velocity are inversely proportional to the respective masses. We see, then, that the question which we propose to test in Exercise XX can be put into this form: *In the case of mutual action between free bodies do the two bodies suffer equal changes of momentum?* As the changes of momentum which the two bodies experience

[1] To say that $m \times v = mass \times velocity$ is to use an abbreviated expression for the explanation given in the text. Such abbreviated expressions are not likely to be stumbling-blocks to the student's understanding and will be used hereafter without further explanation.

must be in opposite directions, this question is equivalent to the following: *Will the sum of the momenta of the two bodies after their mutual action be just the same as before?*

In putting the question into this form we must guard against a possible misunderstanding. Suppose we have a case like that of a man leaping from a small boat, both being at rest before the leap. Here we have before the action no momentum; after the action, momentum in each body, although in opposite directions in the two. This seems at first to answer our last question in the negative, for such cases, but if we agree to call momentum in one direction $+$ and momentum in the opposite direction $-$, with *algebraic sum,* instead of mere *sum,* in our question, there is still a chance that it may be answered in the affirmative, for, in the case supposed, if the man acquires a momentum $+M$, and the boat a numerically equal but algebraically opposite momentum $-M$, we shall have

$$\underset{\textbf{Momentum before}}{0+0} \quad = \quad \underset{\textbf{Momentum after}}{+M-M.}$$

In supposing this case we do not mean to imply in advance of experiment whether the momentum acquired by the man would be or would not be numerically equal to that acquired by the boat. We wish merely to show the advantage for our present purpose of calling momentum in one direction positive and momentum in the opposite direction negative.

We have spoken of the mutual action of free bodies. The nearest approach we can make to a case of free bodies, under proper conditions for observation and measurement, is by allowing bodies suspended by long strings to collide horizontally. This device is therefore used in Exercise XX.

120. EXERCISE XX.

ACTION AND REACTION.

[Preliminary: This exercise makes use of two pendulum-balls so arranged as to swing isochronously and collide when at the middle points of their arcs. For the purposes of the exercise it is necessary to know the relative, though not the absolute, velocities of the two balls just before and just after the collision. These relative velocities may be determined with sufficient accuracy for our purposes by observing the horizontal distances through which the balls have swung or are to swing, for it is a fact that, with suspension of given length, the velocity of a pendulum-ball at the middle point of its arc is nearly proportional to the horizontal projection of the arc, provided this arc is not greater than 30° or 40°. For instance, if two balls having suspensions 2 m. long are pulled aside, one 60 cm. and the other 30 cm., measured horizontally, and then released, the one having the longer arc will pass through its lowest position with a velocity twice as great as that which the other ball will have in its lowest position. (This fact involves the further one that if the two balls are started from their lowest points, one with a velocity twice as great as that of the other, the one will swing very nearly twice as far horizontally as the other, provided the arcs are of moderate size in angular measure.)

To show this fact an experiment has been devised which is now to be described. It is intended for the lecture-room and not for performance by the students.

Apparatus: Two pendulums like those used in Ex. XIX, with suspensions of equal length, the longer the better. One of the boards used in Ex. XVIII with one edge ruled cross-wise in lines 1 cm. apart. Wooden blocks or boxes to stop the pendulum bobs at any chosen points of their paths. A meter-rod.

Suspend the pendulums so that there will be about 4 cm. of clear space between their bobs. Place the board, marked edge up, between the balls with its length at right angles with the line connecting the centres of the balls. Draw one of the balls to one side 60 cm., as measured on the board, and the other ball in the same direction 30 cm. Place the blocks or boxes so that each ball will strike after travelling one fourth of the horizontal distance to its lowest position. Release the balls at the same

instant and listen very attentively to decide whether they strike at the same instant.

Let the bobs be stopped in the same way after having traversed one half, three fourths, then the whole, of the distances to the lowest point. If time permits, the examination may be carried beyond the lowest points of the arcs. If in all cases here suggested and any others that may be tried the blows given by the two bobs come so nearly together that no interval between them can be detected, it may fairly be concluded that the ratio of the velocities of the bobs at any instant, when each is passing through its lowest point, for instance, is nearly, if not quite, equal to the ratio of the horizontal components of the paths in which the bobs travel. (Conclusions for the cases tried must not be extended to cases in which the arcs are much larger parts of circles.) The delicacy of the method of experiment should be tested by finding how small a displacement of one of the obstacles from its proper positions will make a plainly perceptible interval between the two blows.

The method just described is, owing to the limited power of the ear to note small intervals of time, not very accurate. It serves, however, to indicate methods which would be accurate, and it is therefore described to relieve the teacher from the necessity of stating to his class entirely without proof the fact which he wishes to use. Another method, devised in its essentials by Mr. Schobinger, of the Harvard School, Chicago, will now be described. It is suited to individual experimentation rather than to lecture-room use: Suspend the two pendulums, made as isochronous as practicable, some little distance apart, for instance 60 cm. Then stick a tall pin or a pocket-knife upright in the table just in line with the two balls and about 60 cm. distant from the nearer ball, so that when the eye of the observer is placed at the top of the pin or knife the nearer ball hides the other. Then let the two balls be pulled to one side with such displacements that the nearer ball still hides the other from the observer placed as before. Let the balls be released at the same instant. If the nearer continues to hide the other during one whole swing, it is evident that the velocities must have nearly the same ratio throughout the swing.]

Apparatus: Two ivory balls, one weighing about 50 grm., the other three or four times as much, the weight of each being known within one or two per cent. A bar of wood 50 cm. or more long and in width somewhat more than the sum of the radii of the balls.

Some means of fastening this stick in a horizontal position at the greatest height practicable. One of the boards used in Ex. XVIII ruled on one edge in cm. A piece of putty.

Into each end of the stick drive two tacks, making the distance between them equal to the sum of the radii of the balls. Take a piece of No. 33 (B. & S. gauge) naked copper wire, the length of which will depend upon the vertical space available for the suspension, and fasten one end to one of the tacks and the other end to the corresponding tack in the opposite end of the stick. Suspend one of the balls from the middle of this wire. Suspend the other ball from another wire fastened to the other two tacks, so as to make the balls hang just grazing each other with their centres at the same height. Place the board, with its marked edge up, beneath the balls, its length parallel to the line connecting the centres of the balls. Note the point where a vertical line from the centre of each ball would strike the board. Draw the larger ball out, keeping it over the board, about 10° from its initial position, note what point on the board is directly beneath the centre of the ball in its present position, then let the ball swing free and strike the other, which is at rest. Note on the board the extreme position reached by each ball in its first swing after the collision. We have now the means of telling the horizontal range of the swing made (a) by the ball A before the collision, (b) by the ball A after the collision, (c) by the ball B after the collision. Call the first of these horizontal distances D_A; the second, d_A; the third, d_B. Recall the teachings of the preliminary experiment above. It will then be apparent that, if the velocity of A just before the collision is called V_A, the velocity of A just after the collision must have been very nearly $V_A \frac{d_A}{D_A}$, and the velocity of B just after the collision very nearly $V_A \frac{d_B}{D_A}$. Let the mass of A, plus one half the mass of its suspending wire, be called M_A, and that of B, plus one half the mass of its suspending wire, be called M_B.

Then momentum of A before collision $= M_A \times V_A \qquad = M_A D_A \times \frac{V_A}{D_A}$.

" " " " after " $= M_A \times V_A \frac{d_A}{D_A} = M_A d_A \times$ "

" " " B before " $= 0 \qquad = 0 \quad \times$ "

" " " " after " $= M_B \times V_A \frac{d_B}{D_A} = M_B d_B \times$ "

For the purpose of this exercise, which is merely to compare momenta, the common factor $\frac{V_A}{D_A}$ may be left out of the computations.

Try a case in which the small ball strikes the larger at rest.

Try a case in which the balls, coming from opposite sides, meet at their middle positions.

Try a case in which each ball has a thin belt of putty in order to make the collision nearly inelastic, the mass of each belt being added to the mass of its ball in the computations.

Each case should be tried carefully several times under conditions as uniform as practicable, and the average D_A, d_A, etc., of the several trials for each case should be taken for use in the computation of momenta for that case.

Find for each case how the total momentum before collision compares with the total momentum after collision.

Momenta having opposite directions are to be taken with opposite algebraic signs, and the comparison is to be made between the algebraic sum of the momenta before collision and the algebraic sum of the momenta after collision.

Allowance for Experimental Errors.—In considering the result of Exercise XX it should be borne in mind that a pendulum does suffer some slight loss of motion from the resistance of the air. Now as the momentum just before collision is estimated from the length of swing preceding contact and the momentum just after collision from the swing following contact, it is evident that the former estimate will be a little larger, and the latter a little smaller, than it should be. A rough estimate of the error caused in this way may be obtained by studying the rate of decrease of the pendulum-swings.

121. Friction.—When a solid body is made to slide over the surface of another, a resistance to the sliding movement is in many cases observed. This resistance is called *friction.* That it differs greatly in amount with the nature of the surfaces in contact is a matter of every-day observation. The increased difficulty of drawing a sled over spots of

bare ground, on passing from a snow-covered surface is a striking illustration of this difference, and another, even more common, is the diminution of resistance noticeable when an axle or a door-hinge is oiled. The amount of friction experienced in sliding an object over any horizontal surface of tolerably uniform smoothness may be measured directly by applying the horizontal pull necessary to keep the object in motion by means of a spring-balance, and taking the average reading of the balance as the resistance due to friction. This is done in Exercise XIII.

As a preliminary to this Exercise the student should try the experiment of drawing the block along the board by means of a spring-balance at different rates of speed, the rate, however, remaining constant during each particular drawing, in order to see whether the friction depends upon the speed.

122. EXERCISE XIII.

COEFFICIENT OF FRICTION.

[Trowbridge, Exp. 68 and 69. Worthington, Exp. 5, p. 112, p. 115, and Exp. 12, p. 118. Lodge, Art. 104. Goodeve, Arts. 47–49.]

Apparatus: A smooth pine board (see Ex. XVIII). A block of wood (see Ex. VII). Iron or lead weights (see note to Ex. IX) to the amount of 1 lb. The spring-balance of Ex. VII. Blocks, boxes, or other means of supporting the board in an inclined position. A meter-rod.

Lay the board horizontal. Place the block on one of its narrowest sides on the board. Tie a thread around the block and pull on this thread with the spring-balance, held horizontal, with such force as to keep the block moving parallel to its grain with uniform velocity along the board. (This force is likely to vary at different parts of the board. If any part can be found where it is nearly constant for a distance of eight or ten inches, this part of the board should be used throughout the Exercise.) Record the force indicated on the balance.

Then place the block on its broad side and record the force required to maintain uniform motion parallel to the grain of the block. Then load the block, still on its broad side, with ½ lb. and

1 lb. in turn, recording in each case the force required to maintain uniform motion. Weigh the block itself in oz. and calculate the coefficient of friction for each of the four cases described.

Raise one end of the board until the unloaded block, once started, will slide with uniform velocity. Then, keeping the board fixed in position, measure the vertical distance from the under side of the raised end to the table and the horizontal distance from the foot of this vertical to the point where the lower end of the board rests upon the table. From these two measurements the coefficient of friction is again to be calculated.

The coefficient of friction is the ratio $\frac{\text{friction}}{\text{pressure}}$. It is to be obtained, for the first method of Exercise XIII, by dividing the average balance-reading (corrected), in each of the four cases, by the weight of the block and its load (if any). The calculation of the coefficient from the data

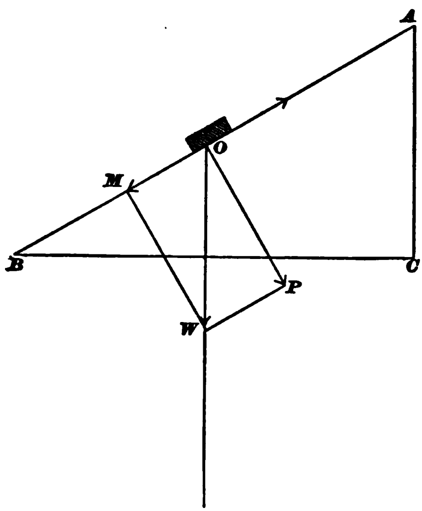


FIG. 44.

obtained by the second method depends upon an application of the principle of resolution of forces (§ 79). In Fig. 44 the board *AB*, the table-top *BC*, and a vertical line

AC let fall from A form a right-angled triangle with the right angle at C. Suppose a vertical line OW, let fall from the block O to represent the weight of the block and the direction in which the force of gravity tends to draw it. Not all of the force OW is available to produce sliding motion down the incline, but only that component of it which lies in the direction AB. If the board AB. were to be raised until it stood vertically, all of OW would lie in the direction AB. If the board were laid flat on the table-top, none of the force OW would avail to produce any tendency to sliding. In any intermediate position of the board, like that shown in the figure, the value of the motion-producing component will be intermediate between the values OW and zero. In any given case, to find the value of the motion-producing component of OW, we must resolve OW into two components, one of which has no tendency to produce motion along the incline, being perpendicular to it, and the other no tendency to produce motion in any other direction than down the incline, being parallel to the line AB. The former component is represented by the line OP, the latter by the line OM, the figure $OMWP$ being a rectangle. An application of simple geometrical principles will show that the triangles OWM (or WOP) and ABC are similar. Then

$$\frac{PW \text{ (or } OM)}{OP} = \frac{AC}{BC}.$$

Now from what has been said concerning equilibrium in § 70 it will be seen that $OM =$ the resistance of friction, whenever the motion of the block down the incline is uniform. To find the coefficient of friction by the second method, then, it is necessary merely to obtain the quotient of $\frac{AC}{BC}$, since this $= \frac{OM}{OP} = \frac{\text{force balancing friction}}{\text{pressure against surface}}$.

123. Laws of Friction.—The results of Exercise XIII should be used by the student to answer for himself the questions:

(1) What is the relation between friction and the extent of the surfaces in contact, other things—that is, total pressure, nature of surface, and velocity—being unchanged?

(2) What is the relation between friction and pressure perpendicular to the surfaces in contact, other things being unchanged?

The answers to these questions constitute two of the most important of the laws of friction. A third law may be derived from experiments like those made as a preliminary to Exercise XIII. It may be that the results obtained will prove somewhat contradictory, since the unavoidable irregularities of the surfaces in contact, and other accidental circumstances, render experiments upon friction somewhat uncertain. In deducing general conclusions it is highly desirable to compare results obtained by different students with various blocks, and in all cases it must be remembered that the experiments have a narrow range and may indicate laws which do not always hold true.

124. Friction in Applied Mechanics.—Friction is one of the most important conditions in the construction and operation of very many mechanical appliances. It enters largely into the list of resistances to be overcome, as in the rolling friction of the car-wheels upon the track or of wagon-wheels upon common roads. Every axle revolves in its bearings with a measurable amount of friction, which can be diminished but not overcome by oiling the surfaces in contact. The resistance of the air is an important obstacle to rapid motion, as in the case of a railroad train, and the frictional resistance of the water to the hull and the propeller of a steamer demands most of the steam-power required to propel the vessel; for when liquids or gases are involved friction increases rapidly with increase

of velocity. On the other hand, many machines and mechanical appliances would be valueless without friction. Upon this the efficiency of belting, of brakes, of nails and screws of every description, is dependent. The driving-wheels of engines or of electric street-cars, the feet of men or of horses, would be unable to produce or maintain locomotion without the aid of friction. If its operation were suspended every river would become a cataract, soon running itself out.

125. Work.—One important object of Exercise XXI (§ 127) is to illustrate and enforce the scientific definition of the word *work*. This word is used by the physicist to signify *the doing of something against opposition.*[1] If a man pushes a saw through a board against friction and cohesion of the particles of wood, if he raises a weight against the force of gravity, he does work. If the saw sticks, so that he pushes in vain, if after lifting the weight a certain distance he encounters some obstacle and ceases to raise it, he is no longer doing work, although he may be making more effort than before. A statement like this frequently arouses a feeling akin to indignation in the mind of the student, who is apt to feel that scant justice is done to the supposed toiler. But the science of physics does not concern itself with emotions and purposes. Looking to the result accomplished, it says that a man who is merely sustaining a weight is doing no more for that weight than a post could do. He is not prevailing over the opposition of gravitation. He is not doing work in the scientific sense of the word.

The nature of a work as a definite mechanical *process* being thus defined, the next point to be considered is the measurement of work. For an English-speaking person the most natural unit of work is the amount necessary to

---

[1] Consult Clerk Maxwell: Treatise on Heat, chapter IV.

raise one pound one foot. This is called the *foot-pound,* and is in almost universal use among English-speaking engineers. Evidently to lift 2 lbs. 1 ft., or 1 lb. 2 ft., requires 2 ft.-lbs.; to lift 4 lbs. 3 ft., or 3 lbs. 4 ft., requires 12 ft.-lbs. In short, to lift m lbs. h ft. requires mh lbs.

French engineers, and others who have the kilogram as the unit of mass and the meter as the unit of distance, take as their unit of work the *kilogram-meter,* that is, the amount of work required to lift one kilogram one meter. Various other units of work which are in common use among physicists will be defined later in this book (§§ 129–131).

Although the foot-pound is defined by reference to lifting a certain mass a certain distance, any kind of work can be expressed in foot-pounds. Thus if a man pushing a carpenter's plane horizontally along a board exerts a horizontal force as great as the vertical force that he would exert in holding up a weight of 10 lbs., we are in the habit of saying that he exerts a force of 10 lbs., and if, exerting this horizontal force all the time, he pushes the plane 5 ft., we say that he does 10×5 ft.-lbs. of work.

Always, *when the force is parallel to the motion maintained or assisted by it,* the work done by the agent exerting the force is reckoned by multiplying the number expressing the force by the number expressing the distance which the point acted on by the force moves during the action of the force.

PROBLEM. How many ft.-lbs. of work must be done in dragging a 100-lb. wt. a distance of 50 ft. along a horizontal surface upon which its coefficient of friction (§ 122) is $\frac{1}{2}$?

126. Work Done by Forces Oblique to the Line of Motion.—When the force applied to any point is not parallel to the motion of that point the calculation of the work done is not quite so simple. Let us deal with the matter by means of a series of questions, the answers to which may be based upon

the previous teachings of this book or upon the student's independent experience.

A street-car moves 10 ft. along its track while a man pushes against the car exactly at right angles with the track with a force of 50 lbs. Does the man's effort assist the motion of the car along the track? Does his effort hinder it, save by possibly increasing the friction? How much work does the man do upon the car during its motion?

Suppose that the car is moving due north and that the man now pushes just as hard as before but in a direction making an acute angle, 45° for example, with the line of motion of the car. How much work does he do while the car moves 10 ft.? To find the answer, resolve the force exerted by the man into two components, one at right angles with the motion of the car, the other parallel to that motion (see § 79, Fig. 22), and find the amount of work done by each component separately.

(Observe—for use in connection with the second part of Exercise XXI—that the product of the whole distance by that component of the force which is parallel to the motion,

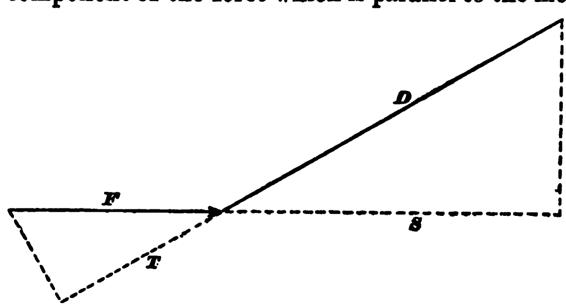


FIG. 45.

is equal to the product of the whole force by that component of the distance which is parallel to the force.

Thus in Fig. 45, where the whole force is represented by

F and the whole distance by D, we see from the similarity of the two triangles that $T : F :: S : D$, whence $T \times D = F \times S$.)

Suppose that the car moves as before and that the man pushes as hard as before but in a direction making an obtuse angle, 135° for instance, with the direction of motion of the car. The man does not now push the car or help to push it. He hinders it. The car pushes the man. We do not in this case say that the man does work. We say that the car does work. Find, by the process already indicated, how much work the car does against the man, while moving 10 ft.

State a general rule for finding the work done by a force F acting continuously upon a body while the body moves a distance D, the angle between the force and the motion having any value from 0° to 90°.

127. EXERCISE XXI.

THE INCLINED PLANE: WORK.

[English units are used in this experiment as the balance is graduated in lbs.]

Apparatus: The two boards described in Ex. XVIII and some means for holding one end about 2⅝ ft. higher than the other. A roller-skate or other small strong carriage. A 20-lb. spring-balance. Some mass weighing 15 or 20 lbs. with which to load the carriage.

FIRST PART.—Find the work done, friction aside, by a force parallel to the incline in drawing the loaded carriage such a distance up the incline as to make the vertical rise 2 ft.

The board used should be supported at the middle as well as at the ends in order to prevent bending. Measure along the incline the distance corresponding to a vertical rise of 2 ft. Then determine the force parallel to this incline which would be necessary to move the carriage with uniform velocity up the incline if there were no friction. To determine and eliminate the friction at any part of the incline, find the pull on the balance parallel to the incline required to move the carriage at a uniform velocity up the incline at the given spot, and then the pull in the same direction

which will allow the carriage to move with uniform velocity down the incline at the given spot. The difference between these two forces will be twice the force required to overcome friction at the spot in question, and the mean of the two pulls will be the pull that would be required if there were no friction.

It is not necessary to draw the carriage the whole length of the distance measured. If the incline were uniform it would be necessary to determine the force at one place only, but as the incline may vary it is best to determine the force at three or four places.

SECOND PART.—Find the work done, friction aside, by a force parallel to the base of the incline in drawing the loaded carriage such a distance up the incline as to make the vertical rise 2 ft.

Support the two boards, with one end elevated, in a parallel position side by side but slightly separated. Let the carriage,

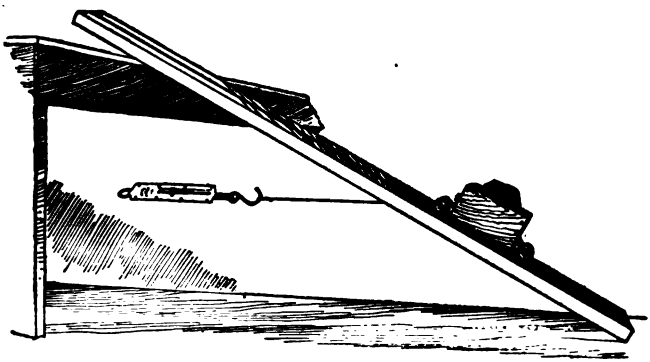


FIG. XIV.

when in position to move up and down the incline, bestride the space between the boards, the pull of the balance being applied by means of a string passing horizontally between the boards [Fig. XIV].

Measure the horizontal distance corresponding to a rise of 2 ft. and the horizontal pull necessary to move the carriage with uniform velocity up the incline, eliminating friction as before by observing the pull going up and the pull coming down.[1]

[1] The method given for eliminating friction is not quite accurate in this second part of the experiment. This inaccuracy is not practically serious if the friction is small.

THIRD PART.—Weigh the carriage and its load on the same balance.

The zero-point error of the balance for the positions in which it is used in the First and Second Parts of this exercise should be determined and used in measuring the forces.

Compare the work done in the 1st case with that done in the 2d, and compare each with the work necessary to raise the given mass, carriage and load, 2 ft. straight up.[1]

128. Energy.—Work and energy are so closely connected that they can hardly be discussed separately. In fact, by definition, *energy is the power of doing work.* As work involves both force and motion, so energy involves not only the power of exerting force upon a body, but also the power of following up the body, for a greater or less distance, if it yields to the force. A compressed spring, a charge of gunpowder, a flying cannon-ball, each of these is a body possessing energy. A weight raised above the surface of the earth is ordinarily said to possess energy, because it can do work during its descent, but strictly the energy in this case belongs not to the weight alone but to the system made up of the earth below and the weight above, which by their mutual attraction tend to approach each other and to overcome any resistance which opposes their approach.

129. Kinetic Energy and Potential Energy.—Two classes of energy may be distinguished. The moving cannon-ball, quite apart from any attraction or repulsion, has energy by virtue of its inertia (see § 110) and its motion. This is called *kinetic energy.*

---

[1] The passages in Lodge, referred to under Ex. XVIII, do not sufficiently emphasize the fact that in reckoning Work as = Force × Distance, we must, when the two are not parallel, take that component only of the force which lies in the direction of the motion, or, on the other hand, that component only of the motion which lies in the direction of the force.

On the other hand, in the case of the raised weight just described, in a bended bow, in an explosive mixture of hydrogen and oxygen, in a Leyden jar charged with electricity, etc., we have energy due to some attraction or repulsion between different parts of the same body or different bodies of a system. This is called *potential energy*.

Each of these forms of energy is the power of doing work, and each is measured in the same units as work, that is, in foot-pounds, kilogram-meters, etc. For example, let us find the number of foot-pounds of energy possessed by a body whose mass is m lbs., and which is moving with a velocity of v ft. per second. Since experience shows that kinetic energy does not depend upon the direction of the motion, we shall for convenience suppose that the body is moving straight upward and that its kinetic energy is to be entirely expended in raising the body itself. If the body rises h ft. from its present position before stopping, its kinetic energy will have done $m \times h$ ft.-lbs. of work. But how great is h? From § 113 we know that h (there called s) $= \frac{v^2}{2g}$, g being equal to $32\frac{1}{6}$ nearly. Putting for h its value we have as a general expression, good for every case in which a body of m lbs. is moving with a velocity of v ft. per second,

$$\textit{kinetic energy} = \frac{mv^2}{2g} \textit{ ft.-lbs.}$$

Everywhere in the preceding paragraph we may without impairing the reasoning in the least write *kilograms* for pounds and *meters* for feet, g becoming in this case 9.81 nearly (see § 113). So we may write *grams* for pounds and *centimeters* for feet without altering the reasoning or the general formula obtained, but g in this case will have the value 981 nearly.

130. Statement of Work in Foot-poundals.—Sometimes work is expressed in foot-*poundals* instead of in foot-pounds. As the earth's attraction for a one-pound mass is g poundals (§§ 112 and 113), it is evident that to lift one pound one foot would require g foot-poundals of work. One foot-pound, then, equals g foot-poundals. Hence, if we use this latter unit in expressing kinetic energy, we have in the case of m lbs. moving with a velocity of v ft. per second,

$$kinetic\ energy = \left(\frac{mv^2}{2g} \times g\right) ft.\text{-}pls. = \frac{mv^2}{2}\ ft.\text{-}pls.$$

131. The Dyne-centimeter or Erg.—Physicists, as distinguished from engineers, are, the world over, in the habit of reckoning in grams and centimeters, rather than in pounds and feet or kilograms and meters. They generally take as their unit of force the *dyne* (see § 112), and as their unit of work the work done by one dyne acting a distance of one centimeter. This unit may be called a dyne-centimeter, but it is used so much that a special short name is given it. It is called the *erg*. If in the paragraph just preceding we write grams instead of pounds, centimeters instead of feet, and dynes instead of poundals, g being in this case 981 nearly, we shall arrive at the general formula

$$kinetic\ energy = \frac{mv^2}{2}\ ergs.$$

In short, when we use a *gravitation* unit of work, that is, a unit of work based upon a gravitation unit of *force* (see § 4), we must write

$$kinetic\ energy = \frac{mv^2}{2g};$$

but when we use an *absolute* unit of work, that is, a unit of work based upon one of the so-called absolute units of force (see § 113), we must write

$$kinetic\ energy = \frac{mv^2}{2}$$

132. Estimation of Potential Energy.—It would be difficult to give here any useful formula for the amount of potential energy contained in a body or system of bodies, in any case except that of bodies raised above the earth's surface. In such a case if m is the mass of the body and if h is its height above the earth's surface, we see that the body in descending to the earth may be made to exert m *gravitation* units of force throughout a distance h. Hence its potential energy, reckoned in gravitation units of work, is $m \times h$. In "absolute" units of work it is $m \times h \times g$.

133. Conservation of Energy.—A body thrown upward spends its kinetic energy in doing work against gravitation, in pulling itself away from the earth, but the work thus done goes to store up potential energy. While the body falls this potential energy is turned back into kinetic energy. This is a familiar example of change from one form of energy to the other which is continually going on about us, sometimes visibly, frequently unseen. Energy takes extremely subtle shapes, in heat, in light, in electric currents, in chemical conditions, but the two classes *kinetic* and *potential* include all forms of which we have knowledge. "All such forms are so related that energy in any one of them may be turned into any or all of the others in succession and back finally to its first shape.

"In actually performing such a circle, or *cycle*, of transformations we observe an apparent loss. We bring back to the original form a smaller amount of energy than we started with. Thus a ball thrown upward returns to the hand with diminished velocity. We must not, however, conclude from such trials that there is any real destruction of energy in the process of transformation.

"If we were to take a quantity of water filling a certain vessel and pour it into a dozen vessels in succession until we reach the first vessel again, this vessel would not now be filled, but we should not infer from this any real destruc-

tion of the substance of the water. We should account for the apparent loss by the amount left clinging to the sides of the vessels, or absorbed by them, or spilled, or evaporated, or possibly changed by chemical action into some different form of matter; and we should have not the slightest doubt that, if all these small amounts could be collected, they would just fill the space now empty in the vessel. This confidence is the result of long experience, which has taught us to believe that matter cannot be destroyed.

"Equally long experience has been teaching the somewhat more subtle truth that energy cannot be destroyed, that its apparent annihilation in its transformation, or wasting away without transformation, is to be explained, like the disappearance of water in the process described above, merely as an escape in various ways from the immediate scope of our observation. But so numerous, and in many cases obscure, are these ways of escape, that their full extent and significance have been perceived only within quite recent years.

"The belief that energy cannot be destroyed or diminished, and the converse belief that energy cannot be created or increased,—which latter belief is practically rejected by those who attempt to 'invent perpetual motion,'—constitute the doctrine of the Conservation of Energy. All the familiar devices for the advantageous application of mechanical power, such as the lever, the inclined plane, and the hydraulic press, are simple examples of the truth of this doctrine, and their laws may be at once deduced from it."[1]

---

[1] Hall's *Elementary Ideas, Definitions, and Laws in Dynamics*, pp. 28, 29.

Questions and Problems.

1. The deck of a steamer is 40 ft. wide. A ball is rolled across the deck at right angles to the steamer's course. If the ball crosses in 6 seconds and the boat's rate of motion is 10 miles per hour, how far does the ball travel in the 6 seconds? 96

2. A car is moving at the rate of 20 miles per hour; how far will a bullet, let fall from the car at a point 7 feet above the ground, move along the track before striking the earth?

3. State what kind of motion is caused by a single constant force. Illustrate your answer.

4. Explain exactly how the experiment on the inclined plane, in Exercise XIII, leads to conclusions regarding the coefficient of friction.

5. Demonstrate by aid of a diagram that the steepness of an incline that will just allow a block to slide down it, is independent of the weight of the block.

6. A body weighing 5 lbs. rests upon an inclined plane 10 ft. long, whose height is 6 ft. and base 8 ft. How great is the pressure which the body exerts normal to (perpendicular to) the inclined plane?

How great a force parallel to the incline is required to prevent the body from sliding down when there is no friction? when the coefficient of friction is $\frac{1}{4}$?

7. If a block weighing 50 gms. is dragged at a uniform rate along a horizontal surface by a pull of 20 gms., what is the coefficient of friction?

8. A block slides with uniform velocity down a board 2 m. long, when one end of the board is raised 50 cm. above the table-top. What is the coefficient of friction?

9. If the coefficient of friction is 1, what angle must the inclined plane make with a horizontal surface to cause a block to slide at a uniform rate?

10. A street-car is moving northeast along a track. The team is pulling exactly north with a force of 800 lbs., and continues to do so while the car moves far enough along the track to go 12 ft. north. How much work does the team do during this movement? (State the unit in which the work is reckoned.)

11. A ball weighing 50 gms. moving horizontally at the rate of 80 cm. per second strikes a ball weighing 200 gms. which is at rest. If the smaller ball rebounds with a velocity of 20 cm. per second, how great is the velocity imparted to the large ball by the collision?

12. A ball weighing 50 lbs. and moving with a velocity of 10 ft. a second strikes squarely a ball at rest weighing 150 lbs. and imparts to it a velocity of 3 ft. per second. What velocity has the smaller ball after the collision?

13. A bullet weighing 10 gms. is shot with a velocity of 300 m. a second from a rifle weighing 6 kgms. How great a backward velocity is given to the rifle by the discharge?

14. Suppose that Exercise XX is performed with balls of soft putty, *C* and *D*. *C* weighs 150 gms. and swings south 25 cm., when it collides with *D* (weight 20 gms.) at rest. How far, approximately, will *D* now swing? 22.

15. Represent by a diagram three of the forces acting on *D* at the instant of collision and name the forces. Draw a dotted line for their resultant.

16. The momentum (§ 119) of a certain body is 600. It encounters a constant resistance of 50 absolute units (§ 112). How long will it continue to move? Can you tell how far it will move?

17. If a mass of 1 mgm. is acted on by a force of 1 dyne for 10 seconds, no other forces being applied to it, how much energy will it acquire in that time if it starts from rest?

18. Define carefully the dyne and erg, the poundal and foot-poundal. Define work.

19. A block weighing 10 lbs. rests upon an incline such that the block must move 5 ft. in order to rise 3 ft. The pressure of the block against the incline is in this case 8 lbs. Let the coefficient of friction be $\frac{1}{4}$. How much work must be done against gravity by a force parallel to the incline in drawing the block 5 ft.? How much work against friction?

20. If a body whose mass is 100 gms., originally moving due north with a velocity of 20 cm. a second on a perfectly smooth horizontal surface, be acted upon by a force of 500 gms. due northeast and an equal force due northwest, what velocity will the body have at the end of one second, and how far will it have moved during that second?

21. An engine pulls with a force of 4000 lbs. upon a train of cars weighing 400,000 lbs. upon a horizontal track. If the friction of the train were zero, how great a velocity would it acquire in one second, starting from rest? If the coefficient of friction were $\frac{1}{200}$, how great a velocity would be acquired in one second?

22. If friction of the cars were zero, how steep an incline could this train ascend, the engine continuing to exert a force of 4000 lbs.? The coefficient of friction being $\frac{1}{200}$, how steep an incline could the train ascend? (Express steepness of incline as a certain number of feet rise for a certain number of feet of track.)

23. Two bodies initially at rest move towards each other in obedience to mutual attraction. Their masses are respectively 20 gms. and 100 gms. If the force of attraction be $\frac{1}{10}$ of a dyne, find the velocity acquired by each mass in 30 seconds.

24. A body weighing 100 lbs. rests upon an incline. How much work must be done in drawing the body 10 ft. up this incline, the vertical ascent being 6 ft. and the coefficient of friction being $\frac{1}{5}$?

25. A hammer hangs vertically, head downward. The

centre of gravity of the head, which weighs 16 lbs., is 3 ft. below the point of support. The centre of gravity of the handle, which weighs 1 lb., is 1 ft. below the point of support. How much work is necessary to lift the hammer into a horizontal position at the level of the point of support?

26. The kinetic energy of a certain body is 600 ergs. It encounters a constant resistance of 50 dynes. How far will it go before coming to rest? Can you tell how long it will continue to move?

27. If the mass of a body is 50 lbs. and it is moving with a velocity of 30 ft. per second, how great is its kinetic energy as reckoned in absolute units? Reckoned in gravi- 22,500 tation units? Name the units of energy in both cases, and explain the difference between the numbers.

28. In a place where the average barometric height is 75 cm., what is the value in dynes of the atmospheric pressure per square centimeter?

29. A bar 20 ft. long is placed horizontally. A force of 100 lbs. due north is applied to the bar at a point 5 ft. from one end and a force of 250 lbs. due south is applied at a point 10 ft. from the same end. How great a force would just neutralize these two forces, and at what point of the bar should it be applied?

30. A uniform bar 12 ft. long and weighing 10 lbs. bears at one end a load of 8 lbs. and at the other end a load of 16 lbs. The bar so loaded is placed horizontal upon a supporting point. How great is the pressure upon the support, and how far must the support be from the end bearing the 8 lbs. load?

31. A beam 12 ft. long and weighing 8 lbs. has

at 9 ft. from one end a force of 7 lbs. acting downward,
" 4 " " same " " " " 5 " " upward,
" b " " " " " " " a " " " ,

and is in equilibrium. Find the magnitude of a and of b.

CHAPTER VIII.

HEAT: TEMPERATURE, THERMOMETRY, EXPANSION OF SOLIDS, OF LIQUIDS, AND OF GASES.[1]

134. Sensation of Warmth.—The bodies which we touch and handle in every-day life might be roughly classified as warm, cold, or in a neutral condition, that is, giving us neither the sensation of warmth nor of cold. This classification is an exceedingly variable one at best, depending largely upon the condition of the observer at the time. To the hands of any one coming indoors on a cold winter's day, a basin of water at the temperature of an ordinary living-room feels warm, while the same water at the same time would feel cool to any one just risen from a warm bed. That something which is capable of producing in us (under favorable conditions) the sensation of warmth is called *heat.*

135. Warm Bodies impart Heat to Cooler Ones.—When a tea-kettle of cold water is placed on a hot stove the water itself soon becomes hot; so does a flat-iron, a soap-stone, or other solid object, placed on the stove. If a hot soap-stone is set on a brick hearth to cool, we shall soon find the stone cooler and the bricks warmer than at first. It would not be safe to assert that the bricks give out no heat to the stone, but the net result of whatever transferences may have taken place is clearly in favor of the bricks. In this

[1] This chapter and the following one can be taken up to the best advantage in the winter. Snow is better than ice for most experiments on heat, and in Exercises XXIV to XXVII inclusive, the steam used may, in a steam-heated building, be taken direct from the radiators, care being taken to ascertain by the thermometer its temperature as used. Much time may be saved by filling "apparatus A" (§ 140) with boiling-hot water at the beginning of experiments in which steam is to be generated. It is well to have this hot water ready at the beginning of the laboratory-period.

case the latter would be a long time in attaining the same degree of warmth that the stone possessed, but in many instances two substances may be so mingled that at the close of the operation their degree of warmth shall be the same. For example, cold water stirred into a body of hot water, or heated shot poured into cold water, with thorough stirring, will leave a mixture of the same degree of warmth throughout.

136. Expansion produced by Heat.—The physicist needs some more trustworthy measure than the unaided senses can furnish of the condition of bodies as regards heat, and a convenient measure for this purpose is furnished by the increase of volume which most bodies undergo when heated.

EXPERIMENT 18.

Take the apparatus known as Gravesande's ring, which consists of a metallic ring (usually of brass) in which a metallic sphere fits so closely as just to drop through by its own weight at ordinary temperatures (Fig. 46). Heat the sphere in the Bunsen flame and try to drop it through the ring. While the sphere is still hot, heat the ring and see whether the sphere will now drop through.

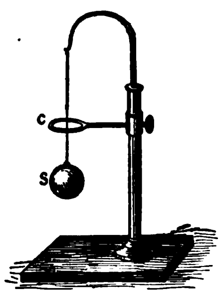


FIG. 46.

EXPERIMENT 19.

Get a straight compound bar, formed of a strip of brass 2 or 3 mm. thick, 2 cm. wide, and 15 or 20 cm. long, riveted or bound with wire to a similar strip of iron (Fig. 47). Move this bar back and

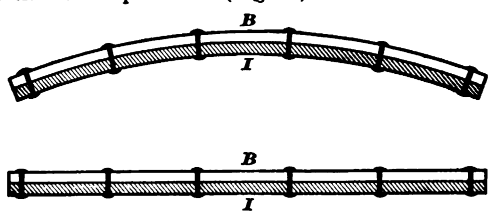


FIG. 47.

forth in the Bunsen flame until it becomes hot throughout;—do

not heat it to redness. Notice whether the bar is still straight while hot, and if not, which metal lies on the convex and which on the concave side. If convenient, after cooling the bar to the temperature of the room, bury it for some minutes in a freezing mixture of ice and salt and again examine its shape. Are the two metals, brass and iron, equally expanded by heat? Give reasons for your answer.

EXPERIMENT 20.

Take an "air-thermometer bulb" of 4 or 5 cm. diameter with a tube of not more than 3 or 4 mm. inside diameter (Fig. 48). Invert this bulb, and immerse the open end of its tube in a glass of water colored red by any of the red "package dyes" sold by druggists. Heat the bulb by applying the hand, or cautiously by means of a Bunsen flame, and note the effect. Then allow the bulb to cool and observe what occurs, the end of the tube remaining all the time in the water.

FIG. 48.

EXPERIMENT 21.

Take two bulbs like Fig. 48 and fill[1] each to a height of 1 or 2 cm. of its stem, one with water and the other with kerosene. Now after marking the height of the liquid surface in each stem, immerse both bulbs in a shallow pan nearly full of almost boiling water, and note the height of each liquid surface in its tube after the bulbs have been immersed some minutes.[2]

137. Conclusions.—We have now shown that certain solids, liquids, and gases expand on the application of heat. (Point out just how this has been shown.) We have found indications, moreover, that different substances expand unequally with the same rise of temperature. The conclu-

[1] A small glass funnel and a short india-rubber tube may be used for this operation. A fine wire thrust down into the stem of the bulb will be of great assistance in removing air-bubbles which might prevent the descent of the liquid.

[2] Do not allow any Bunsen or other flame to remain anywhere near the bulb-tube containing the heated kerosene, since the latter may give off inflammable vapor.

sions which we have reached from the experiments just discussed would only be confirmed by further investigation, and we should find that most solids, liquids, and gases are expanded by being made warmer,[1] and that each solid or liquid substance has a degree of expansion peculiar to itself. It is important to notice the exact meaning of the observed rise of liquid in the tubes attached to the bulbs, filled respectively with kerosene and with water. Since solids in general expand on the application of heat, and glass is no exception to this rule, the interior of the bulbs must have become larger on being placed in the hot water.[2] (This is indeed shown by the fact that at the very instant of placing each bulb in the hot water the liquid surface in the stem fell.) If the interior of each bulb expanded just as much as the liquid which it contains, it is plain that there could be no rise of either liquid surface, so we must conclude that the amount of expansion of each liquid was greater than that of the glass, and that the rise of the liquid surface measured the difference between the amount of expansion of the liquid and that of the glass.

138. The Mercurial Thermometer.—The thermometer furnishes a means of comparing the temperatures, or degrees of warmth, of bodies. The process of making an ordinary thermometer consists of several steps. First a piece of thick-walled small-bore tubing known as thermometer-tubing is taken and the uniformity of the bore is tested by means of a short column, or "thread," of mercury, which is placed in various positions in the bore

---

[1] Exceptions to this rule are mentioned later.

[2] The experiment with ball and ring has shown that the hole in a ring grows larger when the ring is heated. When a glass flask is heated it expands as if it were the outer envelope of an expanding solid lump of glass. The space inclosed by the flask increases in volume just as much as it would if it were filled with a solid lump of glass.

and measured as to length in each position. A bulb, *R* (Fig. 49), is then blown upon one end of the tube, and at the other end a reservoir for mercury, *B*, with a slender prolongation and a minute opening. *B* is heated until the air within it is somewhat rarefied and then plunged into mercury. The cooling effect produced by the latter causes the inclosed air to contract, and considerable mercury enters the reservoir *B*. The end *B* is now raised somewhat higher than *R*, the point is sealed up, *R* and the stem are somewhat heated to rarefy the contained air and then cooled to allow the mercury to sink into *R*. The point is then unsealed by breaking, and the mercury in *R* is boiled so that its vapor shall drive out the air in the stem and *B*. This and similar operations are continued until nothing is left in the bulb and tube but mercury and mercury-vapor. Then finally the tube is strongly heated just below *B*, drawn out to a point and sealed.

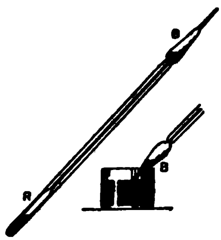


FIG. 49.

Inexperienced students should not try to boil mercury, as its vapor is poisonous.

139. Graduation of the Thermometer.—Upon experimenting with the instrument just constructed we shall find that the mercury column in the tube will rise when the bulb is heated and sink when the bulb is cooled. The amount of rise produced by a given increase of temperature depends on the ratio of the diameter of the tube to the capacity of the bulb, and upon the difference in the amount of expansion of mercury and of glass for equal increments of temperature. Through the range of temperatures within which the thermometer is most used the expansion of mercury is about seven times that of glass.

Besides the mere determination whether one body is

hotter or colder than another, which is all that we can gain from the use of our ungraduated thermometer, there is need of some means for measuring differences of temperature. A practical measure is secured by *graduating* the thermometer. It has been ascertained by an immense number of observations that the point at which ice melts and that at which water boils are substantially invariable, provided certain conditions are maintained.[1] These conditions are chemical and mechanical purity and a standard, unvarying, pressure. Slight chemical or mechanical impurity affects these temperatures but little. Ordinary changes of atmospheric pressure affect the melting temperature extremely little, but they affect the boiling temperature of water very perceptibly. These changes are easily observed by means of the barometer and allowance for their effects is, as we shall see, easily made. Advantage is taken of the peculiarly constant behavior of water to establish "fixed points" on the thermometer-tube. The instrument is first placed upright, with the bulb and the stem plunged in melting snow or melting powdered ice to such a depth that the top only of the mercury-column is visible. The position of the top of the mercury-column, when it has ceased to descend, is marked by a fine transverse scratch on the tube. The thermometer is then removed from the ice and placed upright in a vessel in which the bulb and the greater part of the stem can be perfectly surrounded by an abundant supply of freely escaping steam, the operation being performed, if convenient, at a time when the barometer reads very nearly 76 cm. After the mercury has ceased to rise in the stem, a fine scratch across the latter is

---

[1] A like statement can be made concerning other solids and liquors, but ice and water are the most convenient for use in graduating thermometers. Some solids gradually soften before melting, so that they have no definite melting-temperature.

made at the top of the mercury-column, and the thermometer is then removed from its position. If the thermometer is to be graduated by the Centigrade system (§ 143), the portion of the tube between the melting-point and the boiling-point marks is now to be divided into a hundred equal parts, numbered from 0 at the bottom to 100 at the top, and these divisions engraved on the stem of the thermometer, or ruled on a slip of paper fastened to the thermometer-stem and protected by a larger inclosing tube, or engraved on the metal case to which the thermometer-bulb and tube are fastened, or in some other way fixed in their proper places. The same graduation is continued above and below the melting and the boiling point. If the thermometer is intended for any accurate work the tube before or after filling with mercury should be "calibrated," a short column of mercury being forced through it a few millimeters at a time, and the length of this column measured in successive portions of the tube. By this means, even if the tube is not of equal diameter throughout, it may be divided into successive portions, all of equal *capacity*, and the graduation afterwards made in accordance with these.

140. Apparatus for Heat Exercises.—In the following heat-experiments cork-stoppers, of ordinary sizes, and cork-borers must be used so frequently that it will not be necessary to mention them in the special lists of apparatus for particular experiments. A certain other piece of apparatus will be used several times and may well be described here. It will be called

APPARATUS *A*.

A cylindrical sheet-copper vessel about 15 cm. tall and 10 cm. in diameter, supported on three legs, which are kept from spreading by a circular plate, or a ring, of sheet-copper connecting them all at the bottom. These legs

should raise the vessel about 20 cm. from the table upon which they stand. This vessel is to be used as a boiler, and has, leading out about 2 cm. from the top, a side tube about 5 cm. long and 6 or 7 mm. in diameter, through which the steam may be carried off, in a slightly ascending direction, when the top of the vessel is closed. A conical tube of sheet-copper about 30 cm. tall is made to fit the top of the cylindrical vessel internally, as the cover of a tin pail fits. (It must fit well, for the junction should be, as nearly as practicable, steam-tight. The top of the cylinder should therefore not be wired, but should be left flexible.) The open top of the conical tube is about 2.5 cm. in diameter and of such a shape as to be readily closed by a cork stopper. A side tube, similar to that on the cylindrical vessel but only 2 cm. long, leads out from the cone about 2 cm. from the top. Through this tube, the other apertures being closed, steam escapes when the apparatus is used in testing the boiling-point of a thermometer.

A Bunsen burner, with attached rubber tube, or some other means of supplying heat will, without further mention, be supposed to accompany apparatus *A* in all cases. [See Fig. XV.]

[It seems best here to depart somewhat from the order of the exercises as given in the Harvard College pamphlet.]

141. EXERCISE XXIII.

TESTING A MERCURY THERMOMETER.

[Trowbridge, Exp. 99. Worthington, Exps. 6, 7, and 9, pp. 160 and 161.]

Apparatus and material: The thermometer to be tested. About ½ kgrm. of ice or snow. The "dipper" used in Ex. XXVII, or some other vessel of like size for holding the broken ice. Apparatus A. A barometer.

Break the ice into fine pieces, the finer the better, and fill the dipper with it. Add enough water to fill the space between the lumps of ice. Thrust the bulb of the thermometer down into the middle of the cup until the point marked 0° (the thermometer is

supposed to have the Centigrade scale) is only 1 or 2 mm. above the surface of the water. Give the mercury column time to descend as far as it will and then record the reading. (In this experiment and in most others read the thermometer with care. If its shortest divisions are a millimeter or more in length, try to read to tenths of these divisions. In reading a thermometer place the eye in such a position that a straight line drawn from its centre would strike the thermometer at right angles at the top of the mercury column.)

To test the boiling-point fill apparatus A with water to the depth of 3 or 4 cm., put on the cone, close the side tube leading from the main part of the apparatus, perforate a cork stopper fitting the top of the cone and thrust the bulb and stem of the thermometer down through the cork until the point marked 100° is not more than 2 or 3 mm. above the top of the cork, stopping before this, however, if the bulb of the thermometer approaches within 2 or 3 cm. of the water. Keep the water boiling until the mercury column rises as far as it will and then record the reading.

The water should not be allowed to boil violently, lest the boiling-point be raised by undue pressure of the partially confined steam. The student should test the effect of such confinement upon the temperature of the steam by partially closing the exit tube while the water boils, by holding a wadded handkerchief against its mouth, watching the indications of the thermometer meanwhile. There is also a possibility of error, with the apparatus A, from a superheating of the upper part of the boiler by the stream of hot gases from the flame. This error is not likely to become important, unless the flame is unusually large and the water in the boiler low.

The reading of the barometer should be taken at the time when the boiling-point is tested. In seeking on a thermometer for the boiling-point under standard conditions it is sufficiently accurate for most purposes to deduct from or add to the observed reading of the thermometer in steam at the rate of 1° C. for 27 mm. excess or deficiency in the height of the barometer column, 760 mm. being considered the standard height.

After the boiling-point has been found[1] the thermometer should

[1] As in some experiments the bulb only of a thermometer is surrounded by steam, it is well to find by actual trial how much the reading in such a case will differ from that obtained when the whole stem up to the 100° mark is in steam.

be allowed to cool in air, and should then be placed in ice again for a new determination of the freezing-point. If this point as now found is different from that previously found, the new point should be taken as the true one for any immediate use of the thermometer.[1]

After the zero-point has been finally determined, apply a flame to the metal vessel that contains the ice or snow, and, continually stirring the ice or snow with the thermometer-bulb, note the indications of the thermometer as the melting proceeds.

142. Discussion of Results.—Even if the thermometer used is a pretty good one, considerable errors may be detected in the position of the standard points on the scale,

---

[1] The operation of determining the freezing-point and boiling-point is profitable for the student on account of the light it throws on the theory of the thermometer, but the inaccuracies detected in the operation need not, unless they are greater than 1°, be considered in the heat-experiments of this course.

In estimating, from observations upon the freezing and boiling points, the error of a mark lying between these points, account must be taken of the nearness of the mark in question to the freezing and boiling points. Thus, on a Centigrade thermometer the error of the 50° mark is to be taken as a mean of the errors of the 0° and 100° marks. In estimating the error of the 25° mark three times as much importance should be attached to the error of the 0° mark as to that of the 100° mark, etc.

Besides such errors as have just been considered, and which may come from a change in the glass after the graduation is made, there are usually other errors due to unevenness in the size of the bore of the tube. The evenness of the bore may be tested thus: Hold the thermometer as one holds a pen, but with the bulb uppermost, and strike the palms of the hands together, thus breaking off from the main body of mercury a column 10 or 15 divisions long. Measure the length of this column in divisions of the thermometer at various parts of the scale. If the graduation is correct the column will extend over sensibly the same number of divisions at all parts of the scale. In this way gross errors in graduation may be easily detected; but the complete calibration of a thermometer is a tedious task, and one not demanded by the standard of accuracy which is deemed advisable for this course.

and consequently in the scale as a whole. One such error, known as the elevation of the zero-point, arises from the fact that when the bulb of the thermometer is blown the glass of the bulb cools before its particles become entirely accommodated to each other as they were before the pressure necessary to the operation of blowing was brought to bear upon them. As a result of this the bulb continues to shrink perceptibly for some weeks or months, after which the change is much more gradual.[1] If the thermometer was not filled until a year or more after the bulb was blown, and was not graduated until some months later, the reference-points of the graduation, if once properly placed, will on the whole remain correct enough for ordinary purposes. If, however, the bulb was filled and the stem graduated as soon as made, it is probable that after a time the mercury-column, when immersed in melting ice, will no longer contract to the zero-point of the graduation. On the other hand, even in good thermometers, the expansion of the bulb, produced by heating it to the temperature of boiling water, is somewhat lasting and produces a temporary lowering of the zero-point. The explanation of the directions given for correcting the observed boiling-point with reference to barometric height may be deferred until after the performance of Exercise XXII.

143. Comparison of Thermometer-scales.—Unfortunately there are three thermometer-scales in somewhat general use, the Centigrade, already described, the Reaumur, and the Fahrenheit. The Reaumur scale differs from the Centigrade only in the fact that the space between the two standard points, of freezing and boiling water, is divided into 80 instead of 100 equal parts, so that 1 Reaumur degree equals 1.25 Centigrade degrees. The Fahrenheit

---

[1] Delicate observations have, however, shown the existence of a gradual shrinkage for many years.

scale bears a less simple relation to the Centigrade. Fahrenheit, its inventor, divided the space between the temperature found in melting ice and that found in boiling water into 180 equal parts, and placed the zero of his scale at a point 32 like divisions below that of the melting-point of ice.

The annexed diagram (Fig. 50) shows the relation between the two scales more simply than any verbal statement. In order to find the equivalent of a Centigrade degree in Fahrenheit degrees we need only remember that 100 Centigrade degrees = 180 Fahrenheit degrees. Therefore 1 Centigrade degree is $\frac{180}{100}$ (= $\frac{9}{5}$) of a Fahrenheit degree, or 1 Fahrenheit degree = $\frac{5}{9}$ of a Centigrade degree. In order to reduce *temperatures* from one scale to another we must take account not only of the different values of the degrees of the two scales, but also of their different starting-points. Suppose that it is required to reduce a temperature of 15° C. to the corresponding temperature on the Fahrenheit scale: 15 Centigrade degrees = 15 × $\frac{9}{5}$ (= 27) Fahr. degrees, but the Centigrade temperature was measured from a point coinciding with 32° Fahr., so the required temperature is 27° + 22° (= 59°) Fahr. Suppose that it is required to reduce the temperature 68° Fahr. to the corresponding Centigrade temperature: 68° Fahr. is 68° − 32° (= 36) Fahr. degrees above the Centigrade zero. Therefore the required temperature is 36 × $\frac{5}{9}$

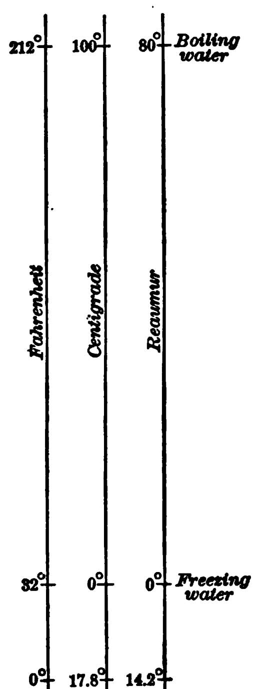


FIG. 50.

(= 20°) C. Stated as formulæ the rules for reduction of temperature would be as follows:

$$F^\circ = C^\circ \times \tfrac{9}{5} + 32^\circ.$$
$$C^\circ = (F^\circ - 32^\circ) \times \tfrac{5}{9}.$$

144. Linear Expansion of Solids.—Nearly all of the solid substances which have been tested are found to expand in all directions as their temperature rises.[1] This statement holds good only so long as the solid remains such;—solids in melting may either expand or contract. If the increase of size is observed or estimated only in one direction, as for example in the lengthening of a bar of metal, the increase is called the *linear expansion* of the bar. In all ordinary cases this lengthening is so slight as to be nearly imperceptible. Although spaces are left between the ends of the rails in laying a railroad-track, to allow of the expansion which may be produced by hot weather, yet to the unobservant eye the spaces appear unchanged through summer and winter. It is only when long pieces or structures of metal or of other solid substances are examined that the lengthening is very manifest. In the spans of long iron bridges the motion of the ends under the influence of changes of temperature may become considerable, as in the case of the iron and steel railroad bridge across the Forth in Scotland. The total length of the bridge, including the approach-viaducts, is 8098 feet, and the allowance for expansion of iron-work over the whole length is 6 feet. The amount of expansion of most metals and of various

---

[1] Perhaps the only observed exceptions are iodide of silver, the compound iodide of lead and silver, Rose's "fusible alloy," garnets, and india-rubber. The last-named substance, if heated when unstretched, expands in all directions, but if it is stretched, rise of temperature increases its already existing tendency to contract. In other words, rise of temperature increases its elasticity (§ 10).

other solids per degree Centigrade has been ascertained by many accurate experiments, and it is found that the expansibility varies greatly with the substance (see Appendix), no two ordinary metals, for instance, expanding the same amount for the same increase of temperature.

145. Modes of Observing Expansion.—In order to measure the trifling increase of length of the rod or bar experimented upon, it is necessary to magnify the actual movement of the free end of the rod, either by a suitably adjusted microscope or by some mechanical device. In the following exercise a rough illustration is given of the latter mode of proceeding.

146. EXERCISE XXIV.

LINEAR EXPANSION OF A SOLID.

Apparatus: A cylindrical tube of brass about 6 or 7 mm. in diameter and about 60 cm. long. A cylindrical tube of sheet-iron, tinned or "galvanized," open at both ends, about 2.5 cm. in internal diameter and 6 mm. shorter than the brass tube which is to be heated in it by the action of steam.[1]

Two large cork stoppers to close the ends of the main tube, and through which the ends of the brass tube are to project. A wooden rack for holding the iron heating-tube in position.[2] [Fig. XV.]

---

[1] Near each end of this tube a short tube about 6 mm. in internal diameter and about 3 cm. long is inserted in such a way that when the main tube is nearly horizontal steam may enter it downward at one end and leave it downward at the other end. A tube about 1 cm. long and 1.5 cm. wide inserted at the middle of the main tube, on the same side as the entrance-tube, gives admission to the bulb of a thermometer.

It has been suggested that this tube can be used as the condenser of a still for obtaining distilled water. For this purpose use a larger brass tube and send through it a constant stream of cold water, leading steam as before into the outer tube.

[2] This has a base-board, protected from *warping* by cleats on each side, about 1 m. long and 15 cm. wide, on which are two uprights

Apparatus A for generating steam. A rubber tube 70 or 80 cm. long for carrying the steam to the iron heating-tube, and a

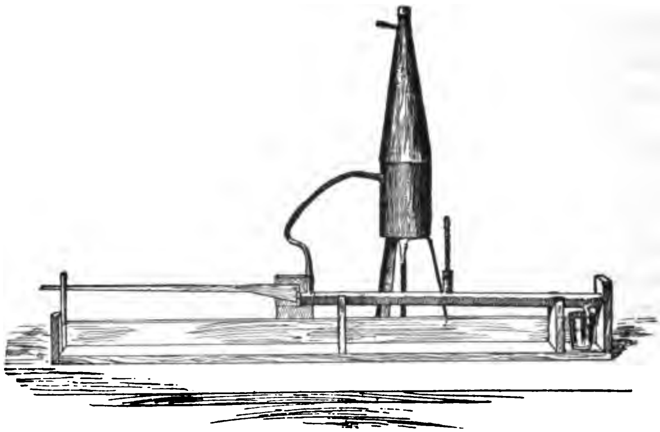


FIG. XV.

small vessel to catch the water which drips from the exit. A thermometer reading to 100° C.

---

about 15 cm. tall, each having a wide-angled V-shaped notch at the top, in which the tube rests. Another upright, somewhat taller, against which one end of the brass tube is to rest while in use, is placed at one end of the base-board. A screw with a broad head is driven horizontally into this upright in such a position that one end of the brass tube may rest against the head. An upright at the other end of the base-board is to hold a mm. scale in a vertical position. The end of the long arm of a right-angled lever is to move up and down along this scale, the vertical short arm, which should be a metal strip, resting against one end of the brass tube, the fulcrum of the lever being the cylindrical part of a screw which is fixed horizontally in another upright attached to the base-board. The horizontal long arm of this lever should be a piece of wood about 45 cm. long, 1½ cm. wide horizontally, and 1½ cm. thick near the fulcrum. The arrangements should be such as to make the movement of the index along the mm. scale about twenty times as great as the expansion of the bar.

A "machine" screw should be used for the fulcrum, and the

Adjust the brass tube along the axis of the main cylinder, the ends passing through the centres of the large corks closing the ends of the cylinder and projecting only 2 or 3 mm. at each end. (As little of the tube as practicable should be imbedded in the corks. Hence the corks should be cut down very short.) Fix the thermometer, passing through a cork, in the short tube at the middle of the main tube. Place the entrance-end of the cylinder next to the lever and depress the exit-end a little (one upright of the rack should be a few millimeters higher than the other) in order that the water which will be formed by condensation in the tube may drain off. After everything is adjusted, but before steam or heated air begins to enter the cylinder, measure the long arm of the lever, i.e., the distance from the centre of the fulcrum-screw to the mm. scale, and the short arm, i.e., the distance from the centre of the fulcrum-screw to the point where the end of the brass tube touches the vertical arm (which point should be in a vertical line with the centre of the fulcrum), read the indication of the lever and note the temperature in the cylinder. Then heat quickly with a generous flow of steam, watching the thermometer and letting the steam flow for a few minutes after the mercury ceases to rise, or as long as the end of the lever continues to rise.[1] When a stationary condition is reached, read the mm. scale and the thermometer again, then detach the tube from the boiler, taking the greatest care not to disturb the cylinder in any way, allow the apparatus to cool, occasionally pressing the back end of the brass tube against the head of the bed-screw, and note whether the index returns in time, as it should, to its original position.

With the data given by this experiment calculate the coefficient of expansion for the brass used, i.e., the ratio of the expansion per 1° C. to the original length of the tube.

(It is well in this experiment to shield the wooden rack, as much as practicable, from the radiation of the flame and the

---

lever should turn upon it readily by its own weight, yet without looseness.

[This apparatus has recently been improved by using a brass casting at the fulcrum end of the lever and by soldering a very short piece of fine steel wire to the brass tube to press against the short arm of the lever.]

[1] It is best to *calculate* the temperature of the steam when a barometer is available, instead of depending upon the thermometer.

boiler. Care should be taken, moreover, to prevent the iron heating-cylinder from rotating. This can be done by making the entrance-tube lean against the post that carries the fulcrum.)

147. Calculation of the Coefficient of Expansion of Brass. — From the data obtained we are now to calculate the *coefficient of expansion of brass for* 1° *Centigrade,* that is, the fraction of itself by which the length of a bar of brass is increased when its temperature is raised 1° C. Our calculation assumes that the expansion is equally great for each and every degree of the rise of temperature. This is not quite true. The quantity which our calculation gives is the average, or mean, value of the coefficient between the limits of temperature used.

We do not know, without calculation, the amount by which the bar was lengthened, but we do know the number of millimeters traversed by the pointer on the scale; and from this, knowing the length of the two lever-arms, we can calculate the actual elongation of the bar, which will be called e.

Let t° = the temperature of the room at the beginning of the experiment;

t'° = highest temperature at which a reading was taken ($= 100^\circ \pm$);

l = length of bar at t°;

l' = length of bar at t'°;

k = coefficient of linear expansion for 1° C.

Then from the very definition of the coefficient of expansion we have

$$k = \frac{e}{(t' - t)} \div l = \frac{e}{(t' - t)l}.$$

We may evidently write also

$$e = k(t' - t)l, \quad \text{and} \quad l' = l + e = l(1 + k(t' - t)),$$

equations which will be found useful in problems relating to expansion. The way in which k is used in the latter equations shows, to those who have studied algebra, why it is called a *coefficient.*

148. Coefficient at Different Temperatures.—For metals between 0° and 100° the coefficient of expansion is usually nearly uniform, but through wider ranges of temperature this uniformity does not hold good in the case of metals or of other solids. It is found that as the temperature grows higher expansion usually proceeds at a more rapid rate. In the case of a specimen of glass whose mean coefficient per degree was ascertained, first from 0° to 100° and then from 0° to 300°, the coefficient k was found in the second case to be about 20 per cent. larger than in the first.

149. Coefficient of Cubical Expansion.—Since each edge of a cube whose length at 0° is 1, becomes $1 + k$ at 1°, the volume of the cube which at 0° was 1, will at 1° become $(1 + k)^3$, or $1 + 3k + 3k^2 + k^3$.

Now the value of k is always a very small fraction (see Appendix), and consequently its square or its cube is so extremely small a quantity that it may for common purposes be altogether neglected, so that the value of $(1 + k)^3$ is practically equal to $1 + 3k$. Suppose, for instance, that for some particular substance $k = .001$. Then if the object experimented upon is a cube 1 cm. on an edge, it will after being heated 1°, have a volume of $(1.001)^3 = 1.003003001$; so that, reckoned to five places of decimals, the cubical expansion, expressed as a fraction of the original volume, is just three times the linear expansion expressed as a fraction of the original length. This conclusion is confirmed by the results obtained from direct measurements of cubical expansion of solids. The coefficient of cubical expansion is defined as the ratio which the increase of volume for 1° rise of temperature bears to

the original volume.[1] In the case supposed it is .003, which is three times as large as the coefficient of linear expansion, .001. If we had not 1 cu. cm. only, but a body of V cu. cm., having k as its coefficient of linear expansion, the volume after a rise of temperature from t° to t'° would be $V' = V\ (1 + 3k\ (t' - t))$, the increase being $3k\ (t' - t)\ V$. We shall represent the quantity $3k$, the coefficient of cubical expansion, by K.

150. Expansion of Liquids.—Liquids, like solids, vary greatly among themselves in the amount of expansion which they undergo for equal increments of temperature (see Appendix). In liquids we generally have to do with cubical expansion, and we may, as appears from § 137, in measuring this take account either of apparent or of real expansion. The real expansion of a liquid may be determined by using the device of balancing columns described in Exercise X. This method properly applied gives the ratio between the density of the liquid cold in one branch of the tube, and of the same liquid hot in the other branch, and from this ratio the rate of expansion is easily found.

Mercury is the liquid that has been most carefully tested in this way. The real expansion of other liquids may be determined by a like test, but the more common method is to measure first their apparent expansion in glass bulbs with slender stems attached, and then add to the apparent expansion the expansion of the bulb. The expansion of the bulb is measured by means of the already determined real expansion of mercury, as follows: fill the bulb and stem with mercury at 0°; raise the temperature to 100° and catch the small quantity of mercury which overflows during

[1] In the case of gases, which are very expansible, it is customary to divide the increase per degree by the volume at 0° C., even when the actual starting-point of the expansion is some other temperature.

the heating. Weigh this amount of mercury, and weigh also the amount which remains in the bulb and stem. Knowing the real expansion of mercury, one can calculate how great the overflow would be if the bulb did not expand. Comparing this with the actual overflow, one can find how much the bulb has expanded.

151. Irregular Expansion of Water and other Liquids.— If a thermometer-bulb and tube of suitable proportions were filled with boiling hot water and then allowed to cool slowly to the temperature of melting ice, the surface of water in the tube would fall slowly until a certain point was reached, after which it would rise until the water became of the same temperature as the melting ice. The turning point, at which contraction ceases and expansion begins, when the water reaches its temperature of maximum density, is very near 4° C. Water taken at this temperature, then, becomes less dense upon being either cooled or heated. Its density at 0° C. is substantially the same as at 8° C. If the volume of a given portion of water is 1.00000 at 0° C., it would be about

0.99987 at 4° C., 1.01180 at 50° C., and 1.04300 at 100° C.

An important consequence of this curious irregularity in the behavior of water on cooling below 4° is that lakes, ponds, and rivers in severely cold weather, when the temperature at the surface approaches 0° C., tend to maintain at the bottom a temperature of 4° C., so that ice does not ordinarily form in their depths. Large bodies of water in which the natural course of things is not disturbed by deep currents or the presence of springs,—deep lakes like Lake Tahoe in California, for instance,—show a bottom-temperature of about 4° C. at all seasons. Salt water continues to contract below 4° C. and, if it is salt enough, down to and below its ordinary freezing-point, which is itself below 0° C.

For equal increments of temperature liquids in general expand much more than solids.

Most liquids expand more rapidly at high temperatures than at low temperatures, according to the mercury thermometer.

It is a consequence of the second fact just mentioned that if a thermometer were made after the fashion of a mercury thermometer, but with a different liquid,—glycerine, for instance,—and if the expansion of this liquid between the freezing and boiling temperatures of water were divided into 100 equal parts, each called 1°, this thermometer would not in general agree with a mercury thermometer, if each were true to itself. According to the mercury thermometer glycerine expands irregularly. According to the glycerine thermometer mercury expands irregularly. We take as a standard for ordinary purposes that thermometer which we find most convenient, the mercury thermometer; but for more refined purposes we find a better standard in some form of gas thermometer (see § 157).

152. Expansion of Gases.—The rapidity with which gases expand when heated was well illustrated by the promptness with which air-bubbles began to escape when the bulb was heated in Exp. 20. The accurate measurement of the expansion of gases is, however, complicated by the fact, already well known to the student, that the volume of a gas is greatly dependent upon the pressure it bears. In fact the pressure upon a gas which is being heated may be so manipulated as to make the rate of expansion anything we please, large or small. It is customary, however, in studying the effect of rise of temperature in gases to follow, as closely as may be, one of two courses: 1st. The pressure may be kept unchanged during the heating, in which case we find the *increase of volume with pressure constant*; 2d. The pressure may be so varied that the

volume remains unchanged during the heating, in which case we get the *increase of pressure with volume constant.* The first method is followed in Exercise XXVI, the second in Exercise XXV.

153. EXERCISE XXV.

INCREASE OF PRESSURE PRODUCED BY HEATING A GAS AT CONSTANT VOLUME.

[Trowbridge, Exp. 36.]

Apparatus: A bent glass tube[1] containing dry air held in by a column of mercury. The heating apparatus and the rack, without the lever, used in Ex. XXIV. A meter-rod. A "galvanized iron" tray about 60 cm. long, 15 cm. wide, and 5 cm. deep, having a tubulure at one end. (This tray is to hold ice-water or snow for cooling the air-column, the tube containing which is thrust through a cork in the tubulure. If the tray have two tubulures at

---

[1] Take a piece of thermometer-tubing about 1 m. long and having a bore 1 or 2 mm. in diameter. (Uniformity of bore is of no consequence for Ex. XXV, but is important for Ex. XXVI, and one properly selected tube will serve for both these exercises.) To test the bore, introduce into it a column of clean mercury about 10 cm. long, and measure the length of this column at four or five places, beginning at one end of the tube and going toward the middle. If this column does not vary more than 2 or 3 mm. in length in moving from one end of the tube to the middle, the tube is satisfactory.

Heat the tube in a narrow flame at a point 2 or 3 cm. from the end of the half that has been tested, and draw it out at that point to a thickness of 1 or 2 mm. Then bend the tube at a right angle near its middle. It will now, if clean inside, be ready to be filled with dry air.

Take a clean bottle having a mouth 3 or 4 cm. wide, fill it to a depth of about 2 cm. with clean mercury, and fit it with a rubber stopper pierced by three holes. Through one of these holes pass that end of the bent tube which has not been drawn out in the flame, and push the tube down until its end is well covered by the mercury. Into the other holes pass short glass tubes, not allowing them to reach the mercury, and connect one of these tubes with an air-pump, the other with a glass tube about 1 m. long dipping

each end, it can be used to cool four air-tubes at once.) A quantity of ice or snow. A barometer.

---

vertically into a cup of mercury, to serve as a pressure-gauge. [See Fig. XVI.]

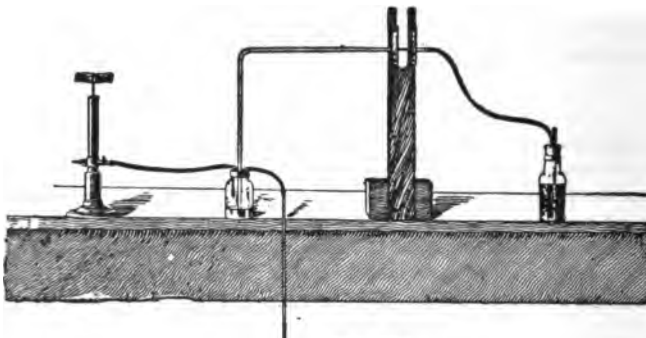


FIG. XVI.

Make a drying apparatus by half filling a wide-mouthed pint bottle with cheap glass beads. Pour concentrated sulphuric acid into the bottle until the beads are covered by it. Close the mouth of the bottle with a rubber stopper pierced by two glass tubes, one reaching nearly to the bottom of the bottle, the other, which should contain a soft wad of cotton to prevent stray particles of acid from passing through it, reaching just through the stopper. Connect this second tube with the free end of the tube which is under treatment, supporting the latter in such a way that it will not accidentally break off, but may be readily *melted* off, at the narrowed section. Now heat this long tube, especially its horizontal part, until it hisses when touched with a wet finger, drawing air gently through it meanwhile by the action of the pump, taking care that the joints are so tight that air can enter the tube only by bubbling up through the sulphuric acid. When this process has gone on for two or three minutes after the tube has become thoroughly heated, melt off the tube at the narrowed portion, thus sealing it at one end and imprisoning within it a column of dry air. Let the tube cool, its open end remaining sunk in the mercury, to the temperature of the room. Then work the pump again and remove so much air from the tube

Pack the air-filled arm of the tube in melting snow or ice and water, keeping the other arm nearly or quite horizontal [Fig. XVII],

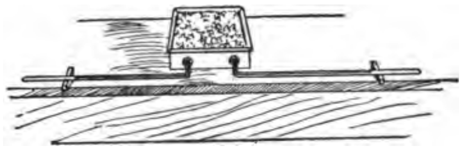


FIG. XVII.

and allow the air to cool until contraction ceases. Make a light, sharply-defined mark with a sharp file on the tube exactly at the inner end of the mercury-column, which should be as near the snow as practicable, in order that the whole of the air-column may be cooled. Measure carefully and record the difference in height of the ends of the mercury-column above the surface, made level, of the table on which the apparatus rests, and note the reading of the barometer. [It is best, if practicable, to have this difference in height zero. In order to have the pressure of the confined air exactly equal to that of the air outside it is only necessary that the two ends of the mercury-column should be at the same height; other parts of the column may be higher or lower without harm (see Ex. V). An inexpensive carpenter's level is very useful in this exercise.] Then heat the air-filled arm, kept nearly horizontal, in steam, using the same heating-apparatus in which the brass tube is heated in Ex. XXIV. As the air is heated, rotate the tube in the cork [there are certain

---

that the remainder will, when subjected to full atmospheric pressure at the ordinary temperature, occupy about 30 cm. of the closed arm of the tube. Then allow full atmospheric pressure to act upon the mercury in the bottle. This will force a column of mercury up the tube and compress the confined air. When this process has ceased, draw the tube carefully out from the mercury bottle. It is then ready for use. When it is not in use the unsealed end should be kept uppermost, and it is well to have it stopped with wax. It is important to handle the tube carefully after it is filled, as jarring is likely to cause a breaking-up of the mercury column. When such confusion occurs it may sometimes be remedied by running a thin iron wire along the bore of the tube. [The iron wire should first be freed from kinks by fastening one end and pulling the other end until some permanent stretching is produced.]

advantages in using a perforated tin cap instead of a perforated cork, where the glass tube enters the large heating-tube] through which it reaches to the interior of the heating-cylinder in such a way as to make the mercury-column in the open arm approach a vertical position, thus increasing the pressure upon the heated air-column and keeping its length the same as when it was in the snow, the inner end of the mercury-column being kept accurately at the mark made by the glass, and this mark being kept just in view at the end of the cork through which the tube extends into the heater. The tube should be rapped gently to enable the mercury to come to its proper position. When the pressure ceases to increase, measure, as before, the difference in height of the ends of the mercury-column. [See Fig. XVIII, in which is shown the use

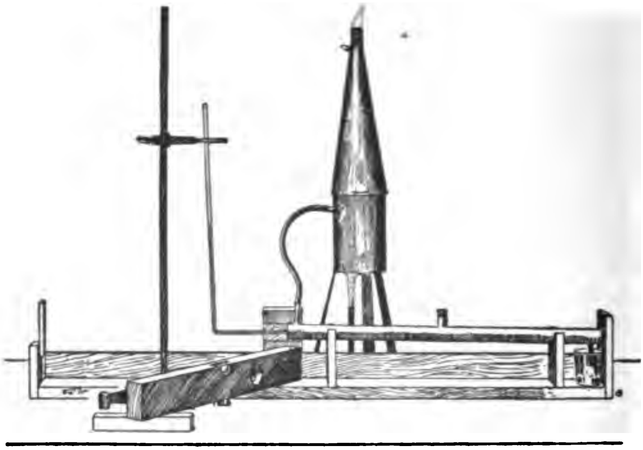


FIG. XVIII.

of a spirit-level, from the upper surface of which the heights may be measured.] At the same time note and record the reading of the barometer.

Calculate the coefficient of expansion of pressure of air at constant volume, i.e., the ratio which the mean increase of pressure per 1° C. throughout the range of temperature used bears to the total pressure at 0° C.

If the two arms of the thermometer were kept level at first, the record of the experiment will read briefly as follows:

Temperature.	Difference of Level.
0°	0 cm.
100° ±	h cm.

Barometer .... H cm.

Coefficient of increase of pressure, as defined in directions for this exercise, $\alpha = \dfrac{h}{100 \pm} \div H.$

154. EXERCISE XXVI.

EXPANSION OF A GAS AT CONSTANT PRESSURE.

[Trowbridge, Exp. 34.]

Apparatus: The same as in Ex. XXV. A straight tube about 50 cm. long may be used, however, instead of the bent tube used in Ex. XXV.

Proceed as in Ex. XXV, but instead of increasing the pressure when the air is heated, keep the pressure unchanged, watching the barometer during the exercise and compensating for any change in atmospheric pressure by proper changes in the inclination of the mercury-column. Measure the increase in length of the air-column, keeping the inner end of the mercury-column during the heating just at the outer end of the cork through which the tube extends into the heater.

Calculate the coefficient of expansion of volume of air at constant pressure, i.e., the ratio which the mean increase of volume per 1° C. throughout the range of temperature used bears to the volume at 0° C.

155. Discussion of Results.—If the two preceding Exercises, XXV and XXVI, have been successfully performed, the student will find that the two coefficients obtained are very nearly equal. In other words, the rate of increase of pressure per degree for air maintained at a constant volume is very nearly the same as the rate of increase of volume per degree for air maintained at constant pressure. An application of Boyle's law (Ex. VI) would have led us to anticipate this conclusion without the performance of Exercise XXV. For we might have reasoned as follows: Suppose a volume V of any gas to be heated to such a

temperature that it would acquire the volume nV, the pressure P remaining unchanged. By Boyle's law (§ 45), temperature remaining unchanged, $V \propto \frac{1}{P}$; therefore to bring volume nV of the heated gas back to volume V without cooling it, a pressure nP must be applied : or in other words, starting with a gas at volume V and pressure P, we may by a certain rise of temperature get, as we may choose, a volume nV at pressure P, or a volume V at pressure nP.

The fact is, however, that Boyle's law, upon which the reasoning just given is based, is not perfectly obeyed by gases. Accordingly the two coefficients described in Exercises XXV and XXVI are not exactly equal, as careful experiments have shown.

156. Behavior of Different Gases; Law of Charles.—It is found that all gases, at temperatures not very near their points of condensation into liquids, expand nearly alike; so that if the student had used coal-gas, carbonic-acid gas, or hydrogen in the two experiments, he would have obtained nearly identical results. The equality of gases in regard to expansion is set forth in a law known from its discoverer as the *law of Charles.* It is variously worded by different authors. Clerk-Maxwell, *Theory of Heat,* states it as follows:

The volume of a gas under constant pressure expands when raised from the freezing to the boiling temperature [*of water*] *by the same fraction of itself, whatever be the nature of the gas.*

157. Relative Merits of Gases and Mercury as Thermometric Substances; Air-thermometer.—The uniformity of the behavior of all gases within a wide range of temperatures is in striking contrast to the behavior of liquids and solids, no two of which, so far as we know, agree exactly in their rates of expansion, or even have rates that maintain a constant ratio at different temperatures. In assuming that the apparent expansion (§ 137) of mer-

cury is regular, as we do in making and using mercury-thermometers, we have to assume that all other substances expand irregularly. Mercury as a thermometric substance has much in its favor. A very low temperature is required to freeze it, and a very high temperature to boil it. Its excellent conductivity enables it to become heated or cooled quickly. It does not wet the tube which incloses it and leave an adhering film, when the column descends, as most liquids would. If a liquid is to be used, mercury is the best one for general purposes. But as all the permanent gases agree very closely in their rates of expansion, it seems better to take a gas as the standard thermometric substance and to test the mercury-thermometer by comparing it with a gas-thermometer. The air-thermometer is therefore considered a better standard than the mercury-thermometer. As sometimes constructed it consists of a bulb *B*, Fig. 51, filled with carefully dried air, connected by means of a slender stem *S* and a short glass tube *S′* with one end of a rubber tube *R* containing mercury, this tube connecting with a glass tube *G* at its other end. The mercury-column confines the dry air and, the outer end being raised or lowered at will, serves to exert upon it the varying pressure required to keep the volume of the air unchanged when its temperature is raised or lowered. The temperature of the air at any time is calculated from its pressure. The bulb may have a capacity of one or two hundred cubic centimeters and the whole instrument is too cumbrous for common purposes. It can

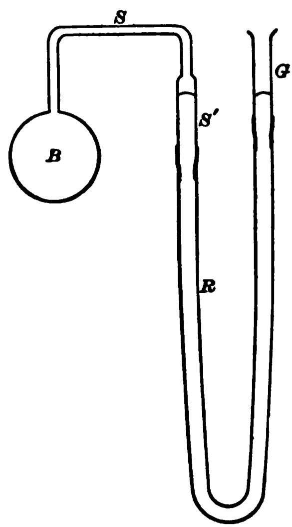


FIG. 51.

be used, however, for testing and correcting mercury-thermometers intended for very accurate work.

158. On the Working of Problems relating to Expansion.—When we have a solid or a liquid of volume V at temperature $t°$ and wish to find what its volume V' would be at $t'°$, we have only to use the formula already familiar (§ 149) and write $V' = V(1 + K(t' - t))$, where K is the coefficient of cubical expansion as already explained.

When the case is that of a gas, we cannot write simply $V' = V(1 + \alpha(t' - t))$ (see § 153 for α), unless the $t°$ is the temperature of melting ice; for in the case of a gas the coefficient is, as we have seen, obtained by comparing the increase of volume with the volume at 0° C. To illustrate, let us suppose that we have 500 cu. cm. of gas at 0° C. and raise it to 1° C.; the increase is .00366[1] $\times$ 500 (= 1.830) cu. cm. For the rise from 1° C. to 2° C. the increase would be the same, .00366 $\times$ 500, *not* .00366 $\times$ 501.83. For every particular degree through which the temperature of the gas may be raised the increase will be the same: not .00366 $\times$ *the volume at the beginning of this particular degree,* but .00366 $\times$ *the volume which the gas has at* 0° C. In the case of solids and liquids it is not necessary to be so precise, for with these substances $K \times$ *the volume at* 0° *C.* would be practically the same as $K \times$ *the volume at any ordinary temperature.*

PROBLEM.—If the volume of a certain quantity of gas is 500 cu. cm. at 50° C. under a pressure of 76 cm. of mercury, what would be the volume at 150° C. under the same pressure?

We can readily solve this problem by finding first the volume at 0° C., which we will call V_0. We know that V_{50}, the volume at 50°, $= 500 = V_0(1 + .00366 \times 50)$. (A)

Whence $$V_0 = \frac{500}{1 + .00366 \times 50} = 422.7 - . \quad . \quad . \qquad \text{(B)}$$

[1] The true value of α is supposed to be between .00366 and .00367.

Then the required volume

$$V_{150} = V_0 (1 + .00366 \times 150) = 422.7 \times 1.549 = 654.8 -. \quad (C)$$

An examination of the equations (A) and (C) shows that we might have written

$$V_{150} = V_{50} \times \frac{1 + .00366 \times 150}{1 + .00366 \times 50},$$

and with this equation have found V_{150} without mentioning V_0. We see, then, that the general formula which may be used in such cases is

$$V_{t'} = V_t \times \left(\frac{1 + .00366t'}{1 + .00366t}\right).$$

This, however, is an awkward formula; it can be greatly improved by dividing both terms of the fraction

$$\frac{1 + .00366t'}{1 + .00366t} \text{ by } .00366.$$

This gives approximately

$$V_{t'} = V_t \times \frac{273 + t'}{273 + t}.$$

This last is a very convenient formula. It tells us that in such a problem as that just discussed we have merely to divide the original volume by the original temperature *plus* 273, and multiply the quotient by the final temperature *plus* 273.

If we had taken a case in which V is constant and P changes, we should have reached the formula

$$P_{t'} = P_t \times \frac{273 + t'}{273 + t}.$$

In cases which involve a change of both V and P with change of temperature we shall do well to imagine the change of volume to be completed before the change of pressure is begun. Let the original temperature, volume, and pressure be $t°$, V, and P, and the final temperature and pressure $t'°$ and P', what is the final volume? If the pressure did not change, the change of temperature from $t°$ to $t'°$ would give

$$\mathbf{V}' = V \times \frac{273 + t'}{273 + t}.$$

The change of pressure from P to P', following the temperature change, would give

$$V' = \frac{P}{P'} \times \mathbf{V}' = \frac{P\,V}{P'} \times \frac{273 + t'}{273 + t},$$

which is the quantity that was to be found. We can write this equation thus,

$$\frac{P'\,V'}{273 + t'} = \frac{P\,V}{273 + t}.$$

This form is easily remembered, and enables us to find any one of the quantities t, V, P, t', V', P', when the other five are given.

Simple as this last equation is, it would be still simpler if, while retaining the Centigrade degree as our unit of temperature, we were to call the temperature of melting ice 273° instead of 0°, the temperature of boiling water 373° instead of 100°, and in general call every temperature 273° higher than we now call it. The interval between the freezing temperature and the boiling temperature of water would still be 100°. All *intervals* would be expressed just as before. If we should adopt the scale of temperature thus described and use the letter T in designating temper-

atures on this new scale, we could write, instead of the equation just given,

$$\frac{P'V'}{T'} = \frac{PV}{T}.[1]$$

This formula is so simple and so easily remembered that for the purpose of making calculations we frequently do use the scale of temperature indicated by T. This scale is called the *absolute* scale of the air-thermometer, and the temperature from which this scale is supposed to start is called the *absolute zero* of the air-thermometer. It is far below any temperature that science has yet produced.

159. Applications of the Laws of Heat-expansion.—Since all known substances change their volume with every change of temperature, numberless occasions arise in physical investigations and in the affairs of practical life for taking account of the expansive action of heat. Standards of length must always be constructed for a particular temperature, and if they are not used at that temperature allowance for their change of length must be made in accurate work. All tables of density or of specific gravity must be stated with reference to a standard temperature. For example, specific gravities are usually reckoned for the substances tabulated taken at 0° C. compared with water at 4° C. Since the determination could not well be made with the water at one temperature and the object in it at another, a calculation is necessary to obtain the specific gravity under the required conditions from that observed under the actual conditions of the experiment. Clocks and watches cannot be made to keep accurate time through variations of temperature unless their pendulums or balance-wheels are so constructed that the expansion of one

[1] In cases which do not involve a change of temperature we can write $PV = P'V'$, which is *Boyle's Law*, with which we are already familiar.

part shall offset the expansion of another part, made of different material. One of the most intelligible of these devices is that in which the pendulum-rod supports, in a sort of stirrup, a narrow bottle or jar containing a tall cylinder of mercury, whose upward expansion just counterbalances, in its effect upon the time of vibration, the lengthening of the rod.

The *force* with which stout rods and beams of metal expand and contract is enormous, since it is precisely equal to that which would have to be applied mechanically to shorten or to stretch the rod enough to undo the change of length produced by the temperature-change. Advantage is taken of this fact to shrink on the tires of wooden and of iron wheels, to rivet the seams of steam-boilers, etc., the tire or rivet being put into place while red-hot, and allowed to draw together, by its contraction on cooling, the parts to which it is applied.

The draft in a stove or an ordinary furnace, the circulation of hot air through furnace-heated houses, and of water through those heated by hot-water pipes; all are due to the unequal density of air or water at different temperatures. Winds are due to unequal heating in different portions of the atmosphere, and ocean-currents to the rise of water heated by a tropical sun and by the flow of this water away from the equatorial regions, while its place is taken by the in-rush of cooler waters from regions more distant from the equator. The importance of this joint circulation of air and water in equalizing the temperature of the earth's surface is incalculable.

Questions and Problems on Chapter VIII.

1. How could you construct a mercurial Centigrade thermometer so as to make a degree very long—for instance, a centimeter or more?

2. The space above the mercury in a good thermometer

is meant to be a vacuum. What advantage has this plan over that of allowing air to remain in it?

3. The standard platinum meter of France is correct at 0° C. What is its length at 20° C.?

4. Indicate the calculations by means of which you found the coefficient of expansion of brass from your experimental data.

5. What weight could be lifted by a rod of brass of 1 sq. cm. area of cross-section and 10 m. long, cooling from 30° to 0°? (See Exercise II.)

6. A glass rod is graduated in millimeters and is correct at 0°; a rod of steel is graduated in millimeters and is correct at 15°. At what temperature (above 15°) will the lengths of the divisions on the two scales be equal?

7. What is the error of the boiling-point in a thermometer that reads, in free steam, 100°.1, when the barometer stands at 77.8 cm.?

8. Explain what measurements in Exercise XXIV must be made with the greatest care, and give reasons for your statement.

9. A glass bulb is just filled by 100 cm. of mercury at 0° C. If the coefficient of cubical expansion of mercury is .00018 and that of glass .000025, what decimal part of the original mass of mercury will remain in the bulb when it is heated to 100° C.?

10. The same bulb, refilled at 0°, is heated until it loses 0.8 cm. of mercury. To what temperature is it raised?

11. The barometer out of doors at 0° stands at 75 cm. What will be the reading after it has been brought into a room in which it attains a temperature of 18° C., the atmospheric pressure remaining unchanged.[1]

12. A certain thermometer is filled to a given height with mercury at 0° and the tube is then graduated. Each

---

[1] Neglect the expansion of the scale.

Centigrade degree is found to measure just 1 mm. The same tube, at 0°, is afterwards filled to the same height with alcohol, and once more graduated. How long are the new degrees?

13. At what temperature would a liter of air weigh 1.419 gm., the barometer reading 76 cm.?

14. A room measures $3 \times 3 \times 3$ m. How many cubic centimeters of air, supposed unexpanded, will escape from it when the remaining air is warmed 1° C.?

15. Under what circumstances would a liter of air weigh just a gram?

16. A balloon contains 500 cu. m. of coal-gas, the barometer stands at 76 cm., and the thermometer at 20° C. The balloon is allowed to rise until the thermometer gives a reading of 19° and the barometer one of 70 cm. What volume does the gas now occupy? (Assume that the gas has the same temperature as the air.)

17. Pressure remaining unchanged, at what temperature would the volume of a quantity of air be twice as great as at 10° C.?

18. Compare the mercury thermometer with the air thermometer, showing the merits of each.

CHAPTER IX.

NATURE OF HEAT ; CALORIMETRY.

160. Nature of Heat.—We have now studied the subject of changes of temperature and the ways of measuring these changes by means of certain effects which they produce in the bodies heated or cooled. We have next to consider the nature of the agent, heat, which must be added to bodies in order that their temperature may rise, and must be taken away from them, in some measure, in order that their temperature may fall. Fifty years ago heat was generally supposed to be a fluid, invisible and without weight, capable of working its way into bodies somewhat as water enters a sponge, thereby causing expansion, and capable, too, of being forced out of bodies by friction or blows. Experiments which should have been fatal to this belief were made about a century ago by Count Rumford, who in boring cannon for the Bavarian government showed that there appeared to be no limit to the amount of heat which continued grinding would bring out of a body, and by Sir Humphry Davy who showed that two pieces of ice could be melted by rubbing them together.[1] The theory survived these seemingly mortal wounds for a long time, but at length, about fifty years ago, the careful experiments of Joule, who measured the amount of heat developed in many different processes, gave it the death-blow, and it steadily, though still gradually, lost its hold upon students' minds.

It is now believed that the extremely minute, singly invisible, particles of which bodies are made up are all the

[1] See Tyndall's "Heat as a Mode of Motion."

time in a state of motion,—that they bump against each other, thus causing a tendency to expansion, and that a rise of temperature is accompanied by an increased tendency to expansion, because it involves an increased violence of the motions and collisions of these particles. In solids and liquids this tendency is supposed to be restrained by the mutual attraction between the jostling particles. In the case of gases this restraint, although there is evidence that it exists, is insufficient to withstand the scattering effect of the collisions between particles, and therefore a gas expands whenever it is not restrained by some force applied from without. As we by doing work can set bodies in motion, thus endowing them with *energy* (§ 128), by means of which they in turn can do work, so, it is believed, we can by friction or blows, or other purely mechanical means, set into more violent motion among themselves the invisibly small particles of which bodies are made up, thus adding to their energy, their power of doing work. We now believe, in short, that *heat is energy, the energy of individual molecules*, as distinguished from the energy of visible bodies. It is like the energy of a mob, each individual of which may be in motion, though the crowd as a whole does not move, while the energy of visible motion is like that of an army moving as a unit. Heat-energy, like any other energy, can be measured in foot-pounds. The experiments of Joule, which have already been alluded to, and of which more will be said later (§ 193), showed very exactly the number of foot-pounds of work which must be done in order to heat a pound of water one degree by stirring, and we have similar information concerning the heating of many other substances.

161. Measure of Heat.—Commonly heat is not measured in foot-pounds or other similar units, the *thermal unit* adopted for convenience being *the amount of heat required to raise the temperature of a certain amount of water one*

degree. The thermal unit used in this book is the amount of heat required to raise the temperature of one gram of water one degree C. This is the unit most used by physicists. It does not have exactly the same magnitude for all degrees, but the difference is small and in this book will be disregarded. Let us now inquire by means of an experiment whether the amount of heat required to raise the temperature of a certain number of grams of another substance is equal to that required to raise equally the temperature of the same number of grams of water.

EXPERIMENT 22.

Make ready 100 gm. of water 10 degrees C. colder than the air of the room, and 100 gm. of mercury 10 degrees C. warmer than the air of the room. Pour both, the water first, into a thin glass beaker which has the same temperature as the air, stir the two liquids thoroughly with a thermometer for one half minute, then note the temperature of the mixture. Which liquid appears to have had the greater influence in producing the final temperature?

Make a similar experiment with water warmer and mercury colder than the air. Which liquid has the greater influence in this case?

162. Specific Heat: Thermal Capacity.—*The ratio which the amount of heat required to raise the temperature of one gram of any substance one degree bears to the amount of heat required to raise the temperature of one gram of water one degree is called the Specific Heat of the given substance.* It is evident from this definition that the specific heat of water is 1. It is evident, too, that the specific heat of any substance is equal to the number of thermal units (§ 161) required to heat one gram of the substance one degree.[1] Water has a greater specific heat than any other distinct substance, either elementary or compound, except hydrogen, that has been examined.

The amount of heat required to raise a given body, large

---

[1] A mixture of water with 20 per cent of alcohol has a higher specific heat than pure water.

or small, one degree is called the *thermal capacity* of that body. If the mass is m grams, and if the specific heat of the material of which the body consists is called h, the thermal capacity of the body is $m\,h$.

163. Calorimetry.—The process of measuring heat, as distinguished from temperature, is known as *calorimetry.*[1] One of the objects of calorimetry is the determination of specific heats. For this determination several methods have been employed. One of the earliest of these was to place a portion of the heated substance in a hole scooped in a cake of ice and cover it with a slab of ice. The amount melted from the ice in this way, the quantity of heat required to melt a given quantity of ice being known (see § 173), served to determine the specific heat required. This general method has been brought to very great perfection by means of an exquisite piece of apparatus known as Bunsen's ice-calorimeter.

The method most commonly used for the determination of specific heats is called the *method of mixtures.* In this method a known mass of the substance to be tested is plunged at a known temperature into a known mass of some liquid, usually water, at a different known temperature, and the resulting temperature, called the temperature of the mixture, is noted. In the use of this method there are various opportunities for error, even if the balances and thermometers are correct and are correctly read. 1st. Each substance when its temperature is taken may not have the same temperature throughout. If the substance is a liquid, or a finely divided solid, it should be thoroughly stirred before its temperature is taken. 2d. A substance may gain or lose considerable heat while it is being poured from one vessel to another. The pouring should be prompt and quick, and through the shortest practicable air-space.

[1] When heat was regarded as a weightless substance this supposed substance was frequently called *caloric*.

3d. The substances may not reach the same temperature before the "temperature of the mixture" is noted. They should be stirred well together, and the thermometer should read the same whether its bulb be near the bottom or near the top of the mixture, before its temperature is taken. 4th. The vessel in which the mixing takes place will probably be heated or cooled by the substance or substances put into it. Allowance must be made for this in the calculations, and in order that this allowance may be small and readily made the vessel should be thin-walled, not unnecessarily large, and, in general, made of metal. 5th. Heat may be lost or gained by the mixture to or from the surrounding air and other bodies before its temperature is taken. An attempt is usually made to keep this loss or gain small by having the liquid into which the heated substance is to be plunged about as much below the temperature of surrounding objects before the mixing as it will be above that temperature at the end of the mixing. As a further precaution, the temperature of the mixture should be taken as soon as it can, by stirring, be made the same throughout.

164. EXERCISE XXVII.

SPECIFIC HEAT OF A SOLID.

[Trowbridge, Exp. 106. Worthington, Exp. 22, p. 177.]

Apparatus and material: About 500 grm. of fine shot, the finer the better. A dipper in which to heat the shot. (This is a sheet-copper cylinder about 10 cm. long and 4 cm. wide, closed at the bottom and encircled, about 2 cm. from the top, by a flat flange of sheet-copper about 4 cm. wide. To this flange is fixed a straight handle.[1]) A piece of pasteboard to cover the top of this dipper. Apparatus A without the cone. A thermometer reading to 100° C. A calorimeter of very thin sheet-brass, about 6 cm. in diameter and 12 cm. tall, brightly polished. (The thin nickel-plated brass vessels, somewhat larger at the top than at the bottom, sold at hardware stores as "liquor-shakers," make excellent calorimeters.) A little ice or snow for cooling water to be used in the calorimeter. A platform-balance.

Put the shot, weighed to 1 gm., into the dipper and cover the top of the dipper with the pasteboard, through a hole in which the thermometer-stem passes, the bulb being plunged into the shot. [See Figure XIX.] Place the dipper in the cylindrical part of Apparatus A, the flange resting upon the top of the cylinder. Close the side tube of the boiler so that the steam will be obliged to pass out under the flange of the dipper. Watch the rise of temperature of the shot, stirring them frequently and thoroughly with the thermometer. (The water in the vessel A should reach nearly to the bottom of the dipper at the beginning of this operation, in order that there may be no danger of boiling it all away before the shot reach their maximum temperature.)

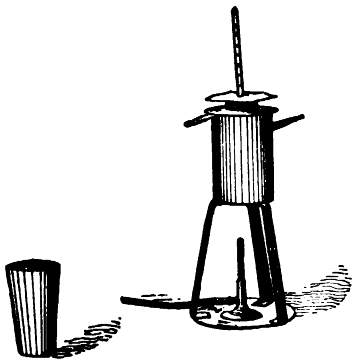


FIG. XIX.

While the shot are being heated about 100 gm. of water should be placed in the calorimeter, which should be conveniently near the heating apparatus, the water being 6° or 8° colder than the air about the calorimeter.

When the shot have attained a practically constant temperature, which will be near 100° C., remove the thermometer, allow it to cool in air to about 40° or 50°, then put it into the calorimeter and find the temperature of the water there, stirring it and giving the thermometer time to come to the proper temperature, meanwhile stirring the shot frequently with any convenient instrument. Then take out the thermometer, and very rapidly take the dipper from the heater, remove the cover, and pour the shot into the calorimeter, taking care to spill neither shot nor water. (It is well to practise a little in pouring the shot, while cold, from the dipper into the empty and dry calorimeter.) Put the dipper aside, at once replace the thermometer in the calorimeter, and, stirring the mixture of shot and water quickly and thoroughly, read the thermometer as soon as the shot and the

[1] This dipper was suggested by a similar one seen in use at the Massachusetts Institute of Technology.

water have reached the same temperature, which will probably be some degrees above the temperature of the room.

The weight of water used, if it is not determined within a few decigrams before the shot are put in, can now be found by weighing the whole and deducting the weight of the shot and the calorimeter. It should not be forgotten in making the subsequent calculations that the calorimeter, in part at least, as well as the water, is heated by the shot. The "thermal capacity," or "water-equivalent," of the calorimeter, if it is of thin brass, can be found with sufficient accuracy for the purpose of this course by multiplying the weight of the vessel, or of that part of it which is below the water-surface, by the specific heat of brass, as given in text-books. The thermal capacity of the thermometer may be neglected.

This exercise may profitably be varied by using sometimes brass-filings instead of shot.

165. Record and Calculation of Results.—Since the heat lost by the shot in cooling to the final temperature is expended in heating the water and the calorimeter to the final temperature, it will be possible to state these losses and gains in the form of an equation.

Let m = mass of water;
m_s = " " shot;
t_w = temperature of water;
t_s = " " shot;
t_m = " " mixture;
m_c = mass of heated part of calorimeter;
h_c = specific heat of material of "
x = " " " shot.

The expression for the amount of heat lost by the shot is

$$m_s \times x \times (t_s - t_m).$$

The amount of heat gained by the water is

$$m_w \times 1 \times (t_m - t_w),$$

the specific heat of water being 1. The amount of heat gained by the calorimeter is $m_c \times h_c \times (t_m - t_w)$, as this is

supposed to rise in temperature just as much as the water does.

Stating the equation, loss of heat by shot = gain of heat by water and calorimeter, we get

$$m_s \times x \times (t_s - t_m) = m_w \times (t_m - t_w) + m_c \times h_c \times (t_m - t_w).$$

Dividing through by the coefficient of x, we have

$$x = \frac{(m_w + m_c \times h_c) \times (t_m - t_w)}{m_s \times (t_s - t_m)}.$$

The quantity $m_c \times h_c$ is frequently called the "water-equivalent" of the calorimeter, because the calorimeter absorbs as much heat as $m_c \times h_c$ grams of water would absorb during the same increase of temperature.

166. Other Calorimetric Experiments.—Exercise XXVII serves as an example of calorimetric work. Other calorimetric experiments will be found in the next chapter in connection with a study of changes of physical state.

Problems on Chapter IX.

1. If the specific heat of mercury is .0333, what will be the temperature of 100 gm. of water taken at 0° C., into which 1000 gm. of mercury at 100° C. are poured and thoroughly stirred?

2. Into 110 gm. of water at 15° C., contained in a vessel the thermal capacity of which is equal to that of 10 gm. of water, are put 200 gm. of a certain solid at 100° C., and the resulting temperature of the whole is 25° C. Calculate the specific heat of the solid.

3. The specific heat of iron being 0.11, how many feet would a mass of iron have to fall in order to be heated 1° C. on striking the ground, if all the heat generated went to raise the temperature of the iron? (To raise one pound of water 1° C. requires 1400 ft.-lbs. of energy. See § 193.)

CHAPTER X.

CHANGES OF PHYSICAL STATE.

167. Change of Properties in Solids by Addition or Subtraction of Heat.—Besides the expansion already mentioned in § 136 as a very familiar effect of added heat on most solids, a number of other changes are usually produced. Solids usually have their rigidity and tenacity lessened by heating. Iron pillars and floor-beams which are amply sufficient to support the floors of buildings become so much weakened upon being heated, if the building becomes thoroughly on fire, that they often yield and fall sooner than fireproofed wooden ones (that is, wooden ones coated with plaster or tiles) would have done under the same circumstances. Zinc, which is not very malleable at ordinary temperatures, may be easily rolled into thin sheets between heated rollers at a temperature of 100° to 150° C., while at a temperature of 200° C. it is so brittle as to be readily powdered in an iron mortar. The power of many solid substances to conduct electricity is diminished by heating so much that this diminution has been used as a means of estimating very high temperatures. Indeed, Sir William Thomson says: "Every known property of a piece of matter, except its gravity and inertia, varies with variation of temperature."[1]

168. Fusion.—Most of the solid *elements*, substances which consist of only one kind of matter, such as the metals, pass at a more or less definite temperature from the solid to the liquid state. So also do many chemical

[1] Article *Heat* in Encyc. Brit., 9th Edition.

compounds and mixtures of compounds, such as common salt, paraffin, beeswax. A comparatively small number of substances, like oxide of arsenic, iodine, and camphor, may pass directly and freely from the solid into the gaseous condition, although iodine and camphor may also be readily melted and then boiled.

EXPERIMENT 23.

Heat a bit of thin soft glass tubing of 1 cm. outside diameter or less, and about 20 cm. long, in the Bunsen-flame or the strongest obtainable flame of a spirit-lamp. Direct the flame on the middle of the tube and rotate the latter so as to heat the entire circumference of the middle portion; then slowly draw out the heated portion until the ends of the tube are pulled apart. Close up each of the drawn-out ends by heating it in the flame and twirling it about as you do so. Now put into one of the closed tubes, after cooling, a bit of oxide of arsenic of the size of a grain of wheat, and into the other about twice this amount of roll-sulphur. Holding the tube horizontal, slowly heat the substance in the closed portion nearly to redness, noting all the changes which the substance undergoes—which substance melts,[1] which is driven off without melting. Does the one which melts suffer any change in appearance during the melting process? What finally becomes of each substance?

169. Melting-points.—The temperature at which a substance melts is called the *melting-point.* The Appendix shows that the melting-points of many substances have been pretty definitely determined.

There is a noticeable difference in the abruptness of the transition from solid to liquid in the case of different substances: ordinary glass becomes plastic at temperatures below redness, while it melts only at an orange or straw-yellow temperature. Wrought-iron and mild steel act in the same way, while cast-iron, antimony, and many other substances, notably water, pass abruptly from the solid to

---

[1] Be careful not to inhale any vapor which may escape from the arsenic-tube.

the liquid condition. The melting-point of a substance is affected by the presence in it of impurities and by change of pressure. Increased pressure lowers the melting-point of a substance which contracts upon melting and raises that of one which expands upon melting (§ 170). The change from this cause is, however, very slight. For instance, one atmosphere additional pressure lowers the melting-point of ice only about .0075 of a degree centigrade.

170. Change of Volume during Fusion or Solidification.—Most solids change their volume during the melting or the solidifying process.

EXPERIMENT 24.

Fill with water and cork very tightly a small bottle (of 50 or more cu. cm. capacity), and bury the bottle in a mixture of snow and salt or pounded ice and salt.[1] Allow the bottle to remain there for half an hour, then remove and examine it to see whether the water has expanded or contracted while freezing.

The fact that ice has a lower specific gravity than water, and will therefore float, co-operates with the fact that the maximum density of water is at or near 4° C. (§ 151) to prevent large bodies of water in cold climates from freezing solid. Most metals and alloys contract in solidifying, but a few, as cast-iron and type-metal, expand, and these alone can be readily and successfully cast when it is necessary to obtain a sharp clear impression of the mould in which the cast is made. In casting steel cannon the melted metal has sometimes been submitted to the action of a powerful hydraulic press, which forces the steel into every portion of the mould and at the same time greatly diminishes the size of any contained air-bubbles.

171. Stability of Temperature during Fusion.—In the concluding portion of Exercise XXIII, in which the mix-

---

[1] For the properties of this mixture see § 172.

ture of snow and water or ice and water was heated, the student probably found that the thermometer rose at first very slowly, and afterwards, as the ice became scanty, more rapidly, until, upon the disappearance of the ice, the rise was very rapid indeed. If the experiment had been conducted much more slowly, allowing merely the heated air of the room to act upon the mixture, there would, as may be seen from the very first portion of the Exercise XXIII, have been no rise of the thermometer at all, as long as a large portion of the ice was found in the mixture. Even with more rapid heating we may feel sure that no rise of the thermometer would be noticeable if each particle of water could be kept in contact with one or more particles of ice.

EXPERIMENT 25.

Half-fill the copper dipper used in Exercise XXVII with fine chips of paraffin. Place this dipper in the cylindrical part of apparatus *A*, nearly filled with water kept at a temperature of about 60° C., and stir the paraffin about with a thermometer without touching the sides of the vessel, taking readings four or five times a minute, as the paraffin melts. If the melting goes on rapidly, lift the vessel of paraffin out of the hot water for a few seconds at a time. When all the paraffin has melted let it rise several degrees in temperature, then take it out of the hot water altogether, and continue to note its temperature as it cools and hardens. How much difference is there between the melting-point and the solidifying point? How does paraffin in this regard compare with ice (or water)?

172. Latent Heat of Fusion; Freezing Mixtures.—It must have become evident to the student by this time that the heat imparted to a melting body produces comparatively little, if any, increase of temperature. A kilogram of crushed ice and a kilogram of water at 0° put in similar vessels and exposed to such equal sources of heat as would be furnished by adjacent lids of an ordinary hot cookstove, would be found, at the end of the few minutes

necessary to melt the ice, to be many degrees apart in temperature, the water in the vessel which contained ice being little above 0°, while that in the other vessel would be hot. The heat which disappears in melting is said to become *latent,* that is, *hidden,* and the phenomenon is not confined to water and paraffin, the only two substances which the student has examined, but occurs whenever a solid is liquefied, either by melting or by being dissolved in a liquid. The student may meet with many apparent exceptions to this rule. Acids in which metals are being dissolved may rise from the temperature of the room to a temperature of 100° C. or over, as is readily shown by putting strips of zinc in strong hydrochloric acid; in all such cases the process is not simple solution, but chemical action accompanied by solution, and the heat produced by the chemical action is here more than enough to balance the cooling which would otherwise be observed. On the other hand, many cases are known in which two solids or a solid and a liquid have sufficient attraction for each other to form a liquid mixture when brought together, but which do not in uniting furnish sufficient heat to provide for the work of liquefaction without fall of temperature. The result is a greater or less cooling, not merely of the mixture itself, but of the surrounding objects as well. Such combinations of substances are therefore known as *freezing mixtures.*

The commonest and most important freezing-mixture is that of ice and common salt (about two parts by weight of the former to one of the latter), so generally employed in ice-cream freezing. This mixture may easily be made to produce a temperature of − 20° C. Fahrenheit took for the zero of his thermometer scale the temperature which he obtained by means of this mixture. With a mixture of properly prepared calcium chloride and snow a temperature of − 48° C. may be reached, and mercury rapidly solidified.

To emphasize still further the demand for heat which

the process of liquefaction involves, one may try the following:

EXPERIMENT 26.

With 50 cu. cm. of water in a thin beaker-glass mix about 10 cu. cm. of alcohol, taking both at the temperature of the room, and note the resulting change of temperature. Then pour into a mass of snow or finely broken ice enough alcohol to moisten it, and note the resulting change of temperature.

173. Exercise XXVIII.—The number of units of heat required to melt the unit mass of a given substance is called the *latent heat of fusion* of that substance. This quantity can most readily be determined by an application of the method of mixtures. It is illustrated in the case of ice in the following exercise.

EXERCISE XXVIII.

LATENT HEAT OF MELTING.

[Trowbridge, Exp. 107. Worthington, Exp. 31, p. 185.]

Apparatus and materials: Ice, about ½ kgm. Warm water. The calorimeter already described. A thermometer. A platform-balance.

Put into the calorimeter about 200 gm. of water at a temperature of about 55° C., and find its weight within 0.5 gm. Check the evaporation by covering the vessel with a piece of pasteboard, leaving a notch at one side to admit the thermometer. Take a lump of clear ice weighing about 150 gm. or more, which should have been previously selected, and pound it up quickly in a cold box into pieces about a large as chestnuts. Observe now and record the temperature of the water, after stirring it, and then immediately put into the calorimeter about 100 gm. of the pounded ice, avoiding the wetter portions. Stir thoroughly, though not violently, with the thermometer, and record the temperature indicated by the thermometer as the last particles of ice melt.

If so much ice has been put in as to cool the water below 5° C., it is well to dip out the ice remaining unmelted at that temperature, taking as little water as possible with it. The exact

weight of the ice is not determined when it is put into the water, but is found by weighing the vessel and its contents after the hurry is over.

The thermal capacity of the calorimeter is to be taken into account in this experiment and in the next, as in the experiment on specific heat.

174. Record and Calculation of Results.—It must be noticed that the ice is first melted, then the water which results from the melting is raised to the final temperature. The heat gained in these two operations must equal that lost by the hot water and the calorimeter which contains it.

Let m_c = mass of cooled part of calorimeter;
$eq.$ = water equiv. of cooled part of calorimeter;
m_w = mass of water;
m_i = mass of ice;
t_w = temperature of hot water;
t_m = temperature of mixture;
x = latent heat of ice.

With these directions the student should be able to form the necessary equation (see § 165), and to find from it the value of x, the quantity which was to be determined.

175. Vaporization; Ordinary Evaporation; Boiling; Dew-point.—Some solids, such as sugar and glue, and some liquids, such as olive-oil, cannot be vaporized[1] without suffering chemical change, by which they are split up into new substances which cannot be reunited by direct means. But the great majority of substances may, like water, exist in all three of the physical states—the solid, the liquid, and gaseous. Vaporization, or evaporation, is not for any particular substance confined to a particular temperature. Water vaporizes at all ordinary temperatures, even below its freezing-point. It is a fact well

[1] A vapor is a very readily condensable gas.

known to housewives that wet clothes hung out in very cold weather will "freeze dry," that is, dry without thawing; and it may be observed that icicles and patches of snow and ice waste away even in severe cold weather, when no thawing could possibly occur. Ordinary quiet evaporation occurs only at the surfaces of liquids or solids where they come in contact with the air, and it may be greatly hastened by increasing this surface. In a process of salt-making formerly a good deal used the brine was concentrated by allowing it to trickle over piles of brushwood, on the surface of which it spread out and rapidly dried away. Increase of temperature, renewing the air in contact with the evaporating liquid, and rarefying the air, all aid evaporation.

The student is probably well aware that a liquid disappears more rapidly when boiling than when not boiling. He has seen, too, in some of the preceding exercises that boiling, in the case of water at least, takes place at a definite temperature, which is, however, somewhat affected by the pressure to which the liquid is subjected. The following exercise is intended to explain the observed facts concerning boiling by revealing the nature of the process. It shows, too, some of the things that occur in the liquid while it is being heated, before it begins to boil. The second part of the exercise is a little study of the process of *condensation,* the reverse of evaporation, the vapor dealt with being that contained by the atmosphere of the room. The *dew-point* is the name given to the temperature at which the water-vapor in the air, when cooling, begins to turn into liquid.

This temperature depends upon the amount of vapor in the air, and differs greatly from time to time. If the room in which this exercise is performed is not very large, it will be interesting to compare the dew-point found before boiling begins with that found after several boilers have been

in operation for a considerable time. Observations of the dew-point are continually made by those whose business it is to predict the weather.

176. EXERCISE XXII.[1]

FIRST PART.

EVAPORATION: BOILING.

Apparatus: A test-tube having a small tube leading out at the side. Two pieces of glass tubing, each about 15 cm. long and in other dimensions nearly like the side branch of the test-tube. A piece of rubber tubing about 15 cm. long, of a size to fit closely over the glass tubing. A solid rubber stopper to fit the test-tube. A tumbler or beaker-glass nearly full of cold water. Some means of heating water at the bottom of the test-tube (a Bunsen burner or a small kerosene lamp). A small plate of glass (a bit of window-pane a few cms. square).

Half fill the test-tube with water. Holding the glass plate just above the top of the test-tube, heat the water somewhat, though not to boiling. Notice and record any change in the appearance of the glass plate. [If the glass gets warm the experiment may fail.]

Now fill the test-tube nearly to the side tube with cold water. Close the top tightly with a cork stopper, pierced however by one of the glass tubes, the end of which should reach 3 or 4 cm. beneath the surface of the water. Fit one end of the rubber tube over the side branch of the test-tube, and fit the other end to the remaining glass tube. Let the other end of this glass tube, which should be kept nearly vertical, dip 1 or 2 cm. beneath the cold water in the vessel provided. Hold the test-tube by means of a thick band of paper near the top and gradually heat the water, applying the heat at the bottom of the tube. [See Fig. XX for a convenient mode of supporting the test-tube by means of a clasp of spring-brass attached to a tumbler containing cold water]. Raise the water thus to vigorous boiling, noting carefully, from

[1] [The student will be much more certain of success in performing the experiment of the last paragraph of the *First Part* (the description of which begins with the words "Remove the cork stopper," etc.) if he substitutes for the test-tube a round-bottomed glass flask of 200 to 400 cu. cm. capacity.]

the start, and recording the appearance and behavior of the bubbles in both the test-tube and the larger vessel alongside, trying to distinguish between bubbles of air and bubbles of vapor.

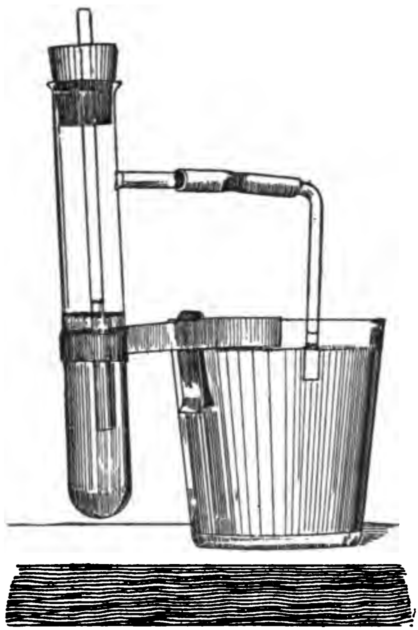


FIG. XX.

Cease heating the water, and note what happens as it begins to cool.

Remove the cork stopper from the test-tube and close the side passage by applying a pinch-cock to the rubber tube. Make the water boil in the test-tube, remove it from the flame,[1] and immediately close it air-tight with the rubber stopper. Cool the seemingly empty top of the test-tube by means of a wet rag or sponge. Note and explain the phenomenon resulting.

---

[1] A dangerous explosion may occur if a vessel containing a boiling liquid is closed tight while the flame is still applied to it.

Second Part.

DEW-POINT.

Apparatus and material: The nickel-plated cup used as a calorimeter in Ex. XXVII brightly polished. A paper-scale thermometer[1] graduated to several degrees below the freezing-point. Ice or snow. Salt.

Pour a little water at the ordinary temperature into the cup, and in this put the bulb of the thermometer. Gradually cool this water by additions of ice-water or ice, and finally of salt if necessary, watching the outside of the lower part of the cup after each addition, until the bright surface becomes dimmed with mist.

Note the highest temperature at which the mist unmistakably appears. Then allow the temperature to rise gradually and note the lowest temperature at which the mist unmistakably begins to disappear. Take as the dew-point the mean of the two temperatures noted. It is interesting to try the effect of breathing upon the cup before the dew-point is reached. During the actual test, however, care should be taken not to let the breath come upon, or near, the cup.[2] The contents of the cup must be thoroughly stirred continually in order that the temperature of the cup may be as nearly as practicable the same as the temperature of the contents in which the thermometer bulb is placed. Note and record the weather and out-door temperature at the time of making this experiment.

---

[1] Paper-scale thermometers are recommended for this course, because they are very easily read, are not easily broken, and are cheap. One hundred of them imported a year or two ago for this course, free of duty, cost the college less than 40 cents each. Their worst fault is a tendency of the mercury to stick in the cavity at the top of the stem after the thermometers have been inverted. The mercury can usually be dislodged from this position by grasping the thermometer firmly at the middle of the stem and giving it a sharp *flirt*, with the bulb outward, so as to cause a strong centrifugal action tending to restore the mercury to its proper place.

[2] Error from this source can best be avoided by keeping some particular spot of the cup turned *from* the observer most of the time and glancing at this spot occasionally in order to detect the appearance or disappearance of mist upon it.

177. Questions on Exercise XXII.—Does the water before it is heated contain any air? If so, in what form is the air, and what does it do while the water is being heated? Are the first bubbles driven over into the cold water air-bubbles or vapor-bubbles? How do you know? Are the bubbles driven over during boiling air-bubbles or vapor-bubbles? What happens to these last bubbles in cold water? Why does the water in the vessel alongside tend to rise into the boiler when the boiling ceases? What prevents it from doing so? What is boiling? Why does it not occur in cold water? Why is the temperature at which it occurs affected by the pressure upon the surface of the liquid?

178. Theory of Evaporation: Maximum Pressure of a Vapor; Saturated Vapor.—It was formerly supposed that air was necessary to the process of evaporation; that, in fact, liquids evaporated because air had a certain attraction for their particles.[1] Evidence in favor of this belief could be drawn from the fact that winds, bringing continually fresh quantities of air into contact with the liquids, assist evaporation. It was shown, however, that the liquids evaporate much more rapidly in a vacuum than in the open air, and therefore the theory that evaporation is caused by the air had to be given up. The present theory is, that, the particles of a liquid being all the time in more or less violent motion (see § 160), some of them near the surface break loose from their fellows even at ordinary temperatures and find their way into the space above the liquid, forming there a vapor which may exist alone or may be mixed with other vapors or with air. If the space in which the vapor forms is the upper part of a closed vessel, the vapor particles within this space may for a time become more and more numerous, the temperature of the liquid remaining constant. But some of them, as they go

---

[1] See Whewell's History of the Inductive Sciences, vol. ii,

knocking each other about in their very brisk motion, plunge back into the liquid, and this return process grows more and more active as the vapor grows more and more dense, until at last it equals the process of liberation. Then evaporation has to all appearance ceased. Particles continue to free themselves from the liquid, but their places are taken by others from the space above, and on the whole the liquid does not waste away, nor does the vapor increase in density or in pressure. It is now said to have its maximum pressure, or "*tension,*" for the given temperature. It is said to be a *saturated vapor.* If one attempts to compress a saturated vapor by putting great pressure upon it, the vapor will, if its temperature remains constant, begin to turn back into the liquid state, to condense, and if the attempt is continued all the vapor will be condensed, maintaining to the last, so long as any part of it remains uncondensed, the same pressure as at the beginning. If one is provided with a mercury-well[1] and with sufficient mercury to fill it, he will find the following experiment interesting and instructive:

EXPERIMENT 27.

Take a glass tube about one meter long, closed at one end; fill it with mercury except half an inch at the open upper end. Fill this last space with ether, then closing the tube tightly with one finger invert it. The ether at once rises to the other end of the tube. Place the lower end under the surface of mercury in the well and then remove the finger. Watch the behavior of the ether, part of which will evaporate. When the mercury column has ceased to fall, measure its height, and by comparing this with the barometer-column find the pressure exerted by the ether vapor. Then slowly,

---

[1] This usually consists of an iron tube about 3 ft. long and $\frac{3}{4}$ in. inside diameter, closed at the bottom and opening at the top into a basin 4 or 5 in. broad. The basin is sometimes of glass and may be made of wood, preferably from a solid block. This well is usually supported by a tripod stand.

taking care not to heat or cool the ether or its vapor, push the tube farther down into the mercury, endeavoring to compress the vapor.[1] Observe whether the mercury column is depressed when the tube descends as much as it would be if the tube contained air. (Apply Boyle's law to this question, or try a separate experiment with air in place of the vapor.) Observe whether the amount of *liquid* ether remaining in the tube changes with ascent or descent of the tube.

Finally, try the effect of warming the tube by rubbing it with a cloth moistened with hot water.[2]

For a table of vapor-tensions see Appendix.

179. Increase of Volume during Evaporation.—1 cu. cm. of water at 100° would make about 1700 cu. cm. of steam, at the standard barometric pressure, at a temperature of 100°. When water is boiled at a temperature higher than 100° C. the saturated steam generated from it has a greater density, as well as a greater pressure, than saturated steam at 100° C.

Most liquids increase in volume less than water upon evaporating, so that their vapors are heavier than that of water.

180. Non-saturated Vapors.—A vapor which is not subject to the greatest pressure which it can bear at its present temperature, that is which can be compressed somewhat at that temperature without suffering condensation, is called a *non-saturated vapor*. Such a vapor, if far from the condition of saturation, behaves under change of pressure and temperature like an ordinary gas. In fact *it is a gas.* Every ordinary gas may properly be considered as such a vapor (see § 183).

181. Temperature necessary for Boiling.—From what has gone before it will be evident that a liquid cannot boil

---

[1] The chief difficulty in this experiment is to remove the air-bubbles from the mercury in the tube. This can be done more or less perfectly by means of an iron wire pushed up and down in the tube.

[2] It should be remembered that ether is a very inflammable substance.

at any temperature below that for which its maximum vapor-pressure is equal to the pressure of the atmosphere, or of any other agent that presses upon the surface of the liquid.[1] The table already referred to shows that the maximum vapor-pressure of water becomes equal at 100° C. to that of 760 mm. of mercury, the standard barometer pressure. Different substances vary greatly in their boiling-points, perhaps no two substances found in nature boiling at the same temperature. The extreme range between the highest and the lowest observable boiling-points has probably not yet been learned: two very wide apart are that of carbonic acid,—78° C., and that of zinc, about 1000° C.

182. Condensation of Ordinary Vapors.—We have already become somewhat familiar with condensation. It occurs whenever an attempt is made to increase the pressure upon a saturated vapor without rise of temperature (§ 178), or to keep the pressure of such a vapor constant during a fall of temperature (Ex. XXII, *Second Part*). Evidently an increase of temperature accompanying an increase of pressure might prevent condensation. Now compression heats bodies, and it is a curious fact that when we try to condense certain saturated vapors by means of pressure, without giving them opportunity to discharge heat to other bodies, they become so much heated that the attempt to condense them fails, and they, on the contrary, become non-saturated vapors (see § 181). Water-vapor is one that acts in this way.

183. Condensation of the so-called Permanent Gases.—The English chemist and physicist Faraday was perhaps the first experimenter who succeeded in liquefying any of the gases which are permanent at ordinary temperatures.

---

[1] If the liquid is deep, the pressure at the bottom, where the heat is usually applied and where boiling commonly begins, will be considerably greater than the pressure at the surface.

He was able by the joint use of cold and pressure to liquefy chlorine, and he afterwards managed by these means to reduce many gases to the liquid state. Six gases, however, —oxygen, nitrogen, hydrogen, marsh-gas, nitrogen dioxide, and carbon monoxide,—had not been liquefied up to the year 1877. In that year a Genevese physicist, M. Pictet, and a French one, M. Cailletet, working independently and by somewhat different methods, succeeded in liquefying all six of these permanent gases. Cailletet's method consisted in confining the gas experimented upon in a very strong glass tube closed at one end, then forcing mercury into the open end by means of a hydraulic press, and finally cooling the condensed gas to the point of liquefaction by a sudden withdrawal of pressure, thus allowing the gas to expand very rapidly (see § 194). The pressure during the experiment rose as high as 300 atmospheres, and the temperature at the moment of expansion fell to a point estimated to be about $-220°$ C.[1] Since air is substantially a mixture of oxygen and nitrogen, it is evident that air is now to be ranked among the condensable gases.

184. Mixtures of Gases and Vapors; Atmospheric Vapor. —When vapors and gases which have no especial attraction for each other are mixed, the pressure of the mixture is the sum of the pressures of its components. For instance, if a cubic foot of oxygen at a certain pressure, a cubic foot of nitrogen at a certain pressure, and a cubic foot of aqueous vapor at a certain pressure, the temperature being the same for all, are crowded together into one cubic foot without change of temperature, the resulting pressure will be equal to the original pressure of the oxygen *plus* that of the nitrogen *plus* that of the vapor. Each of the three constituents is to be regarded as exerting still the same pressure that it exerted before the mixing.

---

[1] Exact determinations of very low temperatures are difficult or impossible to make.

Our atmosphere is a mixture of gases and vapors. When the air contains aqueous vapor in a nearly saturated condition it is commonly called moist air, although strictly aqueous *vapor* is not called *moisture*. The proportion of such vapor in the atmosphere near us may be ascertained by exposing a measured portion of air to the action of a weighed amount of some substance, calcium chloride, for example, which will absorb the water-vapor from the air, and then calculating the amount of this vapor from the increase in weight of the absorbing substance. A more rapid process consists in the use of some sort of *hygrometer*,[1] one of the commonest of which is the dew-point hygrometer (see Exercise XXII). The greater the amount of water-vapor in the air, the higher is the dew-point.

185. Distillation.—A most important practical application of the diversity of substances in regard to their boiling-points is found in the operation of distilling. Any volatile liquid may be separated from less volatile solids or liquids with which it does not combine chemically by heating the mixture to boiling and cooling the escaping vapor until it condenses.

EXPERIMENT 28.

Color some hot water decidedly blue by stirring into it some powdered sulphate of copper. Fill the cylindrical part of Apparatus A about half-full of the solution, put on the conical top of the apparatus, cork the opening at the summit and the tube which proceeds from the side of the cylinder. With a short bit of rubber tubing attach the uppermost tube of Apparatus A to a glass tube some 80 cm. long. Carry this through the stoppered iron tube of Exercise XXIV, and pass a current of cold water through the iron tube. Now boil the solution in Apparatus A violently, and catch in a beaker the water that condenses in the glass tube. Does its color show that any of the sulphate of copper passed over with the steam?

If a liquid, such as crude petroleum, is found to change its boiling-point upon long heating, it is certain that the

---

[1] Greek ὑγρός, *moist.*

substance consisted of a mixture of substances of various boiling-points. These substances may be separated and collected by slowly distilling the mixture, with a thermometer in the path of the escaping vapor, and changing the receiving-vessel every time a rise in the thermometer is noted.

186. Latent Heat of Vaporization: Exercise XXIX.—Every one has noticed how much more than usual the out-of-door cold is felt, especially on a windy day, when the hands are wet. The sensation of cold is still stronger when they are wet with alcohol, which evaporates more readily than water. In the heat of summer the evaporation of perspiration from the skin tends to keep the body cool. When the air contains a great deal of water-vapor, when it is, in common speech, moist, this evaporation from the skin takes place but slowly, and we miss the cooling effect. Weather which is both hot and moist is therefore peculiarly oppressive.

EXPERIMENT 29.

Place a watch-glass, or (better) a thin, small, shallow, metallic vessel, on a drop of water on a large cork. Fill the watch-glass a little more than half-full of carbon disulphide, which is an extremely volatile liquid; blow on the surface of the latter with common hand-bellows or with the breath, and see if the drop of water beneath the watch-glass can be made to freeze by the rapid evaporation of the liquid.[1]

EXPERIMENT 30.

Fill a test-tube half-full of good ether, such as is used by surgeons to produce insensibility, stand a thermometer in the test-tube and then stand the latter in a beaker of hot water, at a temperature of 50°

---

[1] This experiment and the following one should be performed where there is a good draught of air and away from light or flame of any kind. The odor of carbon disulphide is peculiarly disagreeable; its vapor, like that of ether, is highly inflammable, and it is poisonous to breathe.

or 60° C. Note the temperature at which the ether begins to boil, and notice whether the temperature rises after boiling begins.[1]

It is certain that the ether must be continually receiving heat from the surrounding hot water, just as the hot water in Exercise XXIII continued to receive heat from the gas-flame below it; and the fact that neither the ether nor the water rises in temperature after the boiling-point is reached, points to the disappearance of a large amount of heat. In fact the latent heat of vaporization is far greater than the latent heat of liquefaction (§ 172).

EXERCISE XXIX.

LATENT HEAT OF VAPORIZATION.

[Trowbridge, Exp. 108. Worthington, Exp. 33, p. 189.]

Apparatus: The calorimeter already described. A thermometer. Apparatus A. A rubber tube[2] to lead steam from A. A piece of pasteboard to cover the calorimeter, perforated for a thermometer and having a notch at one edge to admit the tube conducting the steam. A platform-balance.

Put into the calorimeter at the start about 275 gm. of water. Condense in it enough steam to raise the temperature about 30° C. Then stop the flow of steam, stir the water thoroughly, take its temperature, and weigh it, doing everything quickly but carefully.

It is well to have the temperature of the water at first about as much below that of the air as it will finally be above it. This device reduces a good deal the error from loss or gain of heat by the calorimeter through external radiation, conduction, and con-

---

[1] If it does, it is because the ether is impure, containing alcohol, etc.

[2] If the tube leading steam from Apparatus *A* to the calorimeter is long and unprotected, considerable condensation will occur in it, and the resulting water will drip into the calorimeter, causing serious error in the result of the experiment. This condensation may be prevented to a considerable degree by the use of a "trap," such as is described by Trowbridge and by Worthington. The side-branch test-tube used in Exercise XXII may well be used as such a trap. If a very short conducting tube is used no trap is necessary, but in this case the calorimeter comes too near the heating apparatus. An effective device (see note by Mr. J. Y. Bergen in the *Journal of Education*, May 23, 1889) is to lead the steam from the boiler to the calo-

vection. The steam should not be admitted to the calorimeter until it is flowing strongly through the conducting tube. The mouth of the tube that admits it should not be plunged far enough beneath the surface of the water to prevent the steam from condensing with a very audible sound. The cover of Apparatus A must fit closely in order to give a good flow of heat through the trap. The steam must not be allowed to play upon the outside of the calorimeter.

187. Record and Calculation of Results.

Let m_c = mass of heated part of calorimeter;
$eq.$ = water-equiv. of heated part of calorimeter;
m_w = mass of water;
m_s = mass of steam;
t_w = temperature of cold water;
t_m = " " mixture;
x = latent heat of steam.

---

rimeter through a glass tube running through the large steam-tube of Ex. XXIV, a flow of steam being maintained in the latter also (see

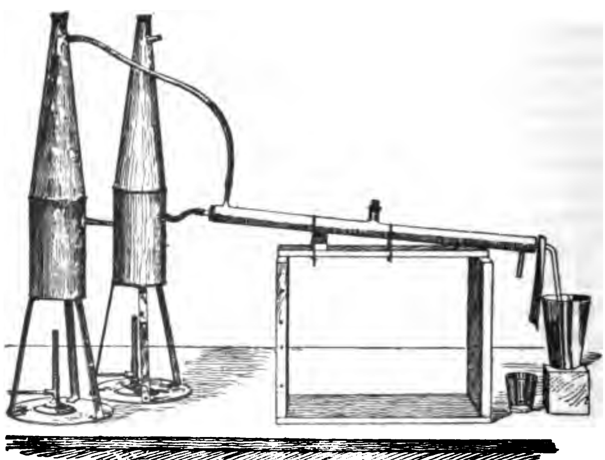


FIG. XXI.

Fig. XXI). The connecting parts outside the steam-jacket should be as short as practicable.

Notice that there are two portions of heat yielded to the cold water, namely, that portion which proceeds from the condensation of the steam and that which is derived from the resulting water at 100°, cooling to the final temperature.

With these directions the student should form an equation between the amount of heat given out on the one hand and taken in on the other, and from this equation find the value of x, the quantity to be determined (see §§ 165 and 174).

188. Steam-heating of Buildings.—The great latent heat of steam makes this fluid an effective agent for the heating of buildings. A comparatively small flow through the radiating pipes is sufficient, for every pound of steam that condenses in these pipes yields a very large amount of heat. (See § 190.)

CHAPTER XI.

TRANSFERENCE AND TRANSFORMATION OF HEAT.

189. Conduction.—Heat may pass from one body to another in more than one way. The way which we shall consider first is that of *conduction.*

EXPERIMENT 31.

Take a rod of copper, a rod of iron and a rod of glass, each about 5 mm. in diameter and 30 cm. long. Support all three on bits of brick or pieces of asbestos laid on a piece of sheet-iron on the ring of a retort-stand. Insert one end of each rod a very little way into the flame of a Bunsen burner, and keep the outer ends of the rods apart. After 15 or 20 minutes test the temperature of each rod by putting on it at successive short intervals, beginning with the outer end, a pinch of powdered sulphur. In the case of each rod stop putting on the sulphur at the point where the latter is seen to melt. The melting-point of sulphur is 111° C. Finally, measure the distance from the 111° point on each rod to the flame.

This experiment will give some rude idea of the relative freedom with which heat travels through the three solids examined. Transmission of heat in this way is called conduction. It is a flow of heat *as such*, that is, without change into any other form of energy, and without the help of any movement, except, of course, molecular heat-movements, in the substance through which the heat is passing. Most ordinary substances have been examined in regard to their power of conducting heat, and it is found that solids, and particularly metals, are the best conduct-

ors, while liquids (except mercury) and gases are extremely poor conductors.

EXPERIMENT 32.

Wind a piece of soft copper wire or a slender strip of sheet-lead about a bit of ice of the size of a hazel-nut, and drop it into a test-tube three-quarters full of ice-cold water. Heat the water at the top of the tube, the ice being at the bottom, by holding the tube inclined in the flame of a Bunsen burner, and notice whether the ice melts rapidly as the upper portion of the water is made to boil.

ILLUSTRATIONS OF CONDUCTION.

Good and bad conductors of heat can frequently be recognized as such by merely touching them. For instance, the coated and uncoated sides of a photographic dry-plate can often be distinguished in the dark by the different degrees of rapidity with which they cool the palm of the hand or the tip of a finger. If a number of bodies all of the same temperature are touched in succession, the good conductors will feel warmer than the bad conductors if all are warmer than the hand, and will feel cooler than the bad conductors if all are, as is generally the case, cooler than the hand. The explanation is that when one touches a bad conductor the spot in contact with the hand, not being freely assisted by the other parts of the conductor, soon rises or falls to a temperature approaching that of the hand at the point of contact. In a good conductor the part touched is not so readily brought to the temperature of the hand. In this test, however, the effect perceived depends upon the specific heat and density of each substance as well as upon its conducting power. The density and conducting power of gases being small, it is possible to hold one's hand for some little time without injury in gases much hotter than the hottest solid or liquid

that one would care to handle. It is a well-known fact that dry steam coming in contact with the skin does less harm than steam mixed with water, although the dry steam may be hotter. Very bad conductors, like cork, felt, and non-metallic powders, are used to impede the flow of heat into or out of bodies, that is, to keep hot bodies hot and cold bodies cold.

Experiment shows that the rate of flow of heat through any conductor from one surface to another is nearly, if not quite, proportional to the difference of temperature of the two surfaces, and is inversely proportional to the distance between them.

190. Convection Currents.—The rapid heating of liquids and gases, so familiar in the case of a kettle put on to boil, or of a room warmed by a stove, is due not merely to conduction of heat, but to this aided by *convection-currents.*[1]

EXPERIMENT 33.

These currents may be shown in air by preparing "touch-paper" by soaking blotting-paper or porous wrapping-paper in a strong solution of saltpetre (potassium nitrate) in water, and allowing the prepared strips to dry. Such paper when lighted glows and emits a good deal of smoke. A strip of the lighted paper may be placed under a large bell-jar, held mouth downward, and currents will rise from the glowing coal, while other currents, outside of the heated region, will descend.

EXPERIMENT 34.

To show convection-currents in water the apparatus shown in

[1] If the laboratory is provided with apparatus for *projection* (§ 281), these may be shown by holding a lighted candle in front of the lens, by means of which a beam of sunlight is spread into a cone and projected upon a white screen in a darkened room. Even the warm hand will produce currents that may in this way readily be rendered visible.

Fig. 52 may be used.[1] Fill the apparatus completely with water, hold it as shown in the figure, and drop in through the tube *a* a very small quantity of any of the aniline dyes soluble in water, —for instance, Hoffman's violet rubbed with a drop of water into a thick paste. Now gradually heat one angle, *b*, of the apparatus by cautiously applying the flame of the Bunsen burner, and note the evidence of currents in the tube.

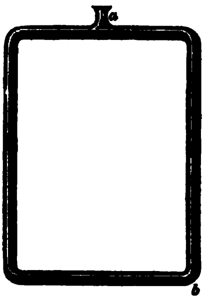


FIG. 52.

This experiment illustrates the method of circulation in the warming of buildings by means of hot water.

191. Currents merely an Aid to Conduction.—Even when currents are set up in a fluid that is being warmed, they do not work independently of conduction, but assist it rather. This they do by constantly bringing colder particles in contact with warmer ones. But the actual transference of heat from one portion of the fluid to another portion is effected to a greater or less extent by conduction.

192. Radiation.—One body may be heated at the expense of another without either convection or conduction between them. The earth is warmed by the sun, but if we start from the earth's surface towards the sun with a thermometer, we shall very soon find that we are not following an ascending scale of temperature in the medium between us and the sun. On the contrary, we get into colder and colder regions above the general level of the earth. If we look for streams of matter coming from the sun to the earth and bringing us heat by convection, we do not find them. Physicists are convinced, by means of experiments not to be described here, that what heat we get from the

[1] A rougher but equally efficient form may be made in the laboratory from large glass tubing and one of the Y-tubes of Exercise X.

sun is transmitted to us by means of a wave-motion of the intervening medium, whatever that medium may be, that fills the space between the heavenly bodies. As waves in deep water travel straight forward, although the single water-particles go but short distances and then return, so waves travel swiftly from the sun to the earth while the particles of the substance that transmits them merely vibrate to and fro. Now regular vibrations, in which every particle repeats the same movements again and again, not jarring against its neighbors irregularly, but moving in system and harmony with them all, are not what physicists call heat-vibrations. We shall therefore speak of the medium between us and the sun as containing, not "radiant heat," as books formerly did, but *radiant energy*, which, coming from the heat of the sun, may be turned back into heat when it strikes the earth. It is not from the sun alone that radiant energy is transmitted. On a small scale the same kind of action is noticeable at moderate distances on the earth's surface, from fires, masses of heated metal, and so on: it is the principal means of heating in rooms warmed by open fires. *Radiant energy may traverse a medium without appreciably warming that medium. Its velocity between us and the sun is about* 30,000,000,000, *or* 3×10^{10}, *cm. per second.*

EXPERIMENT 35.

Hold a convex lens—the larger its diameter the better—in the sunlight above a sheet of dark-colored paper, at such a distance as to throw on the paper as small and bright a spot as possible. Note the result. Try the effect with plain white paper, and again with the same paper blackened with a lead-pencil or covered with lamp-black. See whether a thin piece of polished metal, such as silver, nickel, etc., or a dull piece of sheet-iron, is the more heated by the use of the lens.

The rate at which one body loses heat to another by radiation depends upon the difference in temperature of the

two bodies. If the two are at the same temperature, there is an interchange of radiation between the two; but it is an equal interchange, neither, on the whole, gaining at the expense of the other. If the two are at different temperatures, the hotter loses to the cooler, and loses more and more rapidly if the difference in temperature is increased. When the difference in temperature is moderate, the rate of loss from the warmer body to the other is nearly proportional to this difference; but when the difference of temperature rises to hundreds or thousands of degrees, the rate of loss by radiation increases much more rapidly than the difference of temperature.

193. Mechanical Equivalent of Heat.—The first precise measurements of the amount of work necessary to produce a given amount of heat were made by Joule, an English physicist. His experiments were varied in several ways, but one method which he used was to cause a known weight falling through a known distance to turn paddles inside a vessel containing a weighed portion of water at a known temperature. The friction of the paddles raised the temperature of the water, and from the data obtained in this experiment the value of a thermal unit in foot-pounds could be calculated. Joule found that about 772 ft.-lbs. would raise a pound of water 1° Fahr.; that is to say, a pound of water falling 772 ft. would be raised 1° Fahr., if all the energy of its fall could be turned into heat and all the heat could be kept in the water. To raise a pound of water 1° C., about 1390 ft.-lbs., according to Joule's experiments, were required. Later experiments by Professor Rowland of Baltimore have shown that the quantities 778 and 1400 are more nearly correct than 772 and 1390. Using the C. G. S. system (see Appendix), we find, according to Rowland's experiments, that the mechanical equivalent of 1 gm.-deg. (Centigrade) is 42,690 gm.-cm.

A striking example of the transformation of mechanical

energy into heat is furnished by the operation of the so-called *fire-syringe*, an instrument in which the sudden compression of air heats it to such a temperature as to ignite tinder.

As mechanical work can produce heat, so heat can, on the other hand, be used up in doing mechanical work. For instance, an expanding gas pushing something forward, and so doing work as it expands, becomes cooled by this act.

EXPERIMENT 36.

Place the *bulb* of an air-thermometer[1] under a bell-jar on the plate of an air-pump, the *stem* reaching through a stopper at the top. Rapidly exhaust the air, and note the indications of the air-thermometer. Let the air in quickly after the bell-jar has been nearly exhausted, still watching the thermometer. [The air remaining in the receiver during the exhaustion does *work* in expelling what goes out.]

Experiment shows that one thermal unit used in doing work will yield just so many foot-pounds as in the reverse process would give one unit of heat. This is an illustration of the conservation of energy (§ 133).

194. How a Gas is Cooled in Expansion and is Heated in Compression. — The behavior of gases is best explained (§ 160) by supposing them to consist of very small elastic particles of matter which are flying about in very rapid motion in all directions, frequently bumping against each other and against the walls of the vessel that contains them, but rebounding in a very active manner. The tendency of a gas to expand, shown in the pressure which it exerts upon the walls of the containing vessel, is due to this activity of its particles. The energy of these motions

[1] A so-called "differential thermometer," two bulbs connected by a slender stem bent twice at right angles, and partly filled with colored liquid, will serve here, if one of the bulbs is packed with cotton-wool to make it change temperature less rapidly than the other, the whole instrument being placed under the bell-jar.

is the heat-energy of the gas. To increase the velocity of the particles as a whole is to raise the temperature of the gas, and *vice-versa.* Now when the gas is expanding some part of the containing wall must be moving outward while the air-particles strike it. When it is being compressed, some part of the wall must be moving inward while particles strike it. Why does the gas become cooled in one case and heated in the other case? The following experiment may throw some light on the matter:

EXPERIMENT 37.

Suspend an ivory ball by means of a string several feet long. Hold in one hand a flat weight of two or three pounds. Give the ball a slight push, and on its return through the lowest point of its path let it strike the weight held at rest. Swing the ball again, and on its return let it at the same place strike the *receding* weight. Repeat, letting the ball strike the *approaching* weight. In each case note the effect of the collision upon the velocity of the ball, as indicated by the distance it swings after the collision.

195. Heat-engines.—In all heat-engines, whether they are ordinary steam-engines, hot-air engines, gas-engines, or other forms, we have mechanical work done at the expense of heat, the working substance, whatever it may be, cooling as it expands, because it is doing work, though not always for this reason alone.

196. The Sun our Main Source of Energy.—A consideration of great theoretical interest arises from the fact that nearly all our available sources of energy on the earth's surface are due to the energy of the sun. Wood is produced by the growth of trees, which growth is maintained by solar light and heat. Coal and petroleum are stored-up results of the solar activity of past geological ages. All animals depend for their food either directly or indirectly upon vegetable substances, and therefore upon solar energy. Finally, since running streams depend upon the evaporation of water and its subsequent fall in the

form of rain or snow, and since winds are due mainly to the unequal heating of different portions of the atmosphere, water-power and the motive power of windmills must be referred to the sun as their source.

197. Tendency of other Forms of Energy to become Heat.—Whenever the transformations of any given portion of energy are traced as far as possible it will be found that it tends to take the form of heat. The flight of a cannon-ball, the music of an orchestra, the white light of an electric arc-lamp, the muscular energy of a race-horse,—all end in slightly warming the air and surrounding objects. It may in a general way be said that heat is the lowest form of energy. Other forms run into it as streams run into the ocean.

198. Atoms and Molecules.—*Chemistry and Physics.*—Of all the substances known to us, about seventy are called *elementary substances*, or *elements*. An elementary substance is one which, so far as science has yet ascertained, is not composed of other substances.

The smallest particles of an elementary substance that science has any knowledge of are called *atoms*.[1] The atoms of any one substance are supposed to be exactly alike in all respects.

There is reason for believing that in most elementary substances each atom is, in its ordinary condition, joined in some definite manner to one or more other atoms. These definite combinations of atoms are called *molecules*, and in any particular substance all molecules are supposed to be exactly alike.

The atoms of one element may unite in definite combinations with those of one or more other elements. A group of atoms so formed is also called a molecule, but it

---

[1] The name *atom*, that is, *undivided*, or *indivisible*, should imply merely something that science does not know how to divide.

is a molecule of some new substance, different from either of the elements that go to make it up. Such a substance is called a *compound substance.* It has molecules, but does not have atoms of its own.

The science of *chemistry* deals with atoms and the ways in which they combine to form molecules. *Physics* takes molecules ready-made, and studies their behavior and the phenomena which bodies can show without suffering a break-up of their molecules. Even so great a change as that from ice to water or from water to steam leaves the molecule unchanged in composition, the substance used remaining the same throughout.

Questions and Problems.

1. Make a list of all the common substances that you know may readily be made to appear under all three states, the solid, the liquid, and the gaseous.

2. What conclusion may be drawn from the fact that the inside of the upper part of a bottle partly filled with gum-camphor soon becomes covered with a crystalline deposit? (Solid iodine acts in the same way.)

3. Explain the difference between thermal capacity and specific heat.

4. The climate of islands is more equable than that of the interiors of continents. Explain how this naturally follows from the physical properties of water.

5. If the specific heat of mercury is 0.0333, what will be the final temperature when 1000 gm. mercury at 100° are poured into 100 gm. water at 0°?

6. What is the error of the boiling-point in a thermometer that in freely escaping steam, with the barometer at 77.8 cm., reads 100°.1?

7. Rewrite the formula for calculating specific heat of lead, on the supposition that the shot was cooled to 0° and put into water at 25°.

8. Suppose that you had performed Exercise XXVII exactly as you did, save for the substitution of oil of turpentine ("spirits of turpentine"), sp. ht. 0.47, for the water actually used. Find the final temperature.

9. From an inspection of the table of physical properties in the Appendix what suggestion do you get in regard to the relation between specific gravity and specific heat?

10. If *n* gm. of water taken at 0° C. and *m* gm. of kerosene taken at 20° C. attain when mixed a final temperature of 12° C., find the sp. ht. of kerosene.

11. Tubs or barrels of water in a cellar in winter are said to "keep the frost away" from fruit and vegetables (whose freezing-point is a little below 0°). Explain how this can be true, even if the water used is at 0°.

12. Taking as the value of the latent heat of fusion of ice 80 thermal units[1] and that of the vaporization of water at 100° as 537 thermal units, what will be the number of units necessary to change a kilogram of ice at 0° into steam at 100°?

13. How many inches of rain at 10° would be needed to melt a layer of ice 1 inch thick?

14. From the following data find the latent heat of melting for beeswax:

Weight of water	300 grams.
" " wax	100 "
" " calorimeter	140 "
Sp. ht. of calorimeter material	0.1
Temperature of water just before wax enters	93° C.
" " wax just before it enters water	62° C. (its melting-point).
" " the whole after wax melts	62° C.

(Assume that no heat escapes from the vessel.)

15. Let 100 gm. of ice at 0° C. be put into 300 gm. of water at 50° C., contained in a brass vessel weighing 80 gm.

[1] Gram-degrees (Centigrade.)

Calling the latent heat of water 80 and the specific heat of brass 0.1, calculate the temperature of the mixture.

16. A kilogram of iron at 100° is immersed in a kilogram of ice and water at 0°. The final temperature is 3°. How much ice was present?

17. What is meant by the phrase "the dew-point"? Describe carefully the method by which you determine the dew-point.

18. Describe carefully the nature of boiling, showing how it differs from ordinary evaporation.

19. Describe as fully as you can the transferences of heat that occur in the process of heating a room by means of coils of steam-pipe, beginning with the fire beneath the steam-boiler. Show why the steam in the pipes is more efficient in heating than an equal weight of air or water at the same temperature as the steam.

20. Name all the familiar instances that occur to you of the conversion of mechanical energy into heat, omitting examples given in the preceding chapters.

21. Assuming the average height of Niagara Falls to be 50 m., how much warmer should we find the water at the foot of the falls than at the top? (Assume that all the heat generated by the collision of the falling water with other water and with the river-bed below is imparted to the falling water.)

22. Calculate the number of thermal units necessary to perform the work done in Exercise XXI.

23. How high could a 10-kgm. weight have been lifted by the heat given off during the condensation of the steam in Exercise XXIX?

24. Explain as fully as you can the transformations of energy that take place when steam is generated in a locomotive engine, the engine employed to draw a train at a high rate of speed, and the motion finally checked by applying brakes to the train.

CHAPTER XII.

SOUND.

199. Plan of the following Discussion.—In the study of this subject we shall consider principally—

The means by which sound is brought to our ears;

The velocity with which it travels in its most common medium;

The origin of sounds, and the cause of difference between musical and unmusical sounds;

The means by which musical sounds of a given pitch can be produced.

200. Transmission of Sound.—If an alarm-clock be wound up and placed on some soft substance (wool or cotton-waste) under the receiver of an air-pump, and if the air be then exhausted as perfectly as possible, just before the alarm would begin to sound, the noise of the clock will become so weakened as to be almost inaudible. On repeating the experiment with the receiver full of air the sound is very plainly heard.

EXPERIMENT 38.

Hold one end of a slender rod of wood 3 m. or more in length firmly between the teeth while another member of the class sounds a tuning fork and then holds the stem against the free end of the rod.

The result of this experiment will show the student that other media than air can transmit sound. It is a common and interesting experiment to hold the head under water while a comrade at a distance raps two stones together under water. The loudness of the sound is painful: it may even be sufficient to injure the hearing at a distance

at which the sound produced by rapping the same stones together in air would be only faintly heard.

201. EXERCISE XXX.

VELOCITY OF SOUND IN OPEN AIR.

[Trowbridge, Exp. 168.]

Apparatus: A pendulum beating seconds. A small spy-glass. A board. A hammer or stone.

One experimenter strikes the board sharply just when the pendulum-bob passes the middle point of its arc. Another places himself at the start about 900 ft. distant from the first, in a line at right angles with the plane in which the pendulum swings, and then, looking through his glass, seeks to place himself at such a distance that the stroke made when the pendulum-bob passes through the middle point of its arc in one direction reaches his ear just when the bob passes through the middle point of its arc in the other direction. The distance which the sound travels in one second in one direction is thus roughly determined. As the wind may either help or hinder this movement, the conditions should afterward be reversed by having the experimenter with the glass strike the board, while the other stays at the pendulum and listens for the stroke, signalling to the striker to come nearer or go farther away until coincidence occurs as before. The mean of the two distances now found should be taken as the distance which sound would travel in one second in still air.

This method is not likely to give very accurate results. Its merit lies in its directness and simplicity. A still more direct and simple method is to place two persons a long distance apart, but in sight of each other, and have each in turn discharge a pistol or gun while the other notes, as well as he can, the number of seconds between the sight and the sound of the explosion.

In case this exercise is impracticable, as it would be in most city schools, the velocity of sound may be calculated from the alternative for Exercise XXXII, as described in § 210.

Changes of temperature greatly affect the velocity of transmission in air and other gases, the speed in air increasing about 60 cm. per second for each centigrade degree of increase in temperature. Change of pressure with-

out change of temperature does not affect the velocity. In gases of different density, at the same temperature and pressure, the velocity of sound is found to be inversely proportional to the square root of the density. Experiment shows that the velocity of transmission of sound is greater in solids and in liquids than in gases.

202. Sources of Sound.—If we trace back to its source a sound that comes to our ears, we can usually find it in the movements, often very minute, of some body which is said to *give out* the sound. Often this motion is apparent to the eye, as in the case of vibrations of sounding-strings or long tuning-forks. Frequently the sense of touch will show this state of motion in a sounding body, a short tuning-fork for instance, when sight does not. Sometimes, as in the case of faint sounds of high pitch coming from short thick metal bodies, we do not perceive the motion of the body directly in any way, and only by what we know of other sounding bodies convince ourselves that such motion exists.

203. Sound-waves.—But how is it that the air or any other medium brings the sound from its source to our ears? Do particles of the medium propelled from the sounding body move on until they reach our ears, as a baseball moves from the swinging bat to the hands of the player who catches it? Evidently this cannot be the case when the medium which transmits the sound is a solid, nor is it the case when the medium is a liquid or a gas. Particles of liquid or gas, if sent forth with the velocity of sound, would soon be arrested by the resistance of their fellows, unless the latter were moving with similar velocity and direction.

EXPERIMENT 39.

Take an open tube of tin[1] several feet long and 3 or 4 inches in diameter, but narrowed to a frustum of a cone at one end. Fill the

---

[1] See Tyndall's *Sound.*

tube with smoke, by burning touch-paper[1] in it. Place a lighted candle at the small opening and strike together two books or blocks at the large opening. Can the candle-flame be disturbed in this way without driving the smoke from the tube?

It is a concussion, or jar, not a violent blast of wind, that breaks glass miles away from an exploding powder-mill or dynamite manufactory. Now a disturbance which travels through any medium without permanently displacing the parts of that medium, which is, in fact, merely passed on from one set of particles to the next, is called a *wave*. There are various kinds of waves. With waves at the surface of water all are familiar. The waves which bring us energy from the sun have been alluded to in the preceding pages (§ 192). In these two kinds of waves there is a motion of the particles of the medium to and fro across the direction in which the waves move forward. A sound-wave consists merely of a compression travelling forward, followed by a rarefaction. Every particle of the medium affected by the wave is in turn crowded close to its neighbors and then withdrawn from them, making, on the whole, a slight excursion forward and backward, parallel to the direction in which the sound is travelling. This theory of the nature of a sound-wave is sustained by our knowledge of the properties of bodies and by experimental evidence. We shall not enter upon the argument, but the student will by himself probably find it easy to see how a wave of compression may be sent through a medium such as air. If we, as Professor Tyndall does in his lectures on Sound, explode a small balloon, at the moment of explosion the layer of air immediately about the balloon is violently compressed. By virtue of its elasticity this compressed air will almost instantly transmit the com-

[1] Absorbent paper, e.g., blotting-paper, that has been soaked in a strong solution of potassium nitrate in water and then allowed to dry.

pression to another layer immediately outside of itself, and will at the same time itself become more rarefied than before the explosion. In this way a condensation and a rarefaction—in fact, a series of them—move outward in all directions from the exploded balloon. A single condensation and its accompanying rarefaction together constitute a sound-wave. It is easiest to study such waves in gases; and it is known that the sound-waves produced by a body in mid-air giving off sonorous vibrations, as in the case of a bell rung in a lofty steeple, have, at a distance from the source, nearly the form of hollow spheres having the source at their common centre.

In seeking to form a distinct idea of the movements of particles through which a sound-wave is moving the student will be greatly assisted by the use of *Crova's disk.*[1] This consists (as shown in Fig. 53) of a number of circles drawn so as to be nearly but not quite concentric. The disk is attached to a rotating apparatus, and whirled; while across, in front of either half of it, is held a piece of cardboard, in which is a long slit 2 or 3 mm. wide parallel to any radius of the circles. The short arcs of the circles, seen through the slit, may be taken to represent particles of air, and the way in which they crowd together and separate, as the disk rotates, gives a very vivid idea of the way in which the particles of air actually move when set in motion by a sound-wave The places where many lines

FIG. 53.

[1] The disk may be bought of dealers in physical apparatus, or made according to the directions in Mayer's *Sound*. It is better to buy the rotating apparatus.

are crowded together of course represent the condensed portion, and those where the lines are widely separated, the rarefied portion of a sound-wave. Observe how each condensation is succeeded by a rarefaction and each rarefaction by a condensation.

204. Reflection of Sound: Echoes.—Sound-waves may rebound or be reflected from surfaces against which they strike. They obey the same law of reflection as light-waves (§ 244), that is, the angle of reflection is equal to the angle of incidence.

Thus in Fig. 54 a sound-wave following the line P will after reflection follow the line P', making the angle of incidence, α, equal to the angle of reflection, α'. So for a wave following the lines R and R', the angle of incidence, β, will be equal to the angle of reflection, β'.

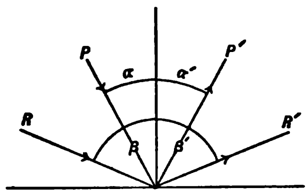


FIG. 54.

A reflected sound is called an *echo*. Echoes occur where a sound travelling through one medium comes abruptly against another medium of different density. Echoes have much to do with the ease or difficulty which a speaker finds in making himself heard in a large hall. The long jarring rumble of a thunder peal is maintained by echoes from strata of air, from clouds, or from the earth.

205. Musical and Unmusical Sounds.—All recognize that some sounds have a musical quality, while others are merely noise.

EXPERIMENT 40.

Upon the whirling apparatus mentioned in § 204 fasten a disk of metal or wood about 6 in. in diameter carrying 2 circular rows of pegs, each row being concentric with the disk. In one row let the pegs be placed at equal intervals of about $\frac{1}{4}$ inch. In the other row let there be an equal number of pegs separated by unequal inter-

vals.[1] Revolve the disk at a uniform rate several times a second, and hold the edge of a card lightly against the regular row of pegs, then hold the card against the other row of pegs. Is there any marked difference in the quality of the sound produced in these two cases? Try the effect of turning the disk very slowly.

From these experiments we conclude that sound-waves striking the ear at regular intervals, and with a frequency greater than a certain limit, produce the sensation of continuous musical sound. Are musical instruments, tuning-forks for instance, capable of giving out sound-waves of regular intervals?

EXPERIMENT 41.

Take a long and tolerably straight piece of clock-spring and fasten it in a vise, leaving about 18 inches projecting horizontally. Set this vibrating in a small arc, and count the number of vibrations which it makes in one half-minute. Then make it swing with a large arc, and count the number of vibrations as before. Does the long vibration take a longer time than the shorter one? Now push the spring farther into the vise, leaving only 6 or 8 inches projecting. Make observations, if possible, as before. Finally, leave only 2 or 3

---

[1] Such an instrument may be constructed as follows: Take a disk of brass 6 in. in diameter and about $\frac{1}{8}$ in. thick. Trace on the face of this disk two circles, one $2\frac{3}{8}$ in. in radius, the other $2\frac{1}{8}$ in. Bore 72 holes about $\frac{1}{16}$ in. in diameter, with their centres at regular intervals on the circumference of the larger circle. Bore 36 holes in the same way on the smaller circle. In these holes cut threads for short machine screws of brass. Fill all the holes of the inner circle with such screws set firmly in. Fill an equal number of holes in the outer circle with similar screws placed at very irregular intervals, but all set firmly in.

The advantage of using screws as pegs is found in the freedom for making variations of arrangement in the irregular row. It would doubtless be cheaper to use pegs fixed once for all and soldered in place.

The irregularities of movement likely to occur with a whirling apparatus driven by hand will make this machine at the best somewhat unsatisfactory as a musical instrument, but properly handled it serves its purpose in this place.

inches projecting. Does the spring now set in vibration yield a musical sound?

Do vibrating strings fastened at both ends, as in violins and pianos, vibrate regularly with a time of oscillation unaffected by the extent of the swing?

EXPERIMENT 42.

Take a rubber tube several feet long, fasten it at each end in such a way that it will be under moderate tension, and then find by trial whether any number of small swings of the tube require less time than an equal number of large swings. If any difficulty is found in making the tube swing slowly enough, it may be filled with sand.

206. Determination of the Length of a Sound-wave.— Since, therefore, a musical sound consists of a series of waves following each other at regular intervals of time, we may call the distance between similar portions of two consecutive waves the *wave-length* of that sound. We may take the distance from the point of greatest condensation in one wave to that of the point of greatest condensation in the next wave, or we may measure from rarefaction to rarefaction. The distance will be the same in both cases.

Suppose any kind of tuning-fork, for instance a common "A" fork of 440 complete[1] vibrations per second, to be set vibrating, and to remain vibrating for one second. In that time we may suppose that at the ordinary temperature the sound will have travelled about 340 m. Then a sphere of 340 m. radius will have been filled with alternate shells, or layers, of condensed and rarefied air. Each complete vibration of the fork produced two layers, or one complete sound-wave. So the length of the sound-wave in this case is about $\frac{34000}{440}$, or 77.3, cm.

[1] A complete, or double, vibration is a movement in which the vibrating body goes over its whole path twice, once from left to right and once from right to left, that is, performs a *round trip*. The word *vibration* used alone will, in this book, mean such a complete vibration.

207. Graphical Representation of a Sound-wave. — A diagram like the following (Fig. 55) is commonly used to represent a sound-wave.

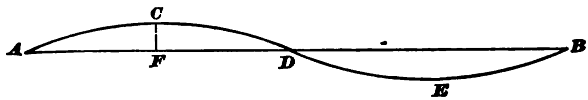


FIG. 55.

The curve *ACD* is taken to represent the condensed part and the curve *DEB* the rarefied part of the wave. Perpendiculars, such as *CF*, let fall from the curve upon the line *AB*, show the relative amount of condensation or rarefaction at any desired point *F* on the line *AB*. Remember that this figure is not a *picture* of a sound-wave, but only a *symbol* for one. This wave is supposed to be travelling from *A* to *B* or from *B* to *A*, and the particles, originally lying on the line *AB*, still lie on that line while the wave-motion is going on. (See § 203.)

208. EXERCISE XXXII.

INTERFERENCE OF SOUND: WAVE-LENGTH.

[Trowbridge, Exp. 160.]

The object of this exercise is to show the fact of the interference of sound in such a way as to demonstrate the existence of waves of sound, and to determine roughly the wave-length of a musical note of a certain pitch.

The method followed is to send the sound from a certain point to another point by two channels of different lengths, and to regulate those lengths in such a way that the result at the second point will be approximate silence, although either channel acting alone would bring a sound of good volume.

In addition to the form of apparatus described in the passage of Trowbridge above referred to, several others are available. Thus Dr. Whiting has shown that it is possible to perform this experiment merely by holding the ends of a rubber tube to the ear while the handle of a vibrating tuning-fork is made to touch the tube at the proper point. It is well to have the two channels of sound converge before reaching the ear. The numerical results

obtained with this apparatus appear to depend somewhat upon the thickness of the wall of the tube.

Another apparatus consists of two brass U-tubes, the arms of one sliding for a distance inside the arms of the other. The arms of one tube are about twice as long as those of the other. Near the middle of each arm of the longer tube a short side-tube is set in. A vibrating tuning-fork is held at the mouth of one of these side-tubes, and a rubber tube attached to the other side leads to the ear of the experimenter. At the beginning the arms of the shorter U-tube are pushed on over those of the other until the two channels from one side-pipe to the other are of the same length. Then while the longer U-tube is held fast the shorter one is drawn out, making the channels unequal in length. That position of the tubes is sought which will make the sound heard a minimum.[1]

No one of the three forms of apparatus above described or referred to is likely to give very accurate values of the wave-lengths used; but nevertheless the exercise, if carefully performed, with any of them is instructive. If the pitch of the fork used is known, the velocity of sound in the tubes may be calculated from the pitch and the wave-length, and this velocity can be compared with that obtained by the more direct method of Ex. XXX.

As an alternative, the familiar experiment with a resonance-tube may be used for determining wave-length. [See § 210.]

[This exercise generally succeeds better with the telescoping brass tube Fig. XXII, than with either of the other combinations of tubes referred to in the next. The tube B may be drawn out at first rather slowly while one student listens at H, the outer end of a rubber tube which connects with the brass tube at E, and an-

[1] For convenient use with a fork making 256 double vibrations per second the arms of the long U-tube should be each about 80 cm. long in the straight part, and should be separated from each other about 10 cm., the bend being a smooth turn. The arms of the shorter U-tube should be each about 40 cm. long. The side-pipe at which the tuning-fork is held has a mouth about 1.5 cm. in diameter, and tapers toward the base. The other side-tube has a mouth about 8 mm. in diameter, and grows larger toward the base. The legs of the tubes must be braced by stiff cross-pieces to keep them in shape. Randolph & Clowes of Waterbury, Conn., make thin telescoping brass tubes suitable for this apparatus.

other holds the sounding-fork at *F*. The blows given the fork to set it in vibration must be as nearly equal as may be, and the formation of harmonic vibrations in the fork (see § 225) as far as possible prevented by striking it on a large rubber eraser or other not too hard object laid on the table-top. When the position of least sound has been roughly ascertained, the experiment should

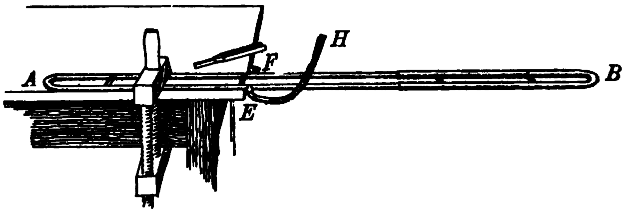


FIG. XXII.

be continued with much care, the tube being drawn back and forth over the region of approximate silence not more than ½ cm. at a time. Each student using the apparatus should take turns in listening at *H*. Only one set of students at a time with one set of apparatus can successfully work at this exercise in a single small room.]

209. Record and Calculation of Results.—Since the sound-waves starting from *F* are equally free to travel to *E* by the route *FAE* or *FBE*, each wave will send about half of its energy by one route and half by the other route. The unequal lengths of the arms *A* and *B* will prevent like impulses from arriving at *E* at the same time by the two routes. Suppose that the part which passes through *B* arrives at *E* one half wave-length behind that which passes through *A*. Then any air-particle at *E*, and therefore any particle at *H*, will be at the same time subject to a compressing impulse by way of one branch and a rarefying impulse of about the same magnitude by way of the other branch. The result of course should be equilibrium, no motion of the air-particles, and consequent silence. Then $FAE - FBE$ would equal one-half *wave-length* for the given fork.

The fact that absolute silence is not obtained in this experiment depends partly on the fact that every fork produces not only a fundamental tone, but also harmonic tones (see § 223) of very different wave-lengths, which are not necessarily to be silenced when the fundamental tone is silenced. Another cause of the slight remaining sound heard may be the power of the metal tube to conduct sound-waves through its own substance. Moreover, some sound may come through the outside air. Finally, the sound which has followed the longer path is not quite so strong as the other, for it has suffered more from friction in the tube.

210. Alternative for Exercise XXXII.[1]

Apparatus: A glass tube, two or three centimeters in internal diameter and 25 or 30 cm. long, fitted with a cork which moves easily, but closely, in the tube, which may be oiled inside. A tuning-fork of about 440 or 528 double vibrations per second (an "A" or a "C"[2]), a stick to use as a ramrod to push the cork back and forth, a meter-rod.

Insert the cork in the tube, pushing it in a few centimeters; sound the fork by striking one prong on a thick piece of rubber, and hold the fork with its long axis perpendicular to the axis of the tube, and with one prong as close to the mouth of the latter as it can be held without touching the glass (see Fig. 56, which, however, illustrates a somewhat different apparatus). Repeat, with cork in different positions, until the length of air-column that gives the maximum resonant effect has been found. Measure this column and the internal diameter of the tube. Repeat with the tube filled with coal-gas

---

[1] The necessity for a certain arbitrary correction (see § 211) makes this experiment of § 210 somewhat unsatisfactory. This necessity can be avoided by using a longer glass tube, finding *two* positions of the stopper that will give a maximum of sound, and measuring the distance between these two positions, which will be half of a whole wave-length. (See Whiting's *Physical Measurements, Part I.*, D. C. Heath & Co.).

[2] Forks an octave lower answer equally well with a glass tube 50 or 60 cm. long.

drawn from a gas-burner or other convenient source, and carried by means of a rubber tube held almost against the cork. The tube must, of course, be held in an inverted position, and the smell of the escaping gas will indicate when all the air has been displaced. The length of the resonant column is then to be determined as before, with the tube held constantly in an inverted position, and the supply of gas from time to time renewed.

211. Calculation of Results.—The resonant effect of the air-column of proper length is due to the fact that a sound-wave coming from the fork is reinforced by the reflection of the preceding wave, coming back to the vibrating fork in such a period as exactly to coincide in phase with the newly-formed wave. In Fig. 56, which shows a common hydrometer-jar used as a resonance-tube, suppose the fork A to have been set vibrating from the position of rest of its lower prong at 1. By the time this prong has reached the position 2 the phase of condensation of the sound-wave will, if the distance BC is properly chosen, have travelled from B to C. While the prong returns to 1 the condensation will travel back to 1. While the prong moves to 3 the condensation will continue to ascend, combining with and strengthening that sent directly from the prong, and so on. It may be inferred that the distance BC is $\frac{1}{4}$ wave-length (for the given fork), from the fact that BC is traversed by the wave while the fork is making $\frac{1}{4}$ vibration, that is, swinging from 1 to 2. It is found by experiment that the diameter of the tube makes some difference in the length of the air-column, so that in calculation $\frac{1}{4}$ of the diameter should be added to the length. (See note to § 210.)

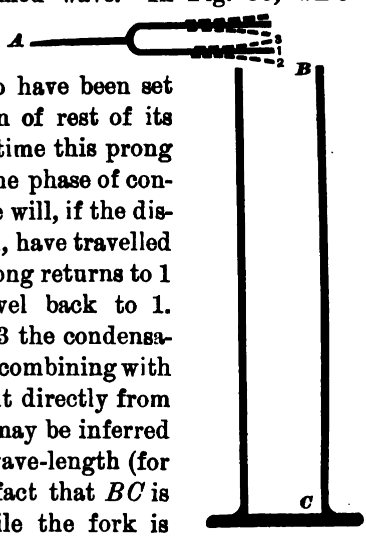


FIG. 56.

We have, then, from the experiment the data for calcu-

lating the length of the sound-wave in air and in coal-gas, and also, when we know the time of vibration of the fork, the velocity of sound in air[1] and in coal-gas.

Let w = length of sound-wave;
v = velocity of sound;
l = length of the resonant column;
d = diameter of the tube;
n = number of vibrations per second of the fork used.

From § 206 the student has already learned how to calculate the length of a sound-wave, knowing the number of vibrations per second which the sounding body undergoes, and knowing also the velocity of sound at the given temperature. Suppose now that the length of the sound-wave has just been learned from the resonant-tube experiment above described, and that v, the velocity of sound, is the unknown quantity.

From § 206 we may obtain the formula $w = \frac{v}{n}$.

But $$w = 4\left(l + \frac{d}{4}\right);$$

therefore $$4\left(l + \frac{d}{4}\right) = \frac{v}{n},$$

and $$v = 4\left(l + \frac{d}{4}\right)n.$$

The value found for the velocity of sound by this method may be compared with that obtained directly in Exercise XXX.

---

[1] The velocity of sound through air contained in an open tube is nearly, but not always exactly, the same as through the open air outside.

212. Relation of Musical Sounds to each Other.—Having now obtained an idea of the essential nature of a musical sound, as consisting of a regular succession of waves of a particular length, let us next inquire what are the peculiarities that distinguish one musical sound from another. These peculiarities are of *loudness, pitch,* and *quality.* We shall discuss the three in turn.

213. Intensity or Loudness of Sound Vibrations.—The rumble of an approaching railroad train, at first faintly heard, gradually increases to a roar as the train nears the listener, and again dies away as the train recedes in the distance. Equally marked variations may be produced by the player upon a large organ, who can at will cause his instrument to murmur softly or to send forth a volume of sound-waves that shall shake the windows and even the walls of the containing building. In the case of the train and of the organ the difference between the lesser and the greater effects produced may be only of degree, not of kind. The vibrating air-particles which transmit the sound to the ear and the jar to all movable objects in the neighborhood are thrown into more violent agitation by the train near at hand and by the loudly-sounding organ-pipes than by the distant train and the feebly-sounding pipes. Plucking a sonometer-wire (§ 222), now gently, and again with greater force, and listening to the resulting sounds, will show to the student that the intensity of sound-waves depends upon the *amplitude* of the vibrations which cause them.

Since sound-waves travel outward from their centres in ever-enlarging hollow spheres, and since the surfaces of spheres are proportional to the squares of their radii, it is plain that a given amount of energy in the shape of a sound-wave must, as it recedes from its source, occupy—if none of it is lost on the way (see § 229)—successive portions of space, which increase with the square of the dis-

tance from the source. Hence the intensity of the wave at any given point must lessen as the point is taken farther and farther away from the source of the wave. More exactly, *the intensity of a sound-wave (and its loudness to the ear) varies inversely as the square of its distance from the vibrating body which produced it.*[1]

214. Speaking-trumpets, etc.—As the spreading of a sound diminishes its intensity, any means which make the sound travel within limited channels tend to maintain its intensity as it moves farther and farther from its source. This explains the effects of speaking-trumpets and speaking-tubes. On the other hand, ear-trumpets, used by deaf people, have a broad mouth, which gathers the sound from a considerable area and concentrates it upon the ear.

215. Acoustic[2] Qualities of Rooms.—The reflection of sound has already (§ 204) been alluded to. In a large room the words of a speaker may come to his hearers once directly and once or several times by reflection from the walls. The effect of the echo may be very confusing, making it difficult to distinguish the words spoken. But if the room is of moderate dimensions the echo comes so soon as to unite with the direct sound, making it practically louder than it would be in the open air. Soft coverings on walls and furniture and the presence of an audience tend to destroy the echo.

216. Increase of Loudness by Resonance.—We have already, in the alternative for Exercise XXXII, seen that

---

[1] This is true only when the distance is so large in comparison with the dimensions of the sounding body that the latter may be considered a *centre* from which the sound-waves proceed along radii. A person ten feet distant from the nearest point of a moving railroad train does not hear four times as loud a sound as one twenty feet distant from the train.

[2] *Acoustic* is derived from a Greek verb ἀκούειν, which means *to hear*.

the sound from a tuning-fork can be greatly augmented by the vibrations of a column of air. Not only air-columns of suitable length, but solid objects also, may serve to increase the amount of sound produced by a vibrating object. The difference in the audibility of a vibrating tuning-fork held in the air, or held with the end of the stem pressed firmly against the table-top, shows this. Still greater increase of volume of sound would be produced by holding the stem of the sounding fork down on the top of

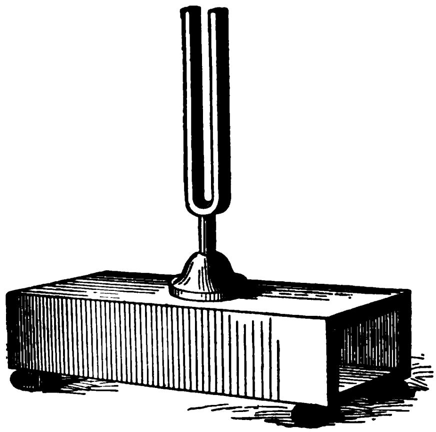


FIG. 57.

a resonance-box, Fig. 57, since this is thinner and more easily set vibrating, and also transmits its vibrations freely to the air. The action of a resonant board may be explained by considering how much larger masses of the surrounding air may be set in motion by it than by the narrow prongs of a tuning-fork or the slender strings of a violin or piano. The student should notice the provisions made in various stringed instruments to secure resonance.

217. Pitch.—The next characteristic to be considered is *pitch*. It is easy to show, by turning with varying speed

the rotating disk used in § 205, that the degree of sharpness changes as the vibrations producing the sound become more or less frequent. An increase of sharpness in a musical sound is called a rise of pitch. The student can readily determine whether an increase of frequency in the vibrations raises or lowers the pitch of a sound. We may naturally inquire how many vibrations per second are needed to yield a tone of given pitch, for instance some familiar note of a musical instrument. This question is taken up in the next exercise.

218. EXERCISE XXXI.

PITCH OF A TUNING-FORK.[1]

[Trowbridge, Exp. 154.]

Apparatus: A tuning-fork not above middle C in pitch, fastened in a nearly horizontal position to some support in such a way that a style fastened to the end of one prong may make, when the tuning-fork is sounding, horizontal movements to and fro a few millimeters above the table upon which the support rests. A pendulum making between 100 and 150 complete double vibrations per minute, and capable of swinging in one plane only, supported in such a way that the lower end of a style fastened to the bottom of the pendulum will swing on the same level as that in which the style of the tuning-fork swings. Some means, preferably a bass-viol bow, for setting the tuning-fork into vigorous vibrations. A piece of plate-glass about 10 cm. wide and 15 cm. long, lightly smoked. A watch or clock.

The method of this exercise is to draw the smoked glass along in a straight line beneath the style of the vibrating tuning-fork, thus tracing on the glass a sinuous curve, and simultaneously to cut this curve crosswise by means of traces made by the style of the pendulum. Arrange the pendulum and fork so that the horizontal part of the pendulum motion shall be parallel to the motion of the fork-prongs, so that the two styles may, when at rest, be in the same vertical plane, with their points as near each other as

[1] The outline of this exercise was given by Professor Hutchins of Bowdoin College.

practicable. Place the piece of smoked glass under the tuning-fork and pendulum, and make such adjustments that the styles will bear lightly upon the glass. Set both pendulum and tuning-fork into vibration, and then draw the glass quickly along beneath them in a direction at right angles to the vibrations. Count the number of vibrations of the pendulum per minute and the number of vibrations of the tuning-fork corresponding to one vibration of the pendulum, then calculate the number of complete double vibrations of the tuning-fork per minute.[1]

---

[1] To make a support for the fork, one may take a board about 30 cm. long, 10 cm. wide, and 2½ cm. thick, put one screw through it near the middle of one end and two screws near the corners at the other end, thus making a tripod, the inclination of which may be controlled by turning the various screws farther in or out. Place the tuning-fork with its prongs parallel to the length of the board, with its handle, which should be of rectangular section, resting upon the board midway between the two corner screws, and fasten it in position. The support for the pendulum may consist of a base-board 75 cm. long, 15 cm. wide, and 2½ cm. thick, having on one side, about 20 cm. from the end, an upright with a projection at the top to carry the pendulum, the upright and the projection being of such dimensions as to make the bottom of the pendulum hang midway between the sides of the base-board and about 1 cm. above it. The pendulum may be a piece of clock-spring having for a *bob* at the bottom a large bullet, to which the style is to be fastened. It should be fastened to its support by some arrangement allowing easy adjustment of the style against the glass plate. The choice of an object to serve as a style is important. Mr. C. W. Parmenter of the Cambridge Latin School, to whom several other good suggestions in regard to this experiment are due, recommends a piece of the hair-spring of a watch (see *Journal of Education* for July 11, 1889). Such a style, some mm. in length, reaching very obliquely downward, and properly adjusted, exerts upon the glass plate an elastic pressure, yet is stiff enough against horizontal bending to make a good mark in the thin brown coating of the plate. A projecting strip of pasteboard attached to one end of the glass assists greatly in drawing it along beneath the styles. The board carrying the tuning-fork is to be placed upon the base-board carrying the pendulum. Under some conditions there would probably be a great advantage in replacing the two corner screws of the former board by hinges fastening it

219. Combinations of Sounds differing in Pitch: Beats.— In Exercise XXXII the intervals of silence occasioned by the interference of sound-waves were well shown, but it frequently happens that periods of reinforcement and of interference succeed each other at regular intervals, giving rise to alternate bursts of sound and periods of comparative silence. This throbbing is called a succession of *beats.*

EXPERIMENT 43.

Tune the two sonometer-wires (§ 222, foot-note) nearly but not exactly into unison, that is, until they produce sounds of nearly the same pitch. This may be done by tightening the wire of lower

---

permanently to the base board. [Fig. 75 shows an arrangement of apparatus very similar to that described in this note, but differing from it in certain details. The clock-spring pendulum is here sup-

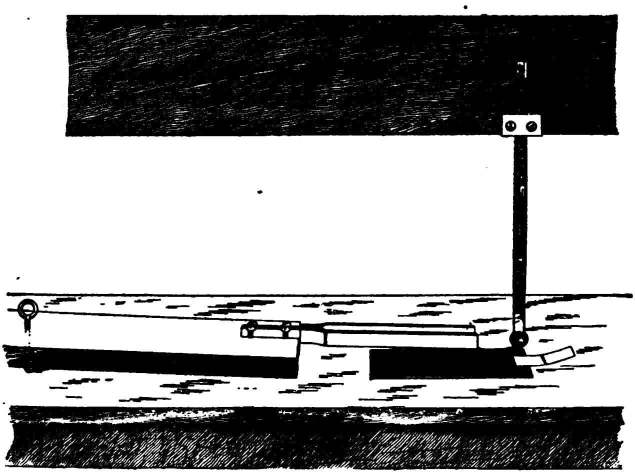


FIG. XXIII.

ported by the movable cross-bar of the apparatus shown in Fig. IV.] The flexible piece of card-board attached to the glass is of great convenience in moving it quickly at the proper time.

pitch or slackening the shriller one. Then sound both at once by plucking them near the middle and allowing them to vibrate. Note the beats that are produced.

Beats may also be caused in a variety of other ways, as by the use of two tuning-forks of very nearly equal pitch mounted on resonant-boxes (see § 216) and sounded together. Two forks may be prepared for this purpose by selecting any two of the same pitch and then loading the prongs of one lightly near the ends with wax, or by cautiously grinding off a little from the ends of the prongs of one upon a grindstone and testing it from time to time with the unaltered fork. Many semi-musical instruments, such as ordinary bells and gongs, nearly always produce beats when sounded.

EXPERIMENT 44.

Hold suspended, mouth downwards, by a cord passed through the hole in the centre, a gong like those used for door-bells, or, better, one of the ornamental Japanese gongs such as are used for call-bells. Strike the edge of the gong a pretty sharp blow with a firm rubber stopper through which a stout wire has been forced so as to form a handle, and, while the gong is still sounding loudly, lower it, with the rim horizontal, until it almost touches the table-top, at the same time listening intently for beats.

If sound-waves are represented, as they may be for convenience, by a diagram of water-waves, the nature of beats is well expressed by Fig. 58, where the regions of high

FIG. 58.

crests and deep troughs stand for phases when the two waves reinforce one another, and the more level intermediate portions for interference-phases.

220. Harmony and Discord; Musical Instruments.—When two or more musical sounds are simultaneously produced, their effect upon the ear may or may not be agreeable. It is certain to be agreeable if there are only two tones whose vibration-numbers have a very simple ratio. For instance, if the two tones are those produced by a "violin-A" fork, of 440 vibrations per second, and another fork of 220 vibrations per second, so that the ratio is 2 : 1, the interval between the two tones is what is called an *octave* and the effect is pleasing. Such a pleasing effect is called *harmony*. But if the fork of 440 vibrations were sounded at the same time with another of 436 or 444 vibrations, distinct beats (§ 219) would be produced, and the result would be unpleasant. It would be called *discord*. Tones an octave apart are produced by any two sonorous bodies in vibration when their vibration-numbers have to each other the ratio 2 : 1. An octave is the simplest possible musical interval, and this is subdivided into seven smaller intervals. The vibration-numbers of the whole series of tones, comprising a given tone, its octave above and the intermediate tones, that constitute the *notes of the gamut*, or *natural scale*, in music, bear to each other relations indicated by the following representative numbers:

do		re		mi		fa		sol		la		si		do
C	:	D	:	E	:	F	:	G	:	A	:	B	:	C
24	:	27	:	30	:	32	:	36	:	40	:	45	:	48

Keeping these relations in mind and knowing the vibration-number of any particular note of the scale, we can calculate the number of vibrations that will be required to produce any given tone. Suppose we are asked to find the number of vibrations per second that will produce the tone G next preceding that of the "violin-A" fork of 440 vibrations. Then we have the proportion $36 : 40 :: x : 440$, from which $x = 396$ vibrations.

221. Quality.—Aside from differences in the loudness of sounds and in their pitch, there is a well-known difference in *quality*. It is this, in part, which enables one to distinguish the voices of acquaintances in the dark or to pick out a familiar voice singing among many others. Still more marked are the differences between the sounds of different musical instruments, the violin and the flute, for instance, even when both are producing sounds of the same pitch. The fact is that a musical sound is usually a combination of several notes differing in pitch, and what is called the pitch of the sound is merely the pitch of its loudest component. How such a combination of notes can be given out by one instrument, and why the effect of the combination is pleasing to the ear, can be better understood after the following experiments with vibrating strings.

222. Transverse Vibration of Strings.

EXPERIMENT 45.

Stretch one of the sonometer-wires[1] pretty tightly, until it gives a clear musical sound on being plucked to one side and then released. While it is still sounding, drop a little Λ-shaped rider made of stiff paper on the wire near the middle. Note the behavior of the rider. Sound the wire again, and look at it in a strong light, while it is sounding. Repeat the experiment with the rider, dropping it upon various portions of the soundingwire, until the trials have been carried quite to one end of the wire. Then holding the finger-tip lightly against the exact middle of the wire, pluck the latter midway between the point where the finger is applied and either end.

---

[1] *Sonometer* is a name given to an instrument consisting usually of a wire or cord stretched over a resonance-box of wood. Such a sonometer as is needed for this and succeeding experiments can be bought of dealers in physical apparatus, or (more cheaply) made under the teacher's direction according to the instructions given in Prof. A. M. Mayer's *Sound* (D. Appleton & Co.). In place of it for individual experiments a wire stretched according to the directions of Exercise XXXIII can be used.

Then quickly remove the finger and put the rider in its place; then put it on some other portion of the wire. Once more place the finger-tip on the wire, twice as far from one end as from the other, sound it by plucking at the middle, remove the finger and put the rider in its place; then put the rider on another portion of the wire.

These experiments will doubtless show that the vibrating wire has regions of greatest vibration and points of little or no vibration. The regions of greatest vibration are called *loops;* points of no vibration, *nodes.* In Fig. 59 the loops

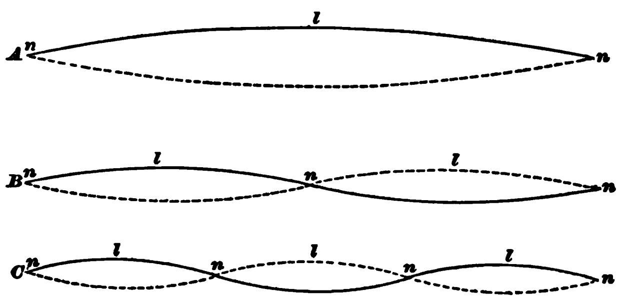


FIG. 59.

and nodes of some vibrating wires are shown. The centre of each loop is marked *l*, and each node is marked *n*.

223. Harmonics and Overtones.—Again pluck one of the sonometer-wires strongly at the centre, then at various intervals toward either end, until finally the end is almost reached. Note the clanging tones of high pitch that are produced as the end of the wire is approached, and observe how these quickly die away, leaving the lowest tone of the wire still sounding. This lowest note is called the *fundamental note,* or simply the *fundamental,* of the wire. It is given by the form of vibration shown in Fig. 59, *A*. The higher tones produced by plucking the wire toward the end, in the way just described, are called *overtones.* There are two kinds of tones among these: the *harmonic over-*

tones, or *harmonics*, and those which are not harmonic. The harmonics may be very numerous, but the vibration-number of each of them bears a very simple relation to that of the fundamental tone of the whole wire. Modes of vibration which give harmonics are illustrated in Fig. 59 *B* and *C*. The ratio of the vibration-number of the fundamental to that of each of the harmonics can be made out, after the performance of Exercise XXXIII, by comparing the length of the whole wire with the length of the vibrating segments which give out the harmonics.

224. Chladni's Figures.—Strings are by no means the only objects that can vibrate in parts. Thin plates of metal or wood may give out a great variety of sounds, corresponding to different modes of vibration. If this were not so, such instruments as the violin and piano would be practically useless. Plates of glass or of metal may be fastened by a clamp and set vibrating by a violin-bow. If sand is sprinkled on the surfaces of such plates while they are vibrating, it will arrange itself in lines which mark the position of the nodes. The figures formed by the sand are known as *Chladni's figures*, in honor of the investigator who first studied and described them. Four forms are shown in Fig. 60. In each case the letter *b* shows where

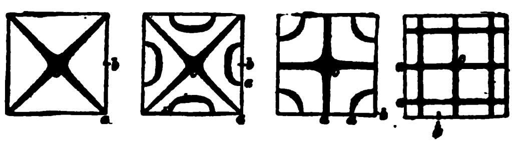


FIG. 60.

the bow is applied to the edge of the plate; *a* indicates the position of the finger held in contact with the plate, to produce nodes; and *c* is the position of the clamp. A form

of clamp adapted to the experiment is shown in Fig. 61. The plates should be as nearly as possible of uniform thickness throughout, and their edges should be slightly rounded.

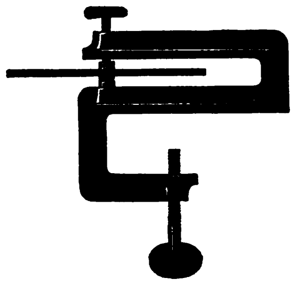


FIG. 61.

A column of air in a pipe does not necessarily vibrate as a whole. This fact accounts for the variety of pitch obtainable from a fife, flute, or other similar wind-instrument.

225. Quality of Sound due to Overtones.—The quality of any musical sound is due to the number, kind, and relative strength of the overtones which, together with the fundamental, constitute the sound. This has been proved both by analyzing sounds such as those of the human voice, of organ-pipes, etc., into their component tones, and by uniting many separate simple sounds to form a compound sound. Any sound which is caused almost entirely by vibrations of a single rate, that is, sound which is nearly destitute of harmonics, is comparatively thin, and lacking in mellowness and richness.

EXPERIMENT 46.

Sound the "middle-C," or the "violin-C," fork alternately with the sonometer-wire tuned to the same pitch. The wire should be plucked about $\frac{1}{7}$ or $\frac{1}{8}$ of its total length from one end, in order to give a full series of harmonics. Compare the rich sound of the wire with the thin sound of the fork.

If the student has a good musical ear, he may be interested to listen to the sound produced by a telegraph-wire set vibrating by the wind. On placing the ear against a telegraph-pole many tones of widely different pitch may be heard.

226. ## EXERCISE XXXIII.

PITCH OF VIBRATING STRINGS.[1]

Apparatus: Two spring-balances such as are used in Ex. I. Two pieces of spring brass-wire about 1½ m. long, one being No. 24, B. & S., the other No. 27. Four of the triangular prisms of wood used in Ex. III. A meter-rod. A tuning-fork about middle C in pitch and another an octave lower.

Drive two screws horizontally into the end of a table-top about 15 cm. apart. Fasten one of the wires to each of the screws and lay the wires out parallel to each other along the table-top. Under each wire near the screw lay one of the wooden prisms with its edges at right angles with the wire, and fasten it in place. To the free end of each wire fasten the hook of one of the balances. To the ring at the other end of each balance attach a strong but flexible wire 40 or 50 cm. long. Within easy reach of each of these wires drive into the table-top a stout screw, leaving it projecting about 2.5 cm. When the balances are under tension, each is to be hitched by its connecting wire to one of these screws and so held fast, being propped in such a way as to lie flat on its back. A second block placed under each of the brass wires can then be moved into almost any position without much affecting the tension, since the wire is held by the pull about as high as the top of the block. Under these conditions the length of the wire subject to vibration is easily varied. It is necessary, however, to hold the wire lightly at this movable block in order to limit the vibration to the length between the bridges. Stretch the [No. 24] wire with a force of 20 lbs., and, keeping this tension constant, find what lengths of it will give notes in unison with the two forks respectively. Then find what length of it will correspond to the lower fork when the tension is only 5 lbs. Find also what lengths of the No. 27 wire correspond to the forks when a tension of 10 lbs. is used. (No. 24, B. & S. gauge, has a diameter of .0201 in., No. 27 a diameter of .0142 in., so that the cross-section of No. 24 is almost exactly twice as great as that of No. 27.)

The strings are set in vibration by plucking them at the middle. The sound is at first rough, owing to the presence of overtones; but these quickly die away, leaving the fundamental note, which

---

[1] This exercise is very like one recommended by Mr. J. Y. Bergen in the *Journal of Education* for Jan. 10, 1889.

is the one to be tested. Even students who have no keen perception of musical pitch can probably secure nearly perfect unison by sounding the tuning-fork and the wire at the same time and guiding themselves by the beats, which become very apparent when the sounds are near unison. The ear should be held near the wire, otherwise the fundamental note may escape detection.

227. Discussion of Results.—From the data obtained in the preceding exercise the student should be able to answer the following questions:

(1) What relation exists between length of the wire and number of vibrations per second?

(2) What relation exists between tension and number of vibrations per second?

(3) What relation exists between the mass of the wire and the number of the vibrations per second? (It should be noticed that mass $\propto$ area of cross-section.)

228. Sympathetic Vibrations.—

EXPERIMENT 47.

Tune the sonometer-wires into unison, sound one strongly and place a rider on the other to see whether it vibrates. If it does, the vibrations of the wire first sounded have been carried through the wood of the sonometer, and in some smaller degree through the air to the second wire.

Try a similar experiment with the wires not in unison or harmony.

If the student has access to a piano, either in the school-building or at home, a more striking experiment may be made with it. Uncover the piano so as to give ready access to the wires. Depress the *forte* pedal, so as to hold the dampers off the wires, then whistle or sing any note, holding the lips close down to the wires. After a moment stop sounding the note, and listen to the piano.

The fact that a body capable of yielding a musical note responds not to all other musical notes indifferently, but

only to such as make harmony with its own, explains the term *sympathetic vibrations*, which is applied to the phenomena here described. As a pendulum or a child's swing is set into vibration by means of small impulses, properly timed and directed, but is on the whole nearly unaffected by equally strong impulses not properly timed and directed, so a stretched cord or other body capable of vibration responds to the impulses received from another body only when these impulses are of the proper frequency.

EXPERIMENT 48.

Lead a string horizontally from one firm support to another several feet distant. From this string suspend two similar balls by threads of equal length,—3 ft., for instance. Leaving one of the balls at rest, set the other swinging in a plane at right angles with the overhead string, and watch the result.

229. Dissipation of the Energy of Sound-waves.—In perfectly elastic and frictionless substances sound-waves might go on forever without loss, that is, the product, *intensity* × *surface of sphere* (§ 213), would be a constant quantity. But if there are any such substances, they are not the media that propagate sound-waves. Even gases manifest a measurable amount of viscosity,[1] and thus, like solids, cause a gradual conversion of the energy of sound-waves which traverse them into heat.

QUESTIONS AND PROBLEMS.

1. Just how could the sonometer-wire be set to vibrating in four loops? In five loops? How many nodes would there be in each case?

2. How long after the flash of a gun is seen would its report be heard if the temperature were 0° and the distance of the observer from the gun were 1.5 kilom.? Light will traverse this distance almost instantaneously.

[1] See Clerk-Maxwell's *Theory of Heat*, pp. 297 to 300, seventh edition.

3. Describe carefully any method which you have used for measuring the velocity of sound.

4. What becomes of the energy of sound-waves? To what fact is this transformation due?

5. Do sounds of high or low pitch travel faster? How does the sound of distant music illustrate your answer?

6. Why does the sound of a circular saw, cutting through a board, grow lower in pitch as the saw enters the board?

7. What will be the length of the resonant air-column that will respond most loudly to a fork of 220 vibrations per second? Neglect the influence of the diameter of the tube.

8. If a tube 4 cm. in diameter and 0.5 m. long responds most loudly to a certain fork, what is the wave-length for the tone of that fork?

9. How would you determine by experiment the time of vibration of a tuning-fork?

10. A sonometer-wire 1 m. long is tuned to unison with a fork of 220 vibrations. Find the lengths of wire to give the first five harmonics of the fundamental tone of the whole wire.

11. Why are violin-strings bowed, and piano wires struck, near an end rather than at the middle?

12. If a certain tone is sung loudly over the sounding-board of a piano, what wires will respond? If there is any difference in the loudness of the responses, what will be the order as regards loudness?

CHAPTER XIII.

LIGHT.

230. Definition of Light; Visibility of Objects.—That agency whereby objects affect our eyes in such a way as to be *visible* is called *light*. We are accustomed to say, and properly, that the light *comes from* the object seen to the seeing eye. We have means for measuring the time required by light to travel a given distance in air, and in many other media.

One object is distinguished *visibly* from others by means of some peculiarity in the light which it sends to the eye. This light may originate at the object, as in the case of a flame; or it may come from some other source, and be merely reflected or transmitted by the object.

231. Light travels in Straight Lines.[1]—It is easy to show in a rough way that light travels in straight lines.

EXPERIMENT 49.

Darken the laboratory and throw sunlight into the room through an opening a centimeter or less in diameter in one of the shutters. The light can be most conveniently managed by means of a *porte-lumière*, Fig. 62, which consists essentially of a hinged mirror

[1] This statement holds good only in cases in which the light travels in a medium or substance of uniform composition throughout. Even under such circumstances there are certain exceptions to the general rule of rectilinear propagation. These occur where light passes close by the edges of objects, but the effects produced, although very interesting and beautiful, are not sufficiently prominent to make their study in this book necessary or desirable.

fastened outside one of the laboratory windows, and arranged to rotate about an axis which passes through the centre of an opening in the shutter outside of which the mirror is attached.[1] Produce sufficient smoke by burning touch-paper to trace the path of the sunbeam across the room, or define it by making a slight cloud of dust by rapping together two dusty blackboard-erasers along the course of the light. Note the sharply-defined, straight edges of the path followed by the light in its passage.

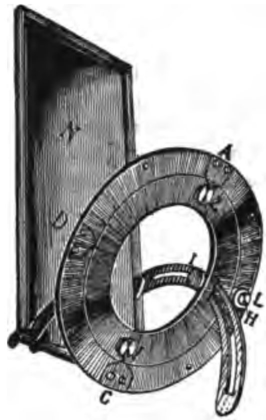


FIG. 62.

All are familiar with many of the practical applications of this principle, that light travels in straight lines. Some of the commonest are found in the practice of *sighting* rifles, cannon, and other firearms; in the method of glancing along the edge of a board, which the carpenter adopts to see whether it is straight; and in the various surveying operations, in which points are located by sighting with the unassisted eye, or by means of fine slits in metal plates, or by the aid of small telescopes.

232. Light Rays and Pencils.—If we, for convenience, consider the case of an imaginary luminous point, this may be regarded as the centre of a sphere of light, and the light will travel from the centre to the surface of this sphere along its radii. Each one of these radii, or lines of transmission, will then constitute what it is convenient to call a *ray of light.* A group or cone of such rays would form a *pencil.* If the source of light were very distant from the observer, as in the case of the fixed stars, the rays of the pencil would be practically parallel. Clusters of parallel rays may be readily produced from ordinary pencils by simple means.

[1] Fig. 62 shows a form of apparatus made and sold by Mr. A. P. Gage, the author of Gage's *Physics*. See also the Appendix.

233. Formation of Images by Means of Small Apertures.

EXPERIMENT 50.

Make of thin wood a box about 10 cm. in depth and width and 30 cm. long, with one end closed and the other open. Fit closely into the box a square piece of thin board, made to slide smoothly, like a piston, from end to end of the box by means of a rod attached to one edge of the board, as shown in the annexed diagram, Fig. 63.

FIG. 63.

The centre of the square board should first have been cut out so as to leave not much more than a centimeter in width at the narrowest portions around the edge, as shown in Fig. 64. Glue over this hole in the board a piece of the thinnest and most transparent tracing-paper, or cloth. Cut a hole 4 or 5 mm. in diameter in the closed end of the box and tack over the hole a piece of very thin sheet-brass, and make a hole with a needle through the centre of the brass. Place the piston in the box, hold the brass-covered end toward a bright gas-flame or lamp-flame in a somewhat darkened room, and look into the open end of the box, toward the flame. Try various positions of the box with reference to the flame; slide the piston back and forth, and notice the various pictures or *images*, of the flame, formed on the paper screen. In the same way images may be formed of any brightly illuminated object, a house for instance, on which the sun is shining.

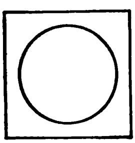


FIG. 64.

234. Explanation of Inversion of Image.—It will be evident at once that the image is upside down, that is, that the bottom of the image represents the top of the flame. This is due to the fact that the light-rays, coming from the flame and traversing the very small aperture in the sheet-brass end of the box, *cross each other in their passage*, as is shown in Fig. 65, where the flame is represented by the

arrow AB. For instance, the cone of rays AA' from the tip of the arrow and the cone of rays BB' from the other end, cross at mn, and appear in the image at the spots A' and B' respectively.

If the aperture mn were gradually to be made larger, the spots A' and B', illuminated from A and B respectively, would grow larger and larger. The same would be true of the spots illuminated from other points of the arrow; and

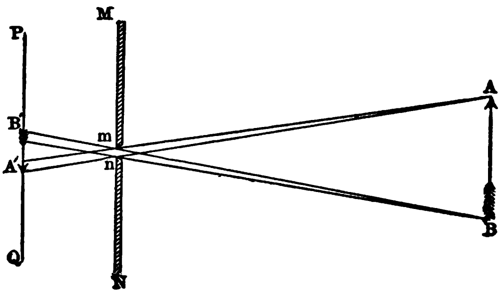


FIG. 65.

at last the growing spots would so overlap each other that the image would be lost in a mere blur of light on the screen.

235. Shadows.—From the fact that light travels in straight lines, it is easy to see that it will be cut off from a portion of space behind any illuminated opaque object, just as waves of water are cut off by a breakwater, leaving a region of calm water behind it. The simplest case is that in which the light-giving object is as small as possible.

EXPERIMENT 51.

Light a bat-wing gas-jet or a kerosene lamp with a very broad thin flame, and cast the shadow of a lead-pencil, held vertical, on a sheet of white paper, having first the edge and then the broad side of the flame toward the pencil. Note the great difference in the sharpness of outline in the two cases.

A shadow with a perfectly well-defined outline could only be obtained by using as the source of light a mathematical point. To illustrate what would be the result if this condition could be secured, the student should examine the annexed diagram, Fig. 66.

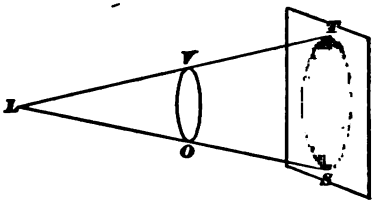


FIG. 66.

Of the light-rays proceeding from the point *L*, the circular opaque object *OV* intercepts all which strike its surface, thus forming a shadow whose shape is in this case the frustum of a cone, *OSTV*.[1] The black space *ST* in the screen is not the whole shadow, but a *section* of the entire shadow *OSTV*. A perfect shadow like this is called an *umbra*.

Suppose now that the source of light is of appreciable size, as for example a candle-flame: then the opaque object *O* cuts off all illumination from some portions of the screen, and from other portions cuts off only a part of the light, as is shown in Fig. 67.

That part of the screen which receives light from part of the flame *AB*, but not from all of it, will appear a partially shaded ring, *P'S'SP*, around the central area of total shadow. This ring forms what is known as the *penumbra* (from two Latin words meaning *almost* and *shadow*). On account of the comparatively large size of most sources of light most shadows are surrounded by a wide margin of penumbra. The student will find the best examples of

[1] That is, a cone with its tip sliced off by a section parallel to the base.

clear-cut shadows in those cast upon near surfaces by opaque bodies exposed to electric arc-lights (if he can get an opportunity to observe these), and he may compare the dim and indistinct shadows of the leaves of shade-trees

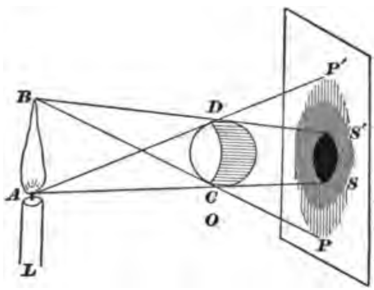


FIG. 67.

exposed to the sun, with those cast by the same objects exposed to the electric light at night, in which even the serrated margins of the leaves are sometimes clearly outlined.

236. Relation between Distance from the Source of Light and Intensity of Illumination.—From the fact that light may be considered to radiate in straight lines from any luminous point, the student can make for himself a shrewd guess as to the law for intensity of light at different distances from the source, and he can, in a rough way, prove or disprove the correctness of his guess by means of the first experiments described in Exercise XXXIV.

If the source of light, however, instead of being a mere luminous point, is of appreciable dimensions, the law may seem to hold good no longer. The fact is that in such a case all parts of the luminous object are not equally distant from the points to which they send their rays, and evidently we cannot take the distance from the nearest point of the object as being the distance from the object as a whole.

237. Photometry; Rumford's Photometer.—It is a matter of great practical importance to compare the illuminating power of different lights. This cannot be done by merely observing them directly; for the eye is unable in this way to distinguish slight differences of power, and if the lights are of somewhat different colors the unaided eye gives only the vaguest indications in regard to their comparative efficiency. One of the simplest devices for measuring the relative power of two lights is Rumford's photometer. This consists essentially of an opaque rod placed in a vertical position in front of a small white screen. A lead-pencil or the upright tube of a Bunsen burner answers well for the rod, and the screen may be of white card-board 12 or 15 cm. square. In making measurements with this photometer the apparatus should be arranged as shown (in ground-plan) in Fig. 68.

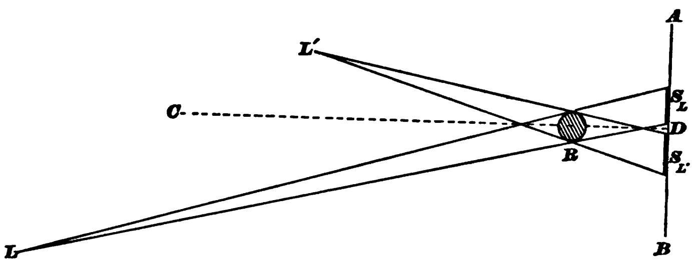


FIG. 68.

L is one of the lights to be compared, and L' the other; R the rod, AB the screen, and S_L and $S_{L'}$ the shadows. The lights should be so arranged that lines drawn from their centres to the centre of R will make approximately equal angles at R with the line CD, a perpendicular to the screen passing through the centre of R, and on this line the observer should be placed. The shadows should be near each other, but must not overlap. It is easy to see that the shadow corresponding to L is illuminated by light

from L', and that the one corresponding to L' is illuminated by light from L.

238. EXERCISE XXXIV.

PHOTOMETRY.

[Rumford's Method. Trowbridge, Exp. 183.]

Apparatus: Five short candles, all alike, one mounted alone, the others standing in a straight row along a board, with centres about 3 cm. apart. A piece of white pasteboard about 20 cm. long and 10 cm. wide. A rod about 20 cm. tall and 1 cm. in diameter. A meter-rod. Two small kerosene-lamps.

Support the pasteboard screen with its long edges vertical on a table, and in front of the screen place the short rod upright about 5 cm. distant. Place the set of four candles, lighted, about 80 cm. distant from the screen and a little to one side of the median line running at right angles with the screen through the rod, the two outside candles of the set being made equidistant from the rod. Place the remaining candle a little to the other side of the median line, and move it back and forth from and toward the rod until the shadow A, which the rod makes by intercepting the light of this candle, appears just as dark as the middle of the shadow B, which the rod makes by intercepting the light from the set of four candles. Then measure and record the distance from the single candle to shadow A, and from the middle of the set of candles to shadow B. Draw any inference that the experiment warrants in regard to the effect which the distance of a source has upon the amount of light received from it.

If this experiment is not performed in a darkened room the apparatus should be so placed, with auxiliary dark screens if necessary, that the rod will cast no shadow on the white screen when the candles are not in use. The candles should in all cases be lighted for some minutes before the test begins, and the wicks should be carefully snuffed occasionally, in order that the flames may be of the same size. In comparing the two shadows the observer should look at them first from one side of the median line and then from the other side.

After the experiment above described, put away the candles and place two small kerosene-lamps equidistant from the upright rod and on opposite sides of the median line. Adjust the flames, turned edgewise to the rod, until the shadows are of equal dark-

ness; then turn one of the lamps about in place until its flame is flatwise to the rod, and compare the shadows again, fixing the attention upon the middle of the more blurred one. If the shadows still appear to be of equal darkness, record the fact. If they do not, move one of the lamps toward or from the rod until the shadows appear equally dark, and then record the distance of each flame from the corresponding shadow.

[For the Alternative, Bunsen's Photometer, see § 241.]

239. Discussion of Exercise XXXIV.—The preliminary experiment with four candles compared with one candle should serve approximately to confirm the law which the student may already (§ 236) have conjectured. The experiment with the kerosene-lamps, with which the exercise closes, may help the student to decide whether flames are perfectly transparent. If they are, there should, of course, be practically no difference in the amount of illumination from the edgewise and the flatwise use of the flame, since the portions of it which are more distant from the screen should send their rays with undiminished intensity through the nearer portions of the flame. It will be found desirable to reverse the mode of using the lamps; that is, if at first L is placed flatwise and L' edgewise to the screen, let L afterward be placed edgewise and L' flatwise, and see whether this change makes it necessary to reverse the relative distances of the lamps from the screen. The student may record his estimate of the relative intensities of the lights used by means of the equation

$$L : L' :: D^2 : D'^2,$$

in which $D =$ distance from L to the shadow S_L (Fig. 68), and $D' =$ distance from L' to the shadow $S_{L'}$.

Rumford's method of photometry cannot be depended upon to yield accurate results unless the observer has had much experience in using it, since the two shadows may differ in *character* so much that it is impossible to be sure when their central portions are equally dark.

240. Bunsen's Photometer. — The form of photometer devised by the German chemist and physicist Bunsen yields, under suitable conditions, more accurate results than the apparatus just described, and is equally simple, but more difficult to use in an undarkened room.

Drop a little spermaceti, stearic acid, or paraffin, on a sheet of heavy, unsized, white paper,—very thin drawing-paper, for instance. Heat the paper by placing on it a moderately hot iron weight or a can of hot water, until the spermaceti, or other substance used, is entirely melted and soaked evenly into the paper, so as to make a roughly circular spot about 3 cm. in diameter. Cut out of the paper a circle about 12 cm. in diameter with the spot just prepared in its centre. It will be noticed that the spot is *translucent*, that is, it allows some light to pass through it, although objects cannot be clearly seen through it. If one looks from a darker portion of the room toward a brighter portion with this screen interposed, the translucent spot will appear brighter than the ring of opaque paper around it, while under the reverse conditions of illumination the opaque ring will appear brighter than the spot.

Mount the screen in any convenient way; for example, in a split stick, fastened to a square bit of board for a base. In making photometric observations with this screen the illumination from one source of light is to be allowed to fall at right angles on one side of the screen, and that from the other source on the other side. The screen is then to be moved back and forth between the two lights until a position is found in which the appearance of the screen, as tested by the contrast between the central spot and the rest of the surface, is exactly the same on both sides when viewed from the same angle. The illumination on the two sides is then equal, and the distances from the lights to the screen will afford a means of comparing the power of the lights, as already indicated in § 239.

ing into another room, similar to the one in which he now is. It is well known, too, that a mirror, a sheet of polished metal, or even a window, may in bright sunlight be itself invisible, while giving off dazzling flashes of light. In all such cases as those above mentioned it is the image of the *reflected objects*, and not the reflecting surface, which is seen; and we may safely conclude that a surface, which reflected light perfectly would itself be absolutely invisible.

On the other hand, objects which transmit light very perfectly are seen with difficulty; so that, for example, it is sometimes not easy for an observer, looking down into a clear, still, and deep body of water, to distinguish its surface at a glance.

And again, the details of rough, black surfaces which absorb incident light very largely, as soft flaky soot or lamp-black does, are not easily seen, especially in a dim light.

So the matter may be summed up by saying that a surface is made visible *in its details* by means of the light which it reflects irregularly, and by that alone. The *outlines* of objects may, of course, be perfectly distinct, by contrast with the background, when all details of their surfaces are invisible.

244. Law of Reflection.

EXPERIMENT 53.

Support a mirror on edge on a sheet of paper laid flat as described in Exercise XXXV (§ 245). Stick a pin in the paper in front of the mirror; sight along the edge of a ruler at the image of the pin formed by the reflection of the mirror, and then draw a line with a pencil along the surface of the paper and the edge of the ruler until it meets the surface of the mirror. Connect this point of meeting with the pin, by another straight line, and draw a third straight line perpendicular to the surface of the mirror, at the point where the latter is intersected by the two lines already drawn. Remove the mirror, and measure with your protractor the angles formed by the oblique lines with the perpendicular to the surface. Call the angle formed with

this perpendicular by the line from pin to mirror the *angle of incidence*, and call the other the *angle of reflection*. A statement of the relation between these two angles constitutes the principal law of reflection of light. State this law.

245. EXERCISE XXXV.

IMAGES IN A PLANE MIRROR.

[Trowbridge, Exp. 181.]

Apparatus: A plane mirror, such as a piece of ordinary looking-glass of good quality, about 15 cm. long and 7 cm. wide. Some means of supporting this mirror with its plane vertical. A meter-rod. A sheet of paper about 50 cm. square.

Fasten the sheet of paper on a table and draw a straight line through the middle of it. Place the mirror over the middle of this line, with its plane vertical and with its horizontal edges parallel to the line. Stick a pin upright into any part of the paper from which the reflecting surface can be seen. Then place the eye in some position from which the reflected image of the pin can be seen, and draw another line on the paper directly toward that point of the table from which the reflection of the pin appears to rise. Without disturbing the pin or the mirror place the eye in some quite different position and draw a line as before. Then draw a line from the actual pin at right angles to the line on which the mirror rests, and continue it until it crosses the other two lines. Repeat everything with the pin in several different positions. Study the results obtained with the purpose of finding how to foretell where the image of the pin placed in any position in front of the mirror will appear to be, the eye of the observer being placed in any position from which the image can be seen.

After the student has in this way learned to place the image of an object so small that the different parts of the image do not have to be separately considered, he should learn to place and form the images of large objects, observing that different points in the outline of such objects are to be treated like small objects taken as a whole.

In following the last paragraph of the directions for Exercise XXXV, one may conveniently use for a *large object* a triangle, or other geometrical figure, drawn on the

paper in front of the mirror, the angles being marked by means of standing pins.

246. Statement of Results.—From the study of the diagrams obtained in Exercise XXXV, and from his everyday experience with mirrors, the student should be able to state the relations which exist between the position of any object and that of its image as produced by a plane mirror. This comparison of the image and object should take account—

(*a*) of their relative size;

(*b*) of their respective positions relative to the mirror;

(*c*) of the question whether the image is upside down, or *inverted*, with reference to the object.

(*d*) of the question whether the right side of the object is opposite to its own reflection, or to that of the left side.

Describe the relations between object and image under the four heads (*a*, *b*, *c*, *d*) above given, but do not forget that the only safe way to locate the image of an object is to locate one by one the images of important points in the outline of the objects.

247. Hinged Mirrors; Kaleidoscope.—As the rays reflected from a plane mirror leave its surface precisely as if they came from an actual object *through* the surface occupied by the mirror, it is evident that such rays may be reflected again from another mirror, in such a way that the observer will see in the second mirror an *image of the first image.* By means of suitable arrangements this process may be repeated indefinitely, each successive image, however, being a little fainter than the one of which it is the reflection.

EXPERIMENT 54.

Two plane mirrors, meeting along one edge, and hinged together by elastic bands so that the angle between them may be varied from 0° to 180°, show interesting effects when a candle-flame or any other

small bright object is placed between them. Let the student make the angle 90°, 60°, etc., in turn, and work out an explanation of the resulting sets of images. This will lead to an understanding of the principle of the kaleidoscope.

248. Two Kinds of Images.—If the rays of light proceeding from a point are by any means brought together again at a different point, then the second point is called a *real* image of the first. (See § 262.) A real image has an actual existence in space, and will show as a *picture* upon a properly placed white screen.

If the rays of light proceeding from a point are by any means made falsely to appear to diverge from a different point, then the second point is called a *virtual* image of the first. (See § 263.) A virtual image has no real existence in space, and would not show upon a screen placed where it *appears* to be.

Is the image formed by a plane mirror real or virtual?

249. Refraction of Light.

EXPERIMENT 55.

Place on a table a tin pan 15 cm. or more in diameter, and with nearly vertical sides 4 or 5 cm. high Place a small coin on the bottom of the pan, and adjust the head in such a position that the side of the pan will just hide the more distant portion of the coin from the eye at *E* (Fig. 70). Maintain the head in this position by resting it against any convenient support; keep one eye closed, and look with the other into the pan, just beyond the farther edge of the coin, while another student slowly pours in water. Have the pouring stopped as soon as the whole of the coin becomes visible, and note the depth of water in the pan.

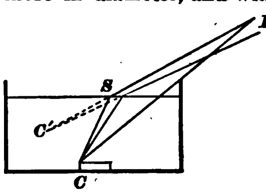


FIG. 70.

Repeat the experiment with the eye held vertically above one edge of the coin, with a slender stick or a stout wire laid across the top of the pan, nearly in the line of vision,

to serve as a point of departure from which to measure the apparent displacement of the coin, if any should be observed.

Empty and wipe the pan and repeat the first experiment, using this time kerosene instead of water. Again measure the depth of the liquid.

250. Interpretation of the Preceding Experiment.—It is a principle in optics, already suggested by Exercise XXXV, and confirmed by subsequent experiments (§ 247), that objects always appear to the eye to be in the direction *from which the rays are travelling at the moment of entering the eye.* Evidently, then, since the coin appeared to rise when the water was poured into the jar, the light-rays which proceeded from the coin must have been bent aside in some way by the water.

In Fig. 70 the straight line *CE*, which passes from the left-hand edge of the coin *C* to the pupil of the eye at *E*, represents the course of a light-ray from that point before the water was poured into the jar. Any ray that passed farther to the right than *CE* would be intercepted by the side of the pan; any ray that passed farther to the left, or more nearly vertical than *CE*, would miss the eye altogether: hence it is evident that, so long as the pan is filled with air only, and the eye kept in the position shown, the coin cannot be seen. But as soon as water is poured into the pan, the rays no longer travel in straight lines from the object *C* to the eye. Each ray suffers an abrupt change of direction at the surface of the water, and from this it follows that such rays as those which take the general course *CS* in the figure are finally brought to meet the eye at *E*. As a result of the bending *C* becomes visible, and its farther edge is seen apparently at *C′*, in a position somewhat raised above the bottom of the pan. Experiment shows that the course *CSE* might be retraced by a ray.

That is, a ray leaving E in the direction ES would reach C by the line SC.

If normals were drawn to the surface of the water, at the points about S where the rays emerge, would the bending, or *refraction*, of each ray be towards or from the normal (in the air)? If the rays were passing from E to C, would the refraction at the surface of the water be toward or from the normal (in the water)? Judging from the depths of water and of kerosene needed to produce the same amount of apparent displacement of the coin, which of these liquids do you think to have the greater *refractive power* (§ 254)?

251. Refraction by a Prism.—The refraction of light produced by a triangular transparent prism is very convenient for study. The prism may be a solid one of glass, or it may be hollow, with the sides made of glass and the interior filled with any desired liquid. If the glass sides are of equal thickness throughout, they will not affect the amount of refraction caused by the prism as a whole. Practically such hollow prisms filled with liquid may be regarded as liquid prisms. When a ray traverses obliquely a parallel-sided medium, such as a sheet of plate-glass, there is refraction of the incident ray at the surface which it enters, and refraction of the emergent ray at the surface from which it passes out; but the incident and the emergent ray are parallel, although not in the same straight line. (Demonstrate by a diagram how this result must follow from the fact that the *amount* of refraction at both surfaces is the same, but its *direction* different.)

EXPERIMENT 56.

Cut out of soft wood, such as pine or white-wood, a wedge-shaped block, two sides of which shall be about 8 cm. square, and shall

make with each other an angle of about 30°, as shown in Fig. 71.

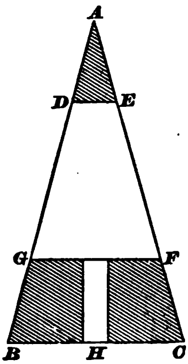


FIG. 71.

Bore through the wedge a hole about 4 cm. in diameter, *DEFG*. Bore a gimlet-hole through from *H*. Soak the finished wedge in very hot melted paraffin until the wood is well saturated with it. Then cement over each of the faces *AB* and *AC* a piece of clear window-glass of even thickness throughout. A beeswax-and-rosin mixture will do for the cement. Fill the prism with water, and stop the hole *H* with a small plug. Then stand the prism on its base *BC*, upon any convenient support, at various heights above the table-top, and look through the water at small objects placed beyond the apparatus, comparing their apparent positions, as seen through the prism, with their real positions.

252. Path of a Ray through a Prism.—From the preceding experiment it is plain that rays passing through the prism are considerably bent out of their course on their way to the eye. A similar experiment made with a slender beam of parallel rays could be so conducted as to show the course of the beam up to the surface of the prism by which it entered, and away from the surface by which it emerged. Since it is a well-established principle that, *in a*

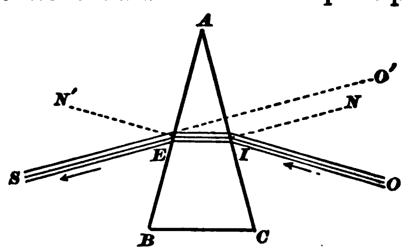


FIG. 72.

uniform medium, light travels in straight lines (§ 231), we may feel sure that *inside the prism* the beam travels in a straight line. Putting together what we are able by suitable arrangements to observe, and what we are justified in

inferring, we may map out the course of a beam of light through a water-prism of 30° as shown in Fig. 72.

The incident beam *OI* is, on entering the prism, refracted *towards the continuation of the normal NI,* and the emergent beam *ES* is, on leaving the prism, refracted *away from the normal N'E.*

If a solid glass prism *DEF*, Fig. 73, were substituted for

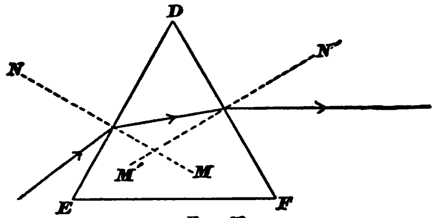


Fig. 73.

the water prism *ABC*, the result would be the same as that just obtained, except that the amount of bending on entering and on leaving the prism would be greater.

253. Position of Minimum Deviation.—It is found that prisms refract light most *advantageously* for purposes of measurement when placed in the *position of minimum deviation,* that is, the position which produces least change in the direction of the ray. This position for any prism whose section, like that of Fig. 74, is an isosceles triangle,

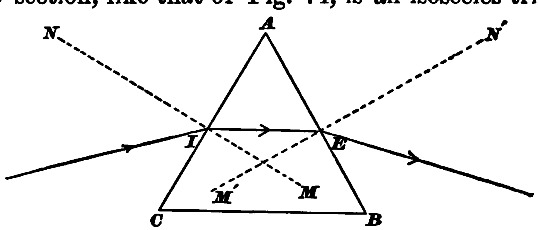


Fig. 74.

is the position in which the course of the rays inside the prism is parallel to the base of its section, that is, in which the line *IE* is parallel to *CB*. In Fig. 74 the prism is placed in the position of minimum deviation; in Fig. 73 the prism is not so placed.

254. Refractive Power: Index of Refraction.—When a ray of light on passing from one medium into another is bent toward the normal to the bounding surface of the two media, drawn from the point of incidence into the second medium, the second medium is said to be *more refractive* than the first medium.

Instead of more refractive and less refractive, the terms *optically denser* and *optically rarer* are sometimes used; but they are objectionable from the fact that of two liquids, for example, the one of greater specific gravity is not infrequently the less refractive, as is the case where water is compared with alcohol or olive-oil.

Every material of which prisms are made is more refractive than air, and it is convenient to remember that a light-ray in entering and on leaving a prism in air will on both occasions be refracted toward the base, or thick end, of the wedge, when the prism is so placed as to give the minimum deviation.

It is convenient to state relative refractive powers not by the ratios of the angles of incidence and refraction themselves, but by those of certain lines which bear definite relations to these angles. Consider the case shown in Fig. 75, in which a ray I suffers refraction at the surface AB. With the point of incidence on AB as a centre describe a circle of radius 1. From the points p and q where this circle is cut by the incident ray I and by the refracted ray R, draw the lines n and v, respectively at right angles with the line NN', which is perpendicular to the refracting surface at the point of incidence.

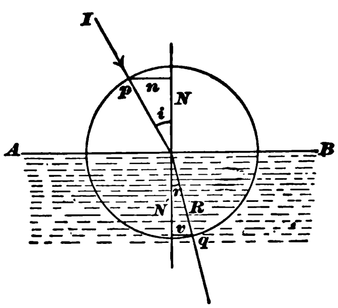


FIG. 75.

The ratio $\frac{n}{v}$ is called the *index of refraction.* If the direction of I were changed, the direction of R would change, and to such an extent that the ratio of the new n to the new v would remain unchanged. The index of refraction is, accordingly, for any two given media, independent of the angle at which the light strikes the surface between the two media. The line n, drawn as here shown within a circle of unit-radius, is numerically equal to what is called in Trigonometry the *sine* of i, the *angle of incidence,* and v to what is called the *sine* of r, the *angle of refraction.* The general law of refraction may, then, be stated as follows: *The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant quantity, for any two given adjoining media, and is called the Index of Refraction for those two media.* (For a table of indices of refraction see Appendix.)

255. Total Internal Reflection, Critical Angle.

EXPERIMENT 57.[1]

Half fill with water[2] an ordinary chemical flask, Fig. 76, and set it

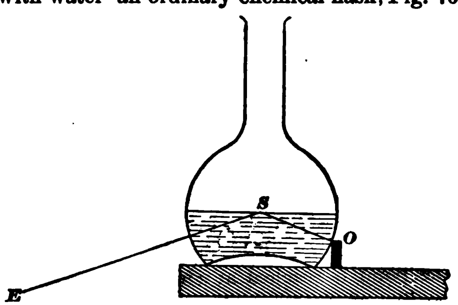


FIG. 76.

on a table, close to the edge. Place against the farther surface of

[1] This experiment is taken from Balfour Stewart's *Elements of Physics.*

[2] Or rather, fill the flask up as high as the centre of its spherical portion.

the flask any small object O, such as a bit of crayon, and, with the eye held at the position shown at E, look up toward S, the centre of the upper surface of the water.

Clearly the ray OS, reflected at S in the direction SE, is transmitted to the eye at E. Further experiments would enable us to show that, practically, all the rays which strike the upper surface of the water from beneath, in lines parallel to OS, are reflected, none escaping through the upper surface of the water. Hence the name *total internal reflection* is given to reflection occurring under such conditions as have just been described.

At this point the question naturally occurs: Would rays coming through a liquid to the surface separating it from air be equally well reflected at every possible angle? This question can probably be answered from the student's previous experience, but the following experiment is instructive.

EXPERIMENT 58.

Cut out of thin wood, such as is used for the backing of pictures and mirrors, a circular piece about 12 cm. in diameter. Make a small hole through the centre with an awl, and push into this hole, in which it must fit just tightly enough not to drop through, a piece of knitting-needle about 12 cm. long.

Float the circular board, with the inserted knitting-needle, in a broad, deep pan, filled to the *brim* with water. Push the needle down until its upper end is nearly level with the upper surface of the board, and look down *obliquely* through the water, close past the margin of the board, at the lower extremity of the needle. Now draw the needle up, little by little, through the floating board until the point is reached at which the needle just vanishes from view, the line of sight being made at last as nearly horizontal as possible. Lift the board from the water, and note how much of needle still projects below the board.

Since it is not possible to see any portion of the needle in the water at the last, it is obvious that the light-rays from it do not succeed in escaping from the upper surface

of the water. As the only difference between the conditions during the earlier and the later part of the experiment depends upon the different angles at which the light-rays from the needle strike the surface of the water, we see that at certain angles the light is, in part at least, transmitted, and passes out, while at other angles it is totally reflected.

A little study of Fig. 77 will serve to illustrate what has

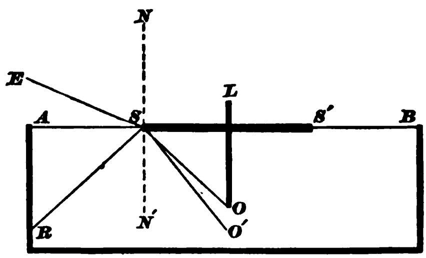


FIG. 77.

already been learned from this experiment, viz., that there is a certain least angle that a ray, as OS, can make with the normal SN' in order to be totally reflected, as indicated by the line SR. This limiting angle OSN' is called the *critical angle* for the media in question,—in the present instance water and air. Any ray which, like $O'S$, makes an angle with the normal less than the critical angle, will pass out of the medium with refraction from the normal SN, as the line SE shows.

256. Cases of Perpendicular Incidence.—When a ray traverses perpendicularly (i.e., at right angles) the surface formed by the junction of two media, there is no refraction, as the student may have inferred from the experiments in § 249.

257. Relation of Prisms to Lenses.—If two prisms are placed as shown in Fig. 78, it is evident that two parallel

rays A and A', both in the plane of the paper, entering respectively the upper and the lower prism, will cross at some point, as P, on the further side of the prisms.

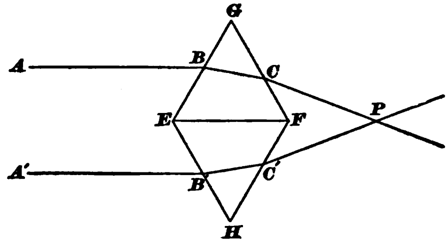


FIG. 78.

Fig. 79 represents the section of a *lens* (see § 258) by a plane so placed as to divide each of the curved surfaces into halves. From the likeness of this figure to the one preceding, it will readily be seen that rays entering the lens *parallel to the line EF* will be so changed in direction by the lens that, after leaving it, they will all cross somewhere the continuation of *EF*. Experiment shows, in fact, that, with ordinary lenses, most such rays will cross this line near one particular point (Fig. 80).

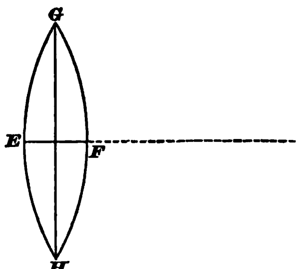


FIG. 79.

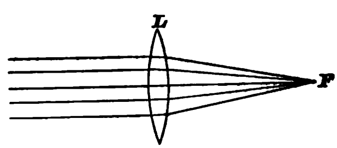


FIG. 80.

258. Definitions relating to Lenses.—The commonest lenses have surfaces which are segments of spherical surfaces, and the simplest case is that in which both surfaces of the lens have the same curvature, i.e., are segments of equal spherical surfaces. (See Fig. 81.)

The *centres of curvature* of a lens are the points *C*, *C'*, about which as centres the curved surfaces *ASB* and *ARB* are respectively described.

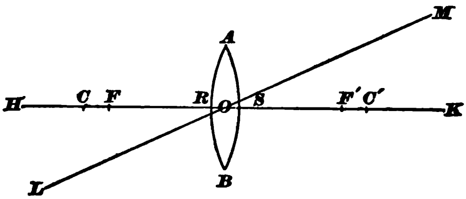


FIG. 81.

The *principal axis* is a straight line *HK* of indefinite length drawn through the centres of curvature.

The *principal foci* (of a convex lens, see § 259) are the two points *F*, *F'*, one on either side of the lens, at which rays entering the other side of the lens parallel to the principal axis will cross each other after leaving the lens.[1]

The *optical centre* is a point *O*, in or near the lens, which is so situated that any ray which passes through this point has the same direction after leaving the lens as before entering it. In Fig. 81 the optical centre coincides with the centre of gravity of the lens.

The *focal distance* is the distance from either principal focus to the surface of the lens. (See Appendix.)

A *secondary axis* is any straight line other than *HK*, as *LM*, drawn through the optical centre. There are, of course, an infinite number of such axes. On the secondary axes foci for rays parallel to each other and to the given axis occur at distances from *O* nearly equal to *OF* and *OF'*, unless an axis is taken which makes a very large angle with the principal axis.

---

[1] The rays do not all cross exactly at a point. See § 270. If the lens is a symmetrical double-convex one, like Fig. 81, and of common glass, the principal foci will lie near the centres of curvature.

259. Forms of Lenses.—Lenses are classed as *convex*, or *converging*, and *concave*, or *diverging*. Convex lenses are all thicker in the middle than at the margin, and cause parallel light-rays to converge. Concave lenses are thinner in the middle than at the margin, and cause parallel light-rays to diverge. Some of the lenses used in the most accurate optical instruments have convex or concave surfaces which are not strictly parts of spherical surfaces. Such lenses possess certain advantages over spherical-surface lenses (see § 270).

260. EXERCISE XXXVI.

FOCAL LENGTH OF CONVERGING LENSES.

[Trowbridge, pp. 325, 326.]

Apparatus: A small "converging" lens, such as is used in spectacles or eye-glasses, about 10 or 15 cm. in focal length, mounted, with the longest dimension vertical, on a cork.[1] A meter-rod upon which the cork bearing the lens rests and slides, the cork being cut so as to bestride the thickness of the rod and hold the lens with its optical axis parallel to the lens of the rod. A small piece of thin, white pasteboard to receive the images formed by the lens, mounted, like the lens, vertically upon a cork prepared to slide along the rod. A pin mounted like the lens and screen.

FIRST METHOD.—Place the lens and the screen upon the meter-rod. Point the rod in such a direction that the sun's rays will pass through the lens and fall upon the screen. Then looking at the back of the screen in order that the eyes may not be dazzled by the image, move either the screen or the lens back and forth upon the rod until an arrangement is obtained that gives upon the screen a sharp image of the sun. Then measure the distance from the centre of the lens [more strictly, from the nearer surface of the lens] to the screen. Make the setting and measurement several times, recording each time. If the two surfaces of the lens are differently curved, repeat the test with the lens reversed.

SECOND METHOD.—Replace the screen by a pin. Let this pin be placed about as far from the end of the rod as the observer usu-

[1] [A better method of mounting, on a small block of wood, is shown in Fig. XXIV.]

ally holds a book from his eyes. Hold this end of the rod to the eye and, looking sharply at the pin, direct the rod and adjust the lens in such a way that the light from some clearly-defined object, a church-spire or a chimney, for instance, several hundred feet distant, may pass through the lens and form an image of the object close to the pin. To decide whether the image is nearer the eye than the pin is, move the eye to and fro, to the right and the left, watching the pin and the image. The more distant of the two will, as compared with the other, appear to move in the same direction in which the eye moves. The rod should not be held in the hands during this test, but should be placed on some more steady support.

Continue the adjustments until this test fails to show which of the two is nearer the eye. Then measure the distance from the centre of the lens to the pin. Repeat the setting and measurement several times, reversing the lens if the sides are differently curved.

261. Discussion of Exercise XXXVI.—In the *First Method* the focal distance of the lens was obtained by allowing the sun's rays to traverse the lens by means of which they were brought approximately to a focus. The position of this focus was located on the surface of the card-board screen, and the measured distance from the surface of the lens to the surface of the screen is the focal distance required. In Fig. 82 the course of the rays is illustrated.

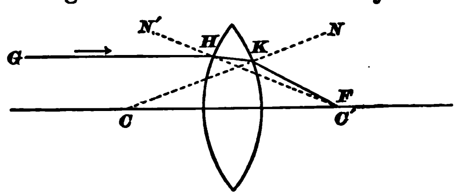


FIG. 82.

Observe that any straight line drawn from either centre of curvature through the lens will be a *normal* to the further surface.

Strictly speaking, the rays from the sun are not precisely parallel, but, owing to its great distance, they are very

nearly so. That is, the rays coming from any particular point of the sun's surface, to the lens are practically parallel *to each other*, and will come to a focus at or very near the principal focus of the lens. Rays from different parts of the surface, from the two ends of the same diameter, for instance, are not practically parallel to each other on reaching the lens, and such rays do not converge to the same point in the image. In fact, if rays from the *different parts* of a luminous body could be converged to the same point, the result would not be an *image* repeating the features of the original objects. It would be a mere point, or very small patch of light.

(Suppose a luminous point to be placed at *F*,[1] Fig. 80, what effect would the lens have upon the rays which proceed from it? How does the common dark-lantern illustrate this?)

The *Second Method* cannot well be discussed until after Exercise XXXVII has been performed. (See § 264.)

262. Formation of a Real Image by a Convex Lens.—A *real image* is (§ 248) an image which may be received and shown on a *screen*, that is, an irregularly reflecting but moderately smooth surface, such as that of a sheet of white paper. The image of the sun produced in Exercise XXXVI was a real one. Such images are shown by the *magic lantern*, § 281, or "stereopticon," but they may be produced much more simply.

EXPERIMENT 59.

Hold the lens used in Exercise XXXVI flatwise between a small flame and a white wall or large sheet of white paper, the laboratory being at least partially darkened. Note the image produced when the lens is held near the flame, and that produced when it is held near the wall. Repeat the experiment with some other lens of very

---

[1] The focus *F* is here represented as coinciding exactly with one of the centres of curvature. This is not necessarily the case.

different focal length. In every case observe whether the image is (*a*) erect or inverted, (*b*) magnified or diminished, (*c*) bright or faint.

The formation of such images as these may be understood by aid of Fig. 83. Suppose the candle to be represented by the arrow AB. Let the course of two rays AC

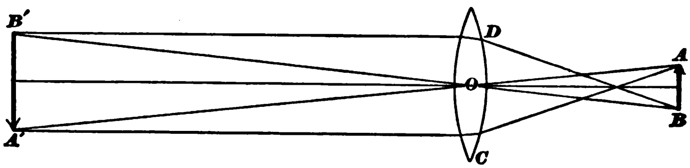


FIG. 83.

and BD be traced from object to image. The ray AC suffers two refractions in passing through the lens, and finally crosses the secondary axis AO at the point A'. In the same way all the rays entering the lens from a point at the extreme tip of the candle-flame will cross the axis AO at or near A'. In consequence of this there will be formed at A' an *image* of the point A. So, too, there will be formed an image of the point B at B', where the rays from B cross the secondary axis from B. In this way the images of any number of points between A and B, the extremities of the candle, may be located at points between A' and B', and the effect of all these images of all the points along AB is to produce at $A'B'$ [1] a complete image or picture of the candle and its flame.

Let the student construct a diagram for the same purpose that is served by Fig. 83, using in place of the ray AC a ray from A parallel to the principal axis, and in place of the ray BD a ray from B parallel to the principal axis.

[1] The image $A'B'$ is here represented, according to custom, as being straight, but the student is expected to determine for himself, in Exercise XXXVIII, whether it is really so.

How would the size and position of the image be affected (*a*) by increasing the convexity of the lens? (*b*) by using a lens of some substance whose index of refraction is greater or less than that of glass?

263. Formation of a Virtual Image by a Convex Lens.—

EXPERIMENT 60.

Hold the lens used in Exercise XXXVI flatwise above a printed page, at first within a centimeter or two of the printed page, and gradually move it toward the eye while looking through it at the print, stopping *as soon as the letters begin to appear blurred.* Is the image of the letters as seen through the lens—

(*a*) right side up or inverted?

(*b*) magnified or diminished as compared with the real letters?

Repeat the experiment with two lenses of different focal distance, using both at the same time side by side (not one over the other), and notice what differences, if any, are to be observed in the images.

The images thus observed are *virtual images* (§ 248). They cannot, as the student will find, be projected upon a screen, and are not formed by the actual intersections of the light rays from the object in the way already explained for real images (§ 262). Fig. 84 may serve to illustrate

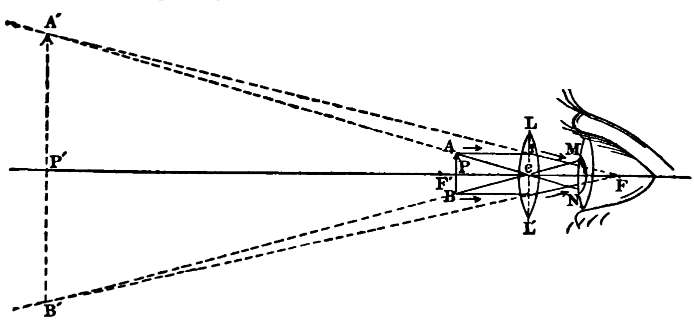


FIG. 84.

the manner in which virtual images are produced. Let *AB* be the object. To find the position of the virtual image of *A*, draw the ray *AI* parallel to the principal axis.

This ray will, after leaving the lens, pass through F, the principal focus, and so will appear to have come along the path MF. Draw another ray, AC, passing through the optical centre of the lens. This ray will (§ 258) after leaving the lens have the same direction as before entering it. If, then, we carry back the line AC until it crosses the line MF, the point A', where the crossing occurs, is a point from which both of the rays considered *appear* to have come. A' is, then, the virtual image of A. By a similar process B' is found to be the virtual image of B.

The image $A'B'$ is evidently larger than the object AB. Whenever a virtual image is formed by a convex lens, this image appears, to an eye placed in any ordinary position on the other side of the lens, larger than the real object would look if held at a comfortable distance from the eye. Hence the name *magnifying-glass*, so commonly given to such a lens.

Draw two diagrams[1] to show why one of the lenses used in Experiment 60 magnified more than the other did.

Draw two others to show why a lens of highly refractive material magnifies more than one of less refractive material.

264. EXERCISE XXXVII.

CONJUGATE FOCI OF CONVERGING LENSES.

Apparatus: The same as in Ex. XXXVI, and in addition a small kerosene-lamp. The identical lens should be taken that was used in Ex. XXXVI.

FIRST PART.—REAL FOCI.—**Smoke the outside of the lamp-chimney at the base and then rub off the soot in one small spot at the height of the bright part of the flame. Place one end of the meter-rod as nearly as may be directly beneath this spot, and place the pasteboard screen at the opposite extremity of the rod [see Fig. XXIV]. Set the lens on the rod near the lamp and then slide**

[1] It will be well to exaggerate the difference in curvature of the two lenses.

it away from the lamp until a distinct image of the illuminated spot on the chimney appears on the screen. Then measure the distance from the centre of the lens to the spot which serves as the object, and call this distance D_o. Measure the distance from the centre of the lens to the image, and call this D_i. Without moving either lamp or screen slide the lens still farther along the rod, and if in any new position it throws upon the screen a distinct image of the spot, measure and record the corresponding distances D_o and D_i. Then place the screen at a distance of 80 cm. from the spot, and starting the lens near the lamp proceed as before. Place the screen in turn at distances of 70 cm., 65 cm., 60 cm., etc., from the

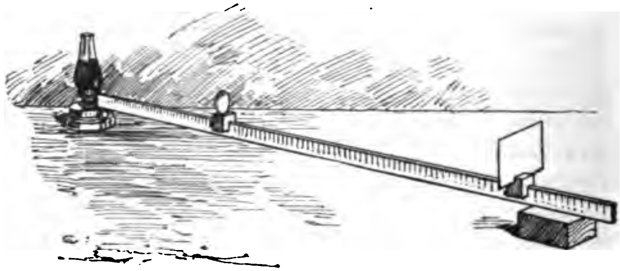


FIG. XXIV.

spot, making settings and readings, until it reaches such a position that the lens can no longer throw a distinct image of the spot upon it. (Five or six positions of the screen should be used in all. If the positions here suggested do not suit the particular lens in use, other positions should be taken.)

If a somewhat darkened room is not available for this exercise, it is well to use a pin as an object, and replace the screen by another pin used as in Ex. XXXVI.

Let F represent the focal length of the lens as already found. Compare in every case the quantity $\frac{1}{F}$ with the quantity $\frac{1}{D_o} + \frac{1}{D_i}$, expressing both quantities in decimals.

SECOND PART.—VIRTUAL FOCUS.—Finally place a pin between the lens and its principal focus, and using another taller pin instead of the screen, endeavor to locate the image of the first pin. The image of the first pin must be seen through the lens while the other pin is looked at over the lens. When the position of the image is

found, measure and record the distances D_0 and D_1 as before. The student should be shown how the results of this experiment, if successful, accord with the laws of conjugate foci already made out.

265. Discussion of Exercise XXXVII.—*First Part.* The mode of formation of the images here obtained has already been explained in § 262. A and A', B and B', in Fig. 83, are called *conjugate foci,* though on secondary axes. In general, *conjugate foci of a lens are any two points so situated that an object at each of them in turn will produce an image at the other.*

Inspection of the observations will at once show that decreasing the object-distance increases the image-distance, so that bringing the object to a position not much farther away from the lens than its principal focus would cause the large image to recede very far on the other side of the lens. And, again, an object at a great distance from the lens has its very small image formed at a point close to the principal focus, but a little farther from the lens. If the object-distance is several hundred meters, the position of the image, in the case of ordinary lenses, practically coincides with that of the principal focus, hence the *Second Method* for focal distance of Exercise XXXVI. The student should illustrate this by a diagram of steeple, lens, and image, drawn according to the directions given in § 262. The formula $\frac{1}{D_0} + \frac{1}{D_1} = \frac{1}{F}$ cannot be explained without the use of more mathematics than can be required of beginners in physics.[1]

266. Discussion of Exercise XXXVII: Second Part.— When, as in the *First Part* of this Exercise, $\frac{1}{D_0} + \frac{1}{D_1} = \frac{1}{F}$,

[1] For a discussion of the formula, see Everett's *Deschanel's Natural Philosophy, Revised Ed.*, pp. 1017, 1018; or Trowbridge's *New Physics*, pp. 322 to 324.

that is, when the reciprocals of object-distance and image-distance must be *added* to produce the reciprocal of the focal distance, it is evident that each of the other fractions must be *smaller* than $\frac{1}{F}$, and therefore each of the other denominators *larger* than F. In fact, all of the real images obtained in the First Part were obtained with the object somewhere beyond the principal focus of the lens.

In the Second Part, however, the object was somewhat inside the principal focus, and therefore the fraction $\frac{1}{D_o}$ is greater than $\frac{1}{F}$. The formula for this part of the exercise compares the *difference* of the other two reciprocals with $\frac{1}{F}$. Write this formula.

From the difficulty which is often experienced in locating the virtual focus, in performing this part of the exercise, the focal distance obtained by calculation from the measurements of the virtual image-distance and its corresponding object-distance cannot always be depended upon as well as can the preceding determinations of focal distance.

267. EXERCISE XXXVIII.

REAL IMAGE FORMED BY A CONVERGING LENS.

[Trowbridge, p. 325.]

In the preceding experiments the object presented to the lens has been small or has been at such a distance as to subtend only a small angle as seen from the place of experiment. In proceeding to the study of cases in which the object subtends a larger angle, the student will be assisted to a considerable extent by what he has already learned, but some new considerations arise which make new experiments desirable.

Apparatus: The lens used in Ex. XXXVI. A sheet of blank paper about 1 m. long and 50 cm. wide. A meter-rod. Several pins.

Spread the sheet of paper smoothly upon a table and fasten it at

the corners. [See Fig. XXV.] Draw a line from the middle of one end of the sheet to the middle of the other end. Near one end of this median line lay off at right angles another line extending 4 cm. on each side, and mark one end of this as the tip and the other as the tail of an arrow. Divide this arrow into four parts of equal length by means of dots marked respectively 1, 2, 3, 4, 5. By means of two pins thrust through the cork that carries the lens, fasten the lens with its edge resting upon the paper, its axis parallel to the median line on the paper, and its centre directly over

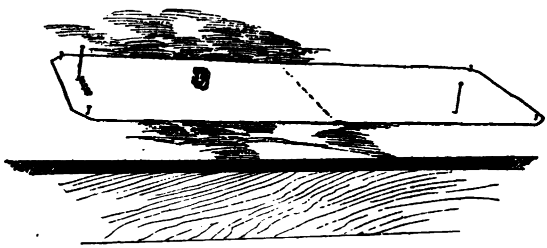


FIG. XXV.

the median line at a point distant from the middle of the arrow about 1⅓ times the focal length of the lens. Stick a pin upright into the point of the arrow, which may be called point No. 1, and then stick another pin upright into the table in such a position as to make it coincide with the image of the first pin. Mark the point where the second pin pierces the paper, I_1. In the same way locate I_2, I_3, etc., corresponding to the points 2, 3, etc., of the arrow. The five I-points outline the image of the arrow as it would be formed by the lens lowered until its centre should be at the level of the paper. Remove the lens, marking that point of the paper which has lain just beneath its centre, and then carefully draw straight lines connecting each of the five marked points of the arrow with its corresponding I-point.

A study of the diagram now obtained upon the paper can hardly fail to be instructive to the student.

268. Discussion of Exercise XXXVIII.—Measure the distance from each object-point, 1, 2, etc., to the lens, and from the latter to the corresponding image-points I_1, I_2, etc.

Measure the length of the object and that of the image. If the latter is not a straight line, measure the distance

straight from I_1 to I_5, and also the actual length of the image, whatever its shape.

Call the distance from point No. 3 on the object to the lens, D_o, and from the lens to the middle point of the straight line I_1I_5, D_1.

Call the length of the object, O, and the length of the straight line I_1I_5, I.

See whether a proportion can be made of these four quantities in the order just given. That is, place them in the form of a proportion, and see how nearly the product of the extremes will equal the product of the means, when the measured values are substituted for your symbols. Repeat the operation, using the distance from object-point No. 1 to lens, and from lens to I_1, together with O and I, to form another proportion. Can you, as a result of this examination, state any law for the relations between the several distances and the dimensions of object and image?

269. Suggestions for Explanation of Form of Image in Exercise XXXVIII.—1. The middle of the object is, as you have seen, nearer the middle of the lens than the ends of the object are.

2. Is the focal length of a lens along a secondary axis equal to the focal length along the principal axis? Test by experiment.

270. "Aberration" with Lenses.—It has already been suggested (§ 259) that lenses whose surfaces are portions of spheres do not bring parallel rays precisely to a point. The cause of this imperfect operation of the lens may be better understood after the student has performed a simple experiment.

EXPERIMENT 61.

Take a double-convex or plano-convex lens of 8 cm., or more, diameter, and of rather short focal distance (e.g., 15 cm.). Cut a circular piece of card-board with a diameter somewhat greater than that of the lens, and cut in the centre of this a circular hole with a diameter somewhat less than one half that of the lens. Holding this

card-board diaphragm against the surface of the lens, find the focal distance of the central portion of the latter by either method of Exercise XXXVI. Then attach a circular piece of card-board of about three fourths the diameter of the lens to a bit of wire to serve as a handle, and with this held against the centre of the lens find the focal distance for the marginal part of the latter.

Which part of the lens brings parallel rays to a focus nearer to the lens itself? Draw a diagram to show how parallel rays will be affected by the central and by the marginal portions of the lens respectively.

The defect in the operation of lenses which this experiment is intended to show is called *spherical aberration.* Lenses ground with surfaces not precisely spherical may be so constructed as to do away with this defect, provided light of one color only is used, but such lenses are very uncommon except in large telescopes.

271. EXERCISE XXXIX.

VIRTUAL IMAGE FORMED BY A CONVERGING LENS.

Apparatus: Similar to that used in Ex. 38.

Place the lens about 20 cm. from one end of the median line. Make the arrow about 5 cm. long, and place it at a distance from the lens equal to about $\frac{2}{3}$ the focal length of the latter. Then proceed as experience gained in Ex. XXXVIII and the second part of Ex. XXXVII shall direct to locate the virtual image of the arrow.

272. Dispersion of Light.—In treating of the course of light-rays through prisms or lenses it has thus far been assumed that all light-rays are equally refracted by the same prism or lens. This, however, is not true. If the paths of a red ray, a yellow ray, and a violet ray, all three incident on the surface of a transparent prism at the same angle, were traced through and out of the substance of the prism, it would be found that the three kinds of rays were unequally refracted; so that they would leave the prism in different directions. This separation of the paths of the rays is called *dispersion.* How great a difference there would be in the amounts of refraction produced would depend upon

the shape and material of the prism and upon the angle of incidence.

273. Solar Spectrum.

EXPERIMENT 62.

By means of the porte-lumière (§ 231) throw a beam of sunlight through a narrow slit at *S*, Fig. 85. A slit in an opaque shutter or

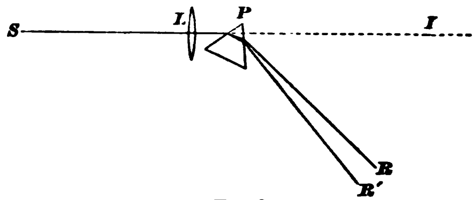


FIG. 85.

curtain may be used without the porte-lumière. Place a lens *L* in the path of the beam, and adjust it so as to throw a distinct image of the slit on a screen at *I*. Now introduce a prism *P* in the position shown in the figure, and then place the screen at *RR'*, making the distance *PR* equal to *PI*. The prism used may be of flint-glass, or, better, may be hollow (§ 251), and filled with the highly dispersive liquid bisulphide of carbon.

Examine the spot of colored light on the screen. (1) How many colors can be distinctly seen? (2) Do they blend, or are they sharply separated from each other? (3) Which color is most refracted? least refracted? Try if possible the effect of passing the emergent pencil through a second prism similar to the first and placed so as to refract the light in the same direction as the first. Try the effect with the second prism so placed as to refract in the opposite direction from the first.

The spot or band of colored light produced by the dispersion of a sunbeam is called the *solar spectrum*. It is customary to divide the spectrum into seven regions, called *red, orange, yellow, green, blue, indigo, violet,* and to call the general colors of these the *primary colors*, to distinguish them from those formed by compounding two or more of them. This division of the spectrum is evidently entirely arbitrary—a mere matter of convenience.

274. Chromatic Aberration with Lenses.—Ordinary lenses, made of a single piece of glass, give rise to colored fringes

or borders about the images which they produce. The student can readily infer the reason for this from what he has already learned in § 257 about the similarity between prisms and lenses, and from what has just been shown in § 273. This defect is called *chromatic aberration,* the word *chromatic* meaning *relating to color.* But little trouble from this source is experienced in the use of lenses of slight convexity, whose images are not to be further magnified; as, for instance, in spectacles and ordinary magnifying-glasses. "Stopping out" the greater portion of the surface of a lens with a circular diaphragm, which allows light to pass only through a small portion of the lens near its centre, improves its performance greatly (see § 270). How much help such diaphragms give by preventing fringes of color, may be learned by taking out some or all of the diaphragms of an ordinary cheap spy-glass and then looking with it at distant objects in bright sunlight.

275. Achromatic Lenses.—Fortunately for the manufacturers and users of optical instruments, it is possible to make an achromatic lens, or one, at any rate, which is practically achromatic. This is usually accomplished by uniting into one lens two separate lenses,[1] one, *A*, of flint-, and the other, *B*, of crown-glass, as shown in Fig. 86. A convex lens made in this way has on the whole a converging effect on parallel rays, while at the same time the superior *dispersive* power of the flint-glass enables the lens *A*, though of less *refractive* power than the lens *B*, just to counteract the dispersive tendency of the latter. Almost all the lenses used in optical instruments of the best quality are achromatic. Eye-pieces (§ 283), however, of the ordinary pattern do not require achromatic lenses.

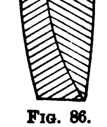


FIG. 86.

[1] Sometimes more than two pieces are employed in making an achromatic lens.

276. Velocity of Light.—Light travels so rapidly that exact measurements of its rate of motion are very difficult. Of the methods adopted to make such measurements the simplest is that of the French physicist Fizeau. It consists essentially of a source of light from which a beam of strictly parallel rays may be obtained, a toothed wheel which may be made to revolve in a plane at right angles to the course of the beam of light, and a plane mirror. Apparatus is provided by means of which the rate at which the wheel revolves can be exactly measured. The beam of light passes through the space between two adjacent teeth of the wheel, travels a distance of several kilometers, is then reflected by the mirror and returned over the same path by which it passed out. If the wheel is at rest, the beam as it returns will repass the aperture between the teeth through which it passed out. It is easy to see that, if the wheel could be revolved fast enough, a tooth might be brought into the path of the returning rays in time to intercept them. Still more rapid revolutions would bring a new gap between teeth into the path of the returning rays, and so on. Now experiment shows that alternate eclipses and appearances of the returning rays are produced when the wheel is revolved at a high and continually increasing velocity. From the rate of motion of the wheel and the distance traversed by the beam it is not difficult to calculate the velocity of light. As a result of measurements made by somewhat different means from those just described, the velocity of light has been ascertained to be about 299,920 kilometers, or 186,360 miles, per second, in a vacuum. The velocity in air is a little less.

277. Relation between Index of Refraction and Velocity of Light.—The velocity of light in any transparent substance depends on the nature of the substance. It is greatest in a so-called vacuum. It is least in the most highly refractive substances, and, indeed, the index of refraction

for any given substance depends upon the rate at which light travels through it. This is sometimes illustrated by an analogy suggested by the march of troops over ground of various kinds. Suppose a column of troops to be marching over smooth ground, as shown in Fig. 87 at the left.

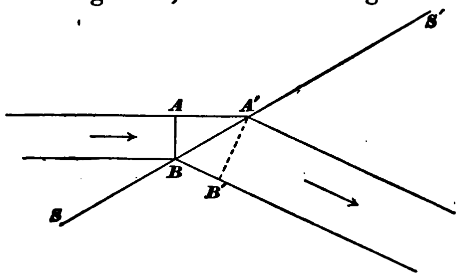


FIG. 87.

The front of the column being at AB, let the line SS' represent the border of a marsh or other difficult ground. Upon entering, the right of the column, B, first encounters the marsh, and the soldiers at B will fall behind those of the rest of the front. In consequence of this the column will, one part after another, wheel to the right until, when the whole front has entered the marsh, it will have the new direction shown by the line $A'B'$. Substitute for the column of troops a beam of light, and for the marsh a highly refractive transparent substance, and it is not difficult to see how refraction may depend upon the retarding effect of refractive substances upon light-rays.

278. Relation between Wave-lengths and other Properties of Rays.—Each of the seven "primary" colors, violet, indigo, blue, green, yellow, orange, and red, is due to a set of rays of nearly equal wave-length. The prism sorts the various colors out from the mixture of all seven which falls as a beam of white light upon the surface of the glass, by means of the fact that rays of short wave-length, like the violet, are refracted much more than those of

greater wave-length, like the red ones. Invisible rays may be detected on both sides of the visible spectrum. Those which are refracted less than the red rays may be recognized by their heating effect, when a thermometer is held in the dark region adjoining the red end of the solar spectrum. Those which are refracted more than the violet rays may be recognized by the promptness with which they affect a sensitive photographic plate held in the dark space alongside the violet end of the spectrum. In fact, pictures have been taken in total darkness by means of these actinic rays, as they are called.

Wave-lengths longer than 812 millionths of a millimeter affect the thermometer readily, but not our eyes. Those of 650 millionths of a millimeter affect the thermometer and give to the eye the sensation of red light; those of 500 millionths of a millimeter have but little heating effect, and give a green color; those of 400 millionths of a millimeter give almost no heating effect, but have a faint violet light. Wave-lengths even shorter than 200 millionths of a millimeter affect sensitized photographic plates.

Questions and Problems on Chapter XIII.

1. What would be the shape of the entire shadow (umbra) formed by a circular disk exposed to the radiation from a luminous point, situated on a perpendicular to the centre of the disk?

2. What would be the shape of the umbra of the same disk illuminated by a sphere larger than itself, and with its centre lying on the perpendicular drawn to the centre of the disk?

3. At what relative distances from a screen would a lamp of 16 candle-power and an electric arc-light of 1600 candle-power illuminate it equally?

4. Let an object be placed between two plane mirrors

meeting at a right angle. Show how many images of the object may be seen in the two mirrors. Discuss the matter as fully as you can, showing, by means of a diagram, how the various images are formed.

5. Let two plane mirrors make with each other an angle of about 60° or 70°, and let an arrow be placed between them. Show by means of a diagram the position and direction of pointing of an image of the arrow after three reflections.

6. From the proportionality of homologous lines in similar triangles, how can you determine the relative size of image and object in the case of images formed by light passing through a small aperture (§ 233)?

7. Applying the principles of § 249 to the art of shooting or spearing fish under water, what precaution should one take in aiming at them when the line of sight is not vertical?

8. In Fig. 88, which of the liquids, *A* or *C*, is optically denser than the water *B*? Explain how you know.

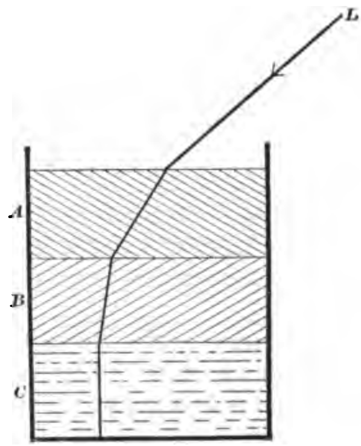


FIG. 88.

9. Show how light could traverse without refraction a prism with the form of cross-section shown in Fig. 89.

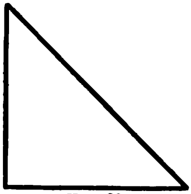


Fig. 89.

10. In your experiments with lenses you sometimes allowed the image of an object to fall upon a card. The image could then be seen from all points from which the face of the card could be seen. When the image was not allowed to fall upon a card, the region from which it could be seen was much more restricted. Explain.

11. Describe carefully the process (Exercise XXXVI, *Second Method*) by which you have found the focal length of a lens, explaining with a diagram of the rays the use of the pin.

12. The focal length of a certain lens is 6 inches. A small object is placed 10 inches from the centre of the lens and on the principal axis.

(*a*) How far from the centre of the lens is the image?

(*b*) Is it on the same side of the lens as the object?

13. If the focal distance of a lens is 24 cm., and one conjugate focus is 72 cm. from the lens, where is the other?

14. Draw a diagram describing the action of an ordinary magnifying-glass, as commonly used for examining small objects, showing the course of the rays and the relative positions of image and object.

15. Why does a magnifying-glass magnify? Show by a diagram that two similar convex lenses used together, with their convex surfaces nearly in contact, will magnify more than one alone.

16. Show by reference to the formula in § 265 what must be the distance (in terms of the focal length) of object and image from the lens in order that they may be of equal size.

17. At what distance from a lens of 24 cm. focal distance must an object be placed in order that the dimensions of the inverted image shall be—

(*a*) half as large as those of the object? 32 - 76

(*b*) twice as large as those of the object? 76 - 32

CHAPTER XIV.

OPTICAL INSTRUMENTS.

279. Importance of Optical Instruments.—Much of the progress of science during the present century has been due to improvements in the construction of optical instruments and their more general use in scientific investigations.

Improvements in telescopes and the invention and perfection of the spectroscope have enabled the astronomer to discover, and even to measure, objects and motions whose existence was unsuspected by the observers of two generations ago. The chemist is to-day able by means of the spectroscope to ascertain in a few minutes the presence, in a substance of unknown composition, of elements which it would have taken him days to detect by purely chemical means.

To the physician, the food-analyst, the manufacturing druggist, and to those engaged in many other professional or technical occupations, the microscope is a necessary piece of apparatus, a tool of daily, almost hourly, use.

Optical instruments comprise a great variety of combinations of mirrors, lenses, and prisms. Only some of the simpler ones can be referred to in an elementary book on physics.

280. The Photographer's Camera.—This instrument consists essentially of a box, in the front of which is fastened a convex lens or a combination of lenses, L, Fig. 90, the distance of which from a ground-glass screen, P, at the other end of the box, may be varied at will. An inverted and usually diminished real image of any outside object not too near L may be formed on P. When this adjust-

ment has been precisely made, the lenses are covered with an opaque cap; a plate of ordinary glass, coated with a film of gelatine made sensitive to light by the presence in it of certain compounds, usually of silver, is substituted for P; the cap is then removed, and the light is allowed to act for a sufficient time upon the sensitive plate, after

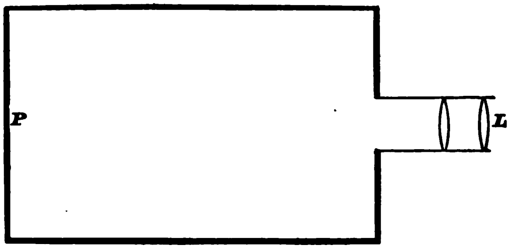


FIG. 90.

which the cap is replaced and the plate removed and "developed" into a photographic "negative."

Those who are interested in practical photography will find in §§ 267 and 269 some explanation of the difficulty experienced in making all parts of the ground-glass screen show clear images at the same time; and in § 270 there is a suggestion as to the effect of "diaphragms" with larger or smaller holes.

281. The Magic-lantern.—This instrument, known also by various other names, *stereopticon*, for instance, requires a powerful source of light, such as a large kerosene-flame, or some form of calcium-light, A, Fig. 91, in which a cylinder of quicklime is heated by a flame formed by burning together oxygen and hydrogen, oxygen and coal-gas, or oxygen and the vapor of ordinary ether (such as is used by surgeons to produce insensibility to pain). By means of a large convex lens B, called the *condenser*, a powerful beam of light from this source is thrown upon the painted or photographed "slide," the image of which is to be exhibited. This slide is pushed into the opening

C, a little outside the focus of a smaller convex lens or a pair of such lenses, *D*, and a greatly enlarged real image of

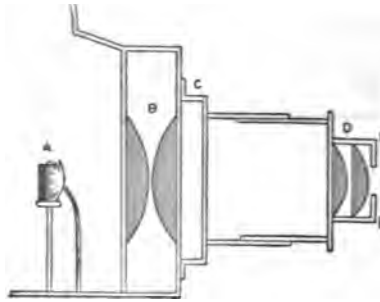


FIG. 91.

the slide is thrown upon the screen in a manner already sufficiently explained in § 262.

282. The Simple Microscope.—In its least complicated form the simple microscope, or magnifying-glass, consists of a convex lens used, as already explained in § 263, to form an upright magnified image of any small object. When much magnifying-power is required, two or even three convex lenses, mounted one over the other with their surfaces only a few millimeters apart, are often used. Such combinations are called *doublets* or *triplets*. They have certain advantages over single lenses of equal magnifying-power.

§ 263 will help the student to see that the magnifying power of a simple microscope is greater as its focal length is less.

283. The Compound Microscope.

EXPERIMENT 63.

Fasten a page of fine print *P*, Fig. 92, upright on the table in a good light. Set up in front of it a convex lens *L*, of 4 or 5 cm. focal length, at a distance of 6 or 8 cm. from the page. Hold another convex lens of about 10 cm. focal distance in various positions farther from the page, as shown at *L'*, until one position is found in

which an inverted, magnified, image of the print is seen through it, this being a *virtual* image of the *real* image formed by the lens *L*. This apparatus is a rude model of the compound microscope.

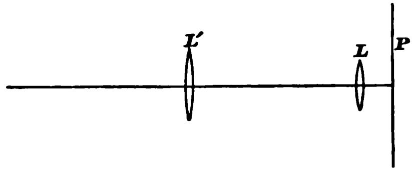


FIG. 92.

For viewing objects under any but the lowest magnifying-powers, that is, in all cases when the apparent diameter of the image is to be anywhere from 50 to 5000 times the actual diameter of the object, the compound microscope is universally employed. The essential optical parts of this instrument, as usually constructed, are, Fig. 93, an *eye-piece,* LL′, here represented as single, but generally consisting of two convex lenses, and an *objective, ll′*, frequently consisting of from two to six plano-convex or double-convex lenses. These lenses are fixed in a brass tube so arranged that the distance between the eye-piece and the objective can be varied at will within certain limits. A mirror, not here shown, which is adjustable to any desired angle, is usually employed for throwing light upon the object.

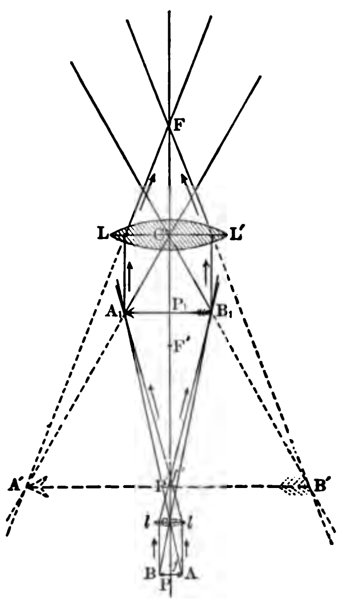


FIG. 93.

The object to be viewed is placed on a platform beneath the objective, and is strongly illuminated by light reflected from the mirror. A real, inverted, magnified image, A_1B_1, of the object is formed within the tube of the instrument at a position somewhat nearer to the eye-piece than its principal focus. This real image is therefore magnified by the eye-piece, which forms an enlarged virtual image, $A'B'$, of it at a position not far from the object.

The foci of the object-glass are at f and f', those of the eye-piece at F' and F.

The total magnifying-power of the instrument is that of the objective multiplied by that of the eye-piece. In general, the shorter the focal length (see Appendix) of a microscope objective, the greater its magnifying power.

An objective of 1 inch focal length will, on a tube 10 inches long, give, with the lowest power eye-piece in common use (the "A" eye-piece), a magnification of about 50 diameters; with an eye-piece of double the magnifying-power ("B" eye-piece) the total magnification will be about 100 diameters, and so on.[1]

284. The Astronomical Refracting Telescope.

EXPERIMENT 64.

Mount upon blocks two convex lenses, one of 30 or 40 cm. focal length, the other of about 5 cm. focal length. Set them up on the table with their principal axes coincident, that is, with their centres on the same straight line at right angles to the centres of their faces. Mount a bit of tracing-paper or greased writing-paper, and place this screen in such a position between the lenses that the one of greatest focal length shall throw upon it a distinct image of some distant bright object. Look at this image on the translucent paper through the 5 cm. lens. Choose such a position and distance as to give a clear virtual image, as much magnified as possible, of the

[1] The makers of microscopes are not perfectly agreed in their mode of designating eye-pieces and objectives. The French and German manufacturers differ much in this regard from English and American ones, and these latter differ somewhat among themselves.

real image on the screen. Now remove the screen and observe that the virtual image of the real image is still visible.

This piece of apparatus illustrates in a rough way the construction and operation of the astronomical refracting telescope. This instrument consists essentially of the long-focus *object-glass*, or *objective*, L, Fig. 94, mounted in one

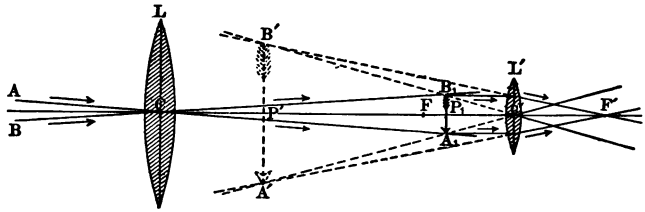


FIG. 94.

end of a tube, at the other end of which is placed an *eye-piece*, L′, precisely similar to that of the compound microscope. The eye-piece can be moved toward or away from the object-glass in order to make the image appear most distinct.

The real image of any distant object is, of course, always formed by the objective very near its principal focus. The foci of the eye-piece are at F and F′.

Astronomical telescopes are always furnished with achromatic object-glasses (§ 282).

285. Efficiency of the Telescope.—The usefulness of the telescope as an aid to vision depends upon the following points: (*a*) the clearness and sharpness of the image, or what is called the *definition* of the instrument; (*b*) the brilliancy of the image; (*c*) the amount of allowable magnification.

Good definition depends upon the accuracy with which the lens is shaped and finished, and upon the quality of the glass, which should be free from flaws.

Brightness depends upon the amount of light which can

be concentrated in the different parts of the image. Hence a large objective will, other things being equal, give the best illumination. In some recent telescopes the objective has a diameter of about 3 feet.

The magnification, with a given eye-piece, is evidently very nearly proportional to the focal length of the objective; but unless the objective is large, and furnishes much light, it is useless to give it great focal length, for the reason that the much-magnified image would be too faint to be seen to advantage.

CHAPTER XV.

MAGNETISM.

286. Magnets, Natural and Manufactured.—There is a certain iron ore of which lumps are occasionally found having the power of attracting particles of iron. Such lumps of ore are called *load*-stones, or *lode*-stones, a word equivalent to *leading*-stones. They are called also natural magnets.

A piece of iron acted upon by a loadstone becomes in turn a magnet, and will attract particles of iron as the loadstone does. Magnets can be made in other and better ways, and are familiar objects to most students. Cobalt and nickel have strong magnetic properties, but are inferior to iron in this respect, and magnets made of them are curiosities rather than articles for real use. Hard steel magnets retain their power better than those made of common iron, and therefore most magnets are made of steel *tempered* very hard.

287. Magnetic Needle; Magnetic Compass.—A slender magnet suspended by a flexible fibre or balanced upon a sharp point, so as to be free to turn in a horizontal plane, is called a magnetic *needle.* It has been known to Europeans for about seven centuries, and possibly to the Chinese for some thousands of years, that a magnetic needle in coming to rest after any disturbance always tends to a position in which its length will be in a general north and south direction. At some parts of the earth's surface the needle points somewhat to the west of north, in others somewhat to the east of north; and its exact direction of

pointing at any one place on the earth's surface varies from century to century.

EXPERIMENT 65.

By means of an observation of the North Star, or any other convenient method, lay off upon a table in the laboratory a true north and south line.[1] Then note how many degrees to the west or east of this line the north end of the magnetic needle points.

An instrument in which a magnetic needle is placed within or over a graduated horizontal circle, so that the turning of the needle from its normal position may be at once read off in convenient divisions of the circle, is called a *magnetic compass*. The mariner's compass, by which the sailor steers his course in the open ocean, is such an instrument, having usually several small magnets fastened to a card-board support balanced upon a point.

288. Induced Magnetization; Temporary and Permanent Magnetization.

EXPERIMENT 66.

Take several small pieces of very soft wrought-iron, horseshoe nails for instance, and test them among themselves to see whether they will exhibit any magnetic power. Then lift one of them by means of a strong bar-magnet and apply one end of a second nail to the lower end of the one so lifted. If the second nail remains suspended by magnetic action attach a third to it and so on until the chain so formed breaks. Finally, after removing all the nails from the neighborhood of the magnet, test them again among themselves for evidences of magnetization.

Magnetization produced in a piece of metal by the action of magnetic force is called *induced* magnetization. A part of such magnetization usually disappears when the producing force is withdrawn. This part is called *temporary* magnetization. The part that remains is called *permanent* magnetization.

[1] For methods of doing this see books on Surveying, e.g., Johnson's *Theory and Practice of Surveying*, John Wiley & Sons, New York.

289. Dipping Needle, the Earth's Magnetic Poles.—If a symmetrical magnetic needle is suspended by its middle so as to be free to turn in a vertical plane as well as in a horizontal plane, one end of the needle will, at most parts of the earth's surface, hang lower than the other end. At most places in the Northern Hemisphere the north-seeking end of the needle will hang low, at most places in the Southern Hemisphere the south-seeking end. In either case the needle is said to *incline*, or *dip*. In general, as one goes north the north-seeking end of the needle will dip more and more, and at some place in the far north will point straight down. This place, which is called the north *magnetic pole* of the earth, is not at the geographic north pole. Somewhere in the far south is a corresponding south magnetic pole, where the south-seeking end of the magnet would point straight downward.

EXPERIMENT 67.

Take a bar of very soft iron and make sure by trial that its ends will not attract bits of soft iron. Then place this bar in a vertical position, and while it is so placed test it again with the bits of iron. Try whether jarring the bar while in a vertical position will affect its power of attracting iron.

This experiment will probably show that the earth has the power of magnetizing iron. The earth is, in fact, a magnet, and the "natural" magnetic power with which the loadstone is found endowed is in all probability imparted to it by the magnetic action of the earth as a whole. What makes the earth magnetic is not known, although various theories to account for its condition have been proposed.

290. Poles of Ordinary Magnets; Magnetic Repulsion.—If one takes an ordinary bar-magnet and presents different parts of it in turn to one end of a magnetic needle, it will be found that the ends have much more effect than the

middle. As in the case of the earth we speak of two magnetic *poles* from which the magnetic influences appear to come, so in the case of the bar-magnet we call the two regions, usually near the ends, in which the peculiar power of the magnet seems to lie, the *poles* of the magnet.

If we again present the two ends of a bar-magnet in turn to the magnetic needle, we shall find that one end attracts the north-seeking point of the needle and repels the south-seeking point, while the other end of the magnet repels the north-seeking point of the needle and attracts the south-seeking point.

If we now float the magnet on water, on a board just large enough to carry it safely, we shall find that the pole which repelled the north-seeking pole of the needle will itself point north. We conclude, then, that *poles which repel each other are alike*, and that *poles which attract each other are unlike.*

291. Law of Inverse Square.—By means of careful and delicate experiments upon the attractions and repulsions exerted by the poles of magnets, it has been found that these attractions and repulsions are, other things being equal, *inversely proportional to the square of the distance between the mutually acting parts.* For instance, if two poles one foot apart attract each other with a certain force F, the same poles when two feet apart will attract each other with a force $F \times \frac{1}{2 \times 2} = \frac{1}{4} F$.

Every magnetic pole that we have to do with is subject to an attraction exerted by one of the earth's poles and a repulsion exerted by the other. Both the attraction and the repulsion tend to carry a north-seeking pole north and a south-seeking pole south. Any movement toward the earth's north magnetic pole strengthens the effect of that pole, but weakens the effect of the south pole. A little consideration, taking account of the law of inverse square,

will show that if this movement is small, 100 ft. for instance, the force experienced by either pole of the magnet will be affected exceedingly little.

PROBLEM.—If two unlike but equally powerful magnetic poles are 100 ft. apart, and if each of the two exerts upon a third pole midway between them a force F, how great would be the total force felt by this third pole if it were moved 1 ft. toward one of the other poles?

292. Opposite Kinds of Magnetism.—The property by virtue of which a magnet attracts or repels is called *magnetism*. To give it a name does not *explain* the property, but the name is a convenience. As there are two kinds of magnetic poles, so we must recognize two kinds of magnetism. The north-seeking pole of a magnet is that part in which the north-seeking magnetism, frequently called *north* magnetism, is more abundant than the south-seeking magnetism, and prevails over it. The south-seeking pole is the part in which the south-seeking magnetism, frequently called *south* magnetism, prevails over the north-seeking magnetism.

EXPERIMENT 68.

Returning to the floating magnet (§ 290), note whether it tends *as a whole* to *drift* either toward the north or toward the south. Try this experiment with a variety of magnets, and note whether any of them move as a whole in either direction, taking care in all cases to have the magnet at rest when the experiment begins, and having the surface of water large enough so that *surface tension* (see any general treatise on Physics) may not urge the float toward the side of the vessel. The float should be the lightest that will bear the magnet with security.

If, on experimenting carefully in this way, we find a magnet which always floats toward the north, we cannot attribute such behavior to the fact that one pole is nearer the north than the other, for we have seen (§ 296) that this would make no perceptible difference in the force to which

the pole is subjected. We shall have to explain it by the supposition that the particular magnet used has more north-seeking magnetism than south-seeking magnetism.

EXPERIMENT 69.

If no magnet can be found in which either kind of magnetism appears to be in excess of the other kind, take some long thin piece of hard steel, an old metal-saw blade for instance, and magnetize it by means of the most powerful magnet at hand, stroking one end of the steel with one end of the magnet, and the other end with the other end of the magnet. It will now be found that one end of the bar will attract the north-seeking end of a magnetic needle, and the other end repel it. Now break the bar in two at the middle. Test each half separately by means of the magnetic needle and on the float. Does each half have both north-seeking and south-seeking magnetism? Does each half have equal quantities of north-seeking and south-seeking magnetism? If these questions are both answered in the affirmative, break one of the halves in two, and repeat the tests just indicated.

In all these experiments make very careful notes of any magnet that is found to have more of one kind of magnetism than of the other kind.

An interesting bar to study is one that has been touched at its middle with one end of a magnet and at each of its ends with the other end of the magnet.

293. Magnetic Field; Lines of Magnetic Force.—Any portion of space in which magnetic force is found is called a *magnetic field.* If, starting at any point of a magnetic field, one notes the direction in which the north-seeking end of a small magnetic needle points, and moves the needle bodily in this direction, and if one continues this process, changing the direction of motion, if need be, continually so as to make it at all times agree with the direction of pointing, the centre of the needle will trace out what is called a *line of magnetic force.*

294.

EXERCISE XL.

LINES OF MAGETTIC FORCE.

[Trowbridge, pp. 167, 168. Worthington, Exp. 45, p. 228; Exps. 48, 49, p. 230.]

Apparatus: Two straight bar-magnets, each 15 or 20 cm. long and about 1 sq. cm. in cross-section, nearly equal in magnetic power. A small compass, with needle 2 or 3 cm. long. Three sheets of paper about 50 cm. square.

FIRST PART.—Fasten one of the sheets of paper on a table and lay one of the large magnets on the middle of the sheet, the north-seeking pole pointing north.[1] Place the small compass at the extreme north-east corner of this magnet and then move it away in the exact direction in which the compass-needle points. Continue this movement, changing direction in such a way as to follow continually the changing indication of the compass-needle, until the path reaches the edge of the paper or returns to the magnet. Trace upon the paper the line thus followed by the middle of the compass, putting arrow-heads at several points to indicate the direction in which the north-seeking end of the compass-needle points at these places. Then place the compass a very little distance from the north-east corner of the large magnet toward the middle, and starting anew trace another line and mark it as before. Then beginning still farther toward the middle of the magnet, do as before. Finally, start not more than 3 or 4 cm. from the middle of the magnet and trace a line. Trace an equal number of lines on the western side of the magnet.

SECOND PART.—Place the same large magnet on another sheet of paper with its north-seeking end pointing south. Then use the compass and trace lines as before, marking them all with arrow-heads to indicate the direction of pointing of the north-seeking end of the compass-magnet.

THIRD PART.—Two magnets. Lay the two magnets parallel on a sheet of paper, the two north-seeking ends pointing north and about 15 cm. apart on an east and west line. Proceed with this system as with the single magnets, taking care to trace several lines between the magnets.

In all parts of this exercise outlines of the large magnets should be marked upon the paper, and other marks should be added that

---

[1] The *north-seeking* end of a magnet is the end that *naturally* points north.

only to such
term *sympat*
nomena here
is set into v
timed and
by equally
so a stretch
sponds to
when these

Lead a s
several feet
threads of
at rest, set
overhead st

229. D
fectly ela
go on f
× *surfa*
But if t
that pr
measur
cause a
which

1.
in fou
there
2.
repor
tance
Ligl

[1] S.
editi

ch particles, with their *north* poles all pointing in the general direction, we can see that in the middle of ɔar the north pole of any particle is sure to be very · the south pole of some other, so that this part of the will have little or no action upon outside bodies. At ıer end of the magnet, however, there is a surface made of poles all of the same kind, and so from the ends ıes the power which affects the needle. Breaking the · at the middle merely separates two surfaces, which, be- made up one of north poles and the other of south les, neutralize each other while in contact, but have each e power of attraction or repulsion when separated.

There is room for doubt concerning the mode of mag- etization of the particles themselves, as to whether their ıagnetism is due to molecular electric currents, whether t is produced in them by the influence which *magnetizes* he bar as a whole, or is natural and permanent in them, so hat the act of magnetizing a bar is merely a process of *ırranging* the molecular magnets of which it is composed.

CHAPTER XVI.

ELECTRICITY.

296. Historical Sketch.—It was known to the ancient Greeks and Romans that certain substances acquire, when rubbed, the power of attracting light objects. Amber was one or these, and from the Greek name of this substance, ἤλεκτρον, the name *electricity* was formed by the English philosopher Gilbert, about 300 years ago.

It appears that Gilbert was the first to make systematic and extensive observations of electrical phenomena. From his time the subject has grown in interest and importance through the invention of frictional electrical machines, of the Leyden jar, Franklin's discovery of the identity between lightning and electricity, the invention of the *galvanic*, or *voltaic*, *cell* (§ 303) about the beginning of this century, the work of Oersted, Ampère, Faraday, and a host of others more recent, until the name *electricity* is now the most popular and fascinating in the whole vocabulary of physical science.

297. Electrification by Friction.—Experiments with electricity produced by friction are very beautiful, and of great theoretical interest, but many of them are troublesome to perform, and their practical importance is comparatively small. Those which follow are selected mainly for the light which they throw on the phenomena of electric currents.

EXPERIMENT 70.

On a cold dry day rub a stick of sealing-wax with a catskin, and then present the rubbed part to small light pieces of paper or bits of thread lying on a table.

Fasten two small pith-balls to the ends of a dry silk thread about

15 cm. long, and suspend them by the middle of the thread from any convenient support. Touch these balls with the freshly rubbed rod of sealing-wax. Note the behavior of the balls with respect to the rod just before they are touched and just after. Note also their behavior with respect to each other after they are touched by the rod. If they act in an unusual manner it is because they have become electrified or charged with electricity by the rod.

Rub a smooth glass rod vigorously with a piece of silk, and present the rubbed part to the suspended pith-balls still charged from the sealing-wax. Note their behavior before and after being touched by the glass.

Do you find in these experiments any evidence that there are two kinds of electrification? If so, is there attraction or repulsion between bodies similarly electrified? between bodies oppositely electrified?

298. Two Kinds of Electricity.—To say that bodies may be oppositely electrified is not necessarily to imply that there are two kinds of electricity. It may be that all bodies in their normal state are endowed with a certain amount of electricity, and that a body possessed of more than its normal amount of electricity will act in one way, and one possessed of less than the normal amount will act in the opposite way.

Some physicists have believed that there is only one kind of electricity. Others have held that there are two kinds. The question is not yet settled. It is extremely convenient, however, to speak of one kind of electrification as caused by a charge of one kind of electricity and the other kind of electrification as caused by a charge of an opposite kind of electricity; and we shall do so.

We shall, following custom, speak of the state into which sealing-wax or any other resinous substance is brought by rubbing with catskin as being due to a charge of *resinous*, or *negative*, electricity, and the state of a glass rod rubbed with silk as due to a charge of *vitreous*, or *positive*, electricity.

There has been, too, a difference of opinion as to whether electricity is or is not a *substance*. A century ago, when heat and light were believed to be weightless fluids, electricity was classed with them as a substance. Later, when it was shown that heat and light were not substances, but mere "modes of motion" in which the particles of matter are involved, the notion gained currency that electricity was a mode of motion, rather than a substance by itself. During recent years belief in the existence of electric substance, or substances, has been growing again.

299. Conductors and Insulators.—If one attempts to repeat Experiment 70, using a very thin wire or a cotton thread or a wet silk thread for suspending the pith-balls on a metal support, it will be found that the balls do not retain their charge as they did before. The explanation is, that the charge escapes along the thread or wire to the support, and so is lost. Materials which, without motion of their own, can serve as avenues of escape for an electric charge, are called *conductors* of electricity. Materials which cannot serve this purpose are called *non-conductors*, or insulators.

Metals are the best conductors, and resinous and vitreous substances are among the best insulators. No perfect insulator is known, and, on the other hand, there is no perfect conductor,—no conductor, that is, which is not somewhat heated by the passage of electricity through it, thus showing that it offers a certain amount of resistance to the movement.

300. The Gold-leaf Electroscope.—The pair of pith-balls suspended by a silk thread used in Experiment 70 is sometimes called an *electroscope*, for the reason that it enables us to detect, or make evident, an electric charge; but for more delicate experiments a more sensitive instrument is needed. This is found in the *gold-leaf electroscope* shown in Fig. 96.

G may be an open-topped glass *receiver*, such as is frequently used with air-pumps, fitted at the top with a cork, *c*; *B* is a disk[1] of metal, usually brass, six or eight inches in diameter, from which a metal rod, *r*, reaches downward through the cork. The upper ends of the strips of thin gold-foil, *l*, *l* (the thinnest used by dentists), are crowded into a narrow slot sawed in the lower end of *r*, so that there is good metallic contact from *B* to *l*, *l*.

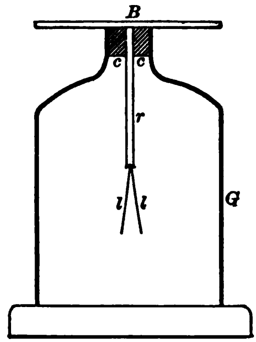


FIG. 96.

EXPERIMENT 71.

Hold a charged rod of sealing-wax or glass over the electroscope, and lower it gradually, stopping an inch or two above *B*. Then raise and lower the rod alternately several times. Are the leaves *l*, *l* affected when the rod approaches the plate? Do they continue to show any peculiar action after the rod is removed? (If the leaves become permanently charged, they may be restored to their normal condition by touching the edge of the plate *B* with the finger. One must take great care in dry cold weather not to rub the glass of the electroscope with the hand even, lest it acquire a charge which will be difficult to remove, and which will greatly disturb the proper course of the experiments. Moreover, one must guard carefully against the action upon the electroscope of possible charges on the clothing. The experimenter should frequently test for a charge upon his sleeve by holding his arm, *without the charged rod*, over the plate *B*, the electroscope being previously discharged (see Exp. 72, second parenthesis)).

301. Electric Induction.—In Experiment 70 pith-balls were charged from an electrified rod by direct communication. In Experiment 71 the electroscope was charged,

[1] The instrument is more sensitive, and therefore better for such experiments as No. 71, if a *ball* about an inch in diameter is used instead of the disk *B*. But the disk is *necessary* for certain experiments that come later.

temporarily, without contact with the electrified rod; but the fact that its charge disappears when the rod is removed indicates that it really receives nothing from the latter save a certain *influence* which throws it, for the time being, into a peculiar state of electrification without really changing the amount of electricity upon it. This operation is called electrification by *induction* (cf. § 288).

One can readily understand that if every body is, in its normal condition, endowed with a certain quantity of positive electricify and a certain quantity of negative electricity, the approach of a charged body may produce in the electroscope, previously uncharged, a redistribution of its electricities, one kind being drawn toward the approaching charged body while the other is repelled into the leaves. If, on the other hand, one holds that there is only one kind of electricity, one may suppose that the approach of a body having more than its normal complement will repel the electricity of the electroscope into the leaves, and that the approach of a body having less than its normal amount will attract the electricity of the instrument into the plate *B*, either event putting the leaves into condition to repel each other. Whichever theory one adopts, experiment shows that a redistribution of electricity, and nothing more, does take place at the approach of the charged body.

But can we now find some means of charging a body permanently by the aid of such induction?

EXPERIMENT 72.

Put the plate of the electroscope into good electric connection with any large conductor (§ 299), the body of the experimenter, for instance, or the earth itself by means of water-pipes or gas-pipes, and then bring the charged rod toward *B*. (Under these circumstances the electricity which is like that of the charge on the approaching rod will, it is reasonable to suppose, be repelled from the electroscope into the large conductor instead of into the gold-leaves, and all the metal of the electroscope may acquire an induced charge opposite in kind to that of the rod.)

Break the connection at the plate B while the charged rod is still held near. Do the leaves now show any evidence of a charge after the rod is removed? If any charge remains, is it of the same kind as that on the charging rod? To answer this question, bring the charged rod again toward B, and notice whether the *first effect* of its approach is an increase or decrease of the divergence of the gold-leaves. If it is an increase, the charge on the leaves is like that on the rod; if a decrease, the charge on the leaves is unlike that on the rod. (If there is a charge on the electroscope, the approach of an *unelectrified* conductor—the hand of the experimenter, for instance—toward the plate B may cause the leaves to approach each other, for the charge already on the electroscope induces a charge upon the approaching conductor, and is itself somewhat changed in consequence. To avoid error from this cause the charged rod should be a long one, so that the hand need not come near B.)

302. The Electrophorus.—This instrument, the action of which depends upon induction, is very convenient for supplying electricity with which to charge conductors of moderate capacity. It consists usually of a shallow metal pan, which may be about 25 cm. in diameter, containing a quantity of resin or other similar material, which has been poured in while hot and has cooled in place, forming a smooth hard surface, and a flat circular plate of metal, somewhat less in diameter than the pan, furnished with an *insulating* handle (§ 299) of glass or hard rubber.[1]

The pan being placed in electrical connection with the earth or with the body of the experimenter, the resinous surface is rubbed with a dry catskin, and thus acquires a charge of negative electricity. The metal disk is then placed upon this surface. If the two fitted each other perfectly the metal plate would become charged with the same

[1] If the handle is of glass it may be found to insulate poorly. In the construction of any apparatus to be used with electricity of rather high pressure it is safest to test any glass that is to be used for insulation before constructing permanent apparatus with it. The jar shown in Fig. 96 may cause disappointment by proving to be a poor insulator. White glass insulates better than green.

kind of electricity as the surface upon which it rests; but the two surfaces really touch at certain points only, and so the negative charge upon the resinous plate *induces* a positive charge on the lower surface of the metal plate and repels the negative electricity to the upper surface. If this upper surface is now touched for an instant by the hand of the experimenter the negative electricity escapes, and when the plate is lifted it carries a positive charge, which may be used to charge other conductors. The resinous plate meanwhile has suffered little loss, and the operation may be repeated many times without recharging it.

303. Electricity developed by Chemical Action; the Voltaic, or Galvanic, Battery.—In the year 1800 an Italian, Volta, following a hint given by an accidental discovery made by another Italian, Galvani, showed how to maintain a continuous supply of electricity by the aid of chemical action. The most famous arrangement which he employed for this purpose consisted of a column made up of disks of copper, zinc, and cloth moistened with acidulated or salt water, the order of arrangement being copper, zinc, cloth, copper, zinc, cloth, etc., the column beginning with one metal and ending with the other. This was called *Volta's pile.* It has the advantage of great simplicity of construction and portability, but loses its power as the liquid evaporates from the cloth disks, and is not so easily cared for as another arrangement, imitated in the following experiment, which arrangement Volta used and called the "*crown of cups.*"

EXPERIMENT 73.

Take fifty small glass tumblers, fifty small strips of zinc, and fifty similar strips of copper (see apparatus for Exercise XLI). Arrange the tumblers in a circle and fill each with water to a point one or two centimeters from the top. In each tumbler place a strip of zinc and a strip of copper, opposite each other. Solder the zinc of one cup to the copper of the next and continue in this way around the whole

circle, leaving, however, a gap between some one zinc and the copper of the next tumbler. At this gap solder a thin copper wire about one-half meter long to the zinc strip and a similar wire to the copper strip.

Now take the electroscope (§ 300) and cover the plate *B* with a sheet of thin paper which has been soaked in melted paraffine. Place on this paper the metal disk of the electrophorus (§ 302) taking care that the paper shall prevent all contact between this disk and the plate *B*. Then touch *B* with one wire leading from the "crown of cups," and at the same time touch the upper metal plate with the other wire. Remove the wires and at once lift the upper plate and the paper from *B*. Do the leaves show any evidence of an electric charge?

The object of the preceding experiment is to show that chemical action—for there is a slight action of the water upon the metals—can produce a state of electric charge similar to that which comes from friction. Further experiments would show that the plate *B* becomes charged *positively* when it is touched by the wire leading from the copper strip, and *negatively* when touched by that leading from the zinc strip.

Far less striking effects in the direction of attractions and repulsions, electric sparks, etc., can be obtained directly from a *Voltaic* (or *Galvanic*) "*battery*," like that just described, than from frictional electric machines. The peculiar usefulness of the battery is its power to maintain a continuous *current of electricity* (§ 306), as we shall see in the following pages.

304. EXERCISE XLI.

SINGLE-FLUID GALVANIC CELL.

[Trowbridge, Exps. 113, 114, 115.]

Apparatus and material: A very small glass tumbler. About 200 grm. of dilute sulphuric acid, 20 parts in volume of water to 1 part in volume of concentrated acid.[1] A strip of sheet zinc and

[1] In mixing *pour the acid slowly into the water*. Pouring water upon the acid is dangerous. A small quantity of acid sinking at once in a comparatively large mass of water is speedily cooled in spite of

a strip of sheet copper, each about 10 cm. long and 1.5 cm. wide, each having 50 cm. of covered copper wire, about No. 20, B. & S., soldered to one end. Mercury for amalgamating the zinc. A galvanoscope.[1]

---

the strong chemical action. A small quantity of water floating for a time upon a comparatively large mass of acid is heated to boiling, and acid is spattered about by this action.

[1] Essentially like that described by Worthington, p. 274, but with a coil about 15 cm. in diameter formed by winding 15 turns of covered copper wire, about No. 20, upon a hoop of pasteboard. [See Fig. XXVI. The base shown in this figure is much heavier than it

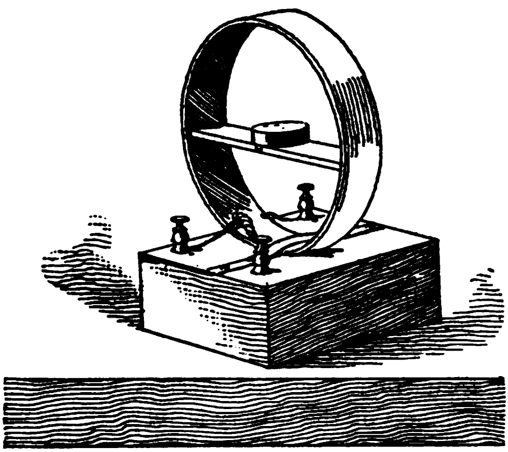


FIG. XXVI.

need be.] The coil should be so wound that 5, 10, or 15 turns may be available at will. The coil may be fastened to the base-block by means of a strip of brass. A light pointer of glass, wood, or quill may be put on the needle at right-angles with the magnetic axis, thus obviating the difficulty of reading when the needle points nearly to 0°.

The form of galvanoscope which has a coil wound close to the needle would do for this exercise, but for the study of the galvanoscope itself in Ex. XLIII it is desirable to have a coil like that here described.

FIRST PART.—Nearly fill the tumbler with the dilute sulphuric acid. Put into the tumbler close to one side the small strip of zinc clean and unamalgamated, and close to the other side the similar strip of copper, both strips reaching to the bottom and being bent at the top so as to clasp the side of the tumbler. Note for about a minute what occurs at the surface of each strip, the strips being prevented from touching each other. Put the two strips into metallic connection, by means of the attached wires, through the 15 turns of the galvanoscope. (In this and in all other electrical exercises of this course take care to have such connections, whether made by means of binding-screws, spring-clamps, or simple twisting together of wires, give firm metallic contact. Again note for a short time what occurs at the surface of each strip. Note, too, the behavior of the galvanoscope needle, recording the position in which it comes to rest, tapping the instrument lightly in order to make sure that the needle has not through friction come to rest in a wrong position.

Disconnect the zinc strip and plunge into mercury the part which has been in the acid. Then, after removing from it any loose drops of mercury, replace it in the acid and, before connecting it with the galvanoscope, note for a short time the behavior of its surface. Then connect it as before, and again note what occurs at the surface of the two strips and what position the galvanoscope needle takes, being careful to have the two strips as far apart as before and immersed to the same depth as before.

SECOND PART.—Leave the circuit closed through the galvanoscope 10 minutes, recording the position of the needle every 2 minutes. If then there are visible bubbles on the strips of zinc and copper, rub them off without removing the strips from the acid, taking care to have no mercury come in contact with copper, and then note the position of the needle. Finally remove the copper strip from the acid, rub it thoroughly, then put it back into the acid and note the deflection of the needle.

Try the effect of amalgamating the copper as well as the zinc. It may be necessary to dip the copper into nitric acid for a moment in order to make the mercury act upon it.

The discussion of Exercise XLI will occupy the next three articles.

305. A *molecule* (§ 198) of sulphuric acid, we are taught by chemists, consists of two *atoms* of hydrogen (H_2), one

atom of sulphur (S), and four atoms of oxygen (O_4). The symbol showing the composition of the molecule is H_2SO_4. When this molecule comes in contact, under proper conditions, with zinc, chemical action takes place, and the two atoms of hydrogen are replaced by one atom of zinc (Zn). The hydrogen is thus set free as a gas, while the compound that remains is $ZnSO_4$, called *zinc sulphate*, or *sulphate of zinc*. This change, or *reaction*, as it is called, may be represented briefly by the following equation :

$$Zn + H_2SO_4 = H_2 + ZnSO_4$$

in which the sign = is equivalent to the word *yields*.

If this reaction takes place while a battery is not in use, it is evidently wasteful of zinc, which is a rather expensive metal. Chemically pure zinc, not in contact with other metals, is but little affected by sulphuric acid, but its high cost forbids its use in batteries. Impure zinc when *amalgamated* acts much like chemically pure zinc.

Copper is, under ordinary circumstances, but little affected by sulphuric acid. Any action seen at the surface of the copper strip in Exercise XLI, even when the circuit is closed, is due to hydrogen bubbles, freed by chemical action which leaves the copper unaffected.

Various injurious actions are likely to take place in a Daniell's cell when it is left for any long time, over night, for instance, in open circuit. Even when the plates are taken out, the porous cup may be spoiled by the deposition of copper upon it, owing to the presence of metallic particles in its wall. The cell should therefore be taken apart when not in use.

306. We have seen in § 303 that the wire leading from the copper end of a galvanic battery is charged positively, and that leading from the zinc end charged negatively, when these two wires are not connected with each other. From what we have learned concerning the mutual attrac-

tion of positive and negative electricity, and from what is said in § 299 about *conductors*, we can readily understand that when these two wires, charged as first described, are made to touch or are connected by means of a wire, the two charges will mingle, so that the two wires will afterward be charged alike, unless the action of the battery is able to maintain a certain difference in their condition in spite of the metallic connection between them.

EXPERIMENT 74.

Fill the cells of the "crown of cups" (§ 303) with such liquid as is used in Exercise XLI, and then connect the terminal wires through a resistance of some hundreds of ohms (§ 316). While the circuit is still closed in this way connect plate B of the electroscope, arranged as in Experiment 73, by means of a wire with a point of the main wire near the terminal zinc, and connect the upper plate in the same way with a point of the main wire near the terminal copper. Then break these connections with the electroscope, and at once lift the upper plate and the paper. Do the gold leaves give any evidence of electric charge? If so, test, as in § 301, to see whether this charge is positive or negative. Repeat the experiment, connecting now the plate B with a point near the terminal copper of the battery, and the upper plate with a point near the terminal zinc. Test the character of any charge that may be left on the electroscope by this arrangement.

If these experiments show a permanent difference of charge between the tested points of the circuit while they remained connected by a wire, we shall be led to the conclusion that the battery maintains a steady flow of electricity along the wire connecting its terminals. If we believe that there are really two kinds of electricity, the positive and the negative, we shall think of this wire as transmitting all the time a current of positive electricity in one direction and a current of negative electricity in the other direction. It is customary, however, to speak of the positive current as *the* current, and to say that with such a cell as that used in Exercise XLI the current flows from cop-

per to zinc *outside* the cell, and from zinc to copper *inside* the cell.

Faraday called the two solid conductors of a cell *electrodes;* that by which the current leaves the cell, the *kathode,* and that by which it returns to the cell, the *anode.*[1] The kathode is more commonly called the *positive pole,* and the anode the *negative pole* of the cell. To the liquids of the cell Faraday gave the name *electrolytes,* by which he meant substances *dissolved,* or rather *decomposed,* by the electric current.

The action of the electric current upon a magnet is plainly shown in Exercise XLI, but it need not be discussed here, as it is to be made the subject of Exercise XLIII.

307. The temporary weakening of the current when the circuit is closed for a considerable time, and its recovery when the plates are thoroughly rubbed, is a phenomenon that demands serious attention. It is evidently not due to an exhaustion of the chemical energy of the battery, but rather to the condition that the action of the cell produces at the surface of one or both of the metal strips.

EXPERIMENT 75.

Fill an ordinary tumbler with such liquid as is used in Exercise XLI, and place in it, near to but not touching each other, two pieces of sheet lead as large as can be conveniently used. Solder to each sheet of lead a piece of copper wire about one-half metre long. Connect these wires with the galvanoscope, using all the windings, and see whether any effect is produced upon the needle. Free one of the lead-cell wires from the galvanoscope and put in its place one wire from the two-fluid cell of Exercise XLII, or any more powerful cell. Connect the other wire from this cell with the other lead-cell wire, and note the behavior of the needle for two or three minutes. Then quickly throw out of the circuit the two-fluid cell and again connect the lead-cell alone with the galvanoscope. Does the needle indicate any current from this cell as now used? If so, is the current a

---

[1] *Anode* is from Greek ἀνά (upward), and ὁδός (way); *kathode* from κατά (downward) and ὁδός.

permanent one or does it decrease rapidly while the circuit remains closed? Consider whether the current which passes through the lead-cell as now arranged is in the same direction as that which was sent through it by the two-fluid cell.

Any current which the lead-cell, used alone, may yield is due to the peculiar state into which its plates are thrown by the action of the current from the two-fluid cell. This state is called battery *polarization*. It does not merely offer *resistance* to the electric current by which it is produced, for it tends to send a current *backward* through the cell, and will do so, as the experiments just described make evident, when opportunity is given. It was such polarization that gradually reduced the strength of the current in Exercise XLI. It is a serious disadvantage in many galvanic cells, but it is not without its uses, for upon it depends the action of all the so-called *storage batteries*, which within the last ten years have come to fill an important place in the industrial applications of electricity. It is to be noted that the thing *stored* in such cells is not, according to the usual terms of science, electricity, but chemical energy in a form peculiarly available for the production of an electric current.

308. EXERCISE XLII.

TWO-FLUID GALVANIC CELL.

Apparatus and material: A strong glass tumbler about 10 cm. tall and 7 or 8 cm. wide. A cup of unglazed porcelain about 10 cm. tall and 4 or 5 cm. wide. About half a litre of dilute sulphuric acid, 20 parts in volume of water to 1 part in volume of concentrated acid. About half a litre of a saturated solution of sulphate of copper. A piece of zinc 10 cm. long, 2.5 cm. wide, and 0.5 cm. thick, having 50 cm. of covered copper wire, about No. 20, B. & S., soldered to one end. A piece of sheet-copper 10 cm. square, with a wire like that on the zinc. Mercury for amalgamating the zinc.[1] A galvanoscope like that used in Ex. XLI.

[1] It is well to have enough mercury to fill one of the porous cups nearly to the top. A zinc may then be amalgamated in an instant by dipping it into the mercury.

Put the zinc into the porous cup and then pour in dilute acid until the cup is full to a level 2 cm. below its top. Put the cup containing the zinc and acid into the tumbler and pour into the tumbler sulphate of copper solution until this liquid stands as high in the tumbler as the acid stands in the porous cup. Then remove the zinc from the acid, amalgamate it by dipping it into the mercury, wipe it to remove loose drops of mercury, and then weigh it to 0.1 grm. Weigh the copper sheet, well washed, with equal accuracy. Replace the zinc in the porous cup and put the copper, bent somewhat so as partly to encircle the cup, into the sulphate of copper in the tumbler. Put the cell in circuit with the 5-turn section of the galvanoscope. Note the position of the needle when it comes to rest, and every five minutes thereafter for half an hour. Then disconnect the cell, dry the zinc and copper without rubbing them and weigh them again.

This exercise, if properly performed, accomplishes several objects: 1. It teaches the construction of one of the most useful two-fluid cells, the Daniell. 2. It illustrates the great merit of two-fluid cells as compared with one-fluid cells, viz., constancy of current. (The increase of current which is likely to occur at the beginning of the half-hour run is due to the fact that at first the porous cup is not thoroughly wet through by the solutions.) 3. It shows, roughly at least, what change in weight, increase or decrease, takes place in the plates during the action. It is evidently necessary to guard against rubbing or shaking mercury from the zinc plate between the two weighings.

The fact that the cell of this exercise gives a stronger current than the cell of Ex. XLI is due to the difference in size more than to the difference in construction. See Ex. XLV.

309. Discussion of Exercise XLII.—The chemical action in the two-fluid cell of this exercise is somewhat more complicated than that in the single-fluid cell of Exercise XLI, but if we consider only the *results* of the reactions they are not difficult to understand.

Within the porous cup we have, as in Exercise XLI, zinc replacing the hydrogen of sulphuric acid. The hydrogen thus freed does not appear in the form of bubbles, but in the outer part of the cell we find that hydrogen is replacing the copper of the sulphate of copper, and the

copper thus freed is deposited upon the copper plate. It is not to be supposed that the identical atoms of hydrogen replaced by the zinc find their way to the outer cell, but the outcome of the reactions is represented by the following equation (See § 305):

$$Zn + H_2SO_4 + CuSO_4 = ZnSO_4 + H_2SO_4 + Cu.$$

It will be seen from this account that the *polarization* which occurs in Exercise XLI, cannot occur with the two-fluid cell of Exercise XLII, the plates of which are not changed in chemical condition by the operations that increase or diminish their weight.

310. Other Galvanic Cells.—There are many other cells in common use. In nearly all of them zinc is employed as one of the electrodes (§ 306), and whenever used it is the *anode.* It is usually, but not always, placed in sulphuric acid, and from this frees hydrogen. We have, in § 309, seen one chemical device for preventing this hydrogen from accumulating upon the other electrode. There are various other devices, the general plan of which is to render the hydrogen harmless by bringing it into combination with oxygen.

One of the most effective oxidizing agents is nitric acid, and one of the most powerful galvanic cells, called *Bunsen's* cell, is that which is formed by replacing the sulphate of copper in the Daniell cell (Exercise XLII) with nitric acid, and the copper plate by a plate of carbon. This carbon is not affected by the chemical action of the cell. The choking and corrosive fumes that come from the nitric acid make the Bunsen cell objectionable, and it is now used but little.

Another cell is that which may be formed from the Bunsen by replacing the nitric acid with a mixture of sulphuric acid and bichromate of potash. This is known as the *Poggendorff* cell, and by various other names. It is in very common use.

Within the last few years a cell has been coming into favor which is like the Poggendorff cell with this exception, that it employs bichromate of soda instead of bichromate of potash. It is more enduring in its action than the Poggendorff cell, and is likely to replace the latter to a great extent. It is called, in some places at least, the *volta-pavia* cell.

All the cells just described are, when in good condition, more powerful than the Daniell cell, but none of them equal it in constancy of behavior.

The *Leclanché* cell uses zinc in a solution of sal-ammoniac, and for the kathode a bar of carbon packed in crushed carbon mixed with peroxide of manganese. This cell polarizes rapidly when the circuit is closed, but is very useful in furnishing occasional currents of short duration, such as are needed for ringing door-bells and sounding alarms.

311. Electromotive Force.—The power which a galvanic cell has of charging one of its terminals positively and the other negatively (§ 303), so that a current of electricity will flow from one to the other when they are connected by a wire, is called its *electromotive force.*

For testing the electromotive forces of single cells an instrument similar in principle to the gold-leaf electroscope, but more sensitive, may be used. Such an instrument is called an *electrometer.*

The strength of current which is maintained by any galvanic battery depends upon its electromotive force, but not upon this alone, as will be seen in connection with Exercise XLV.

The electromotive force of a cell depends upon its chemical composition and, to some extent, its physical condition. The following experiment will serve to show whether it depends upon the size of the cell:

EXPERIMENT 76.

Take one very small Daniell cell and one very large one. Join together the copper terminals of the two cells. Then connect the zinc terminal of one cell with one terminal of the galvanoscope and the zinc terminal of the other cell with another terminal of the galvanoscope, thus forming a circuit consisting of the two cells, *opposed to each other*, and the maximum number of galvanoscope windings. Is there now evidence of any current through the galvanscope? What does the experiment indicate as to the comparative electromotive force of the two cells.

The following experiments will give some idea of the electromotive force of various combinations of galvanic cells:

EXPERIMENT 77.

Take half a dozen similar Daniell cells. Connect the copper of one with the zinc of a second, and the copper of the second with the copper of a third. Connect .he zinc of the first cell and the zinc of the third with the terminals of the galvanoscope, using all the windings. With this arrangement the first and second cells, *connected in series*, as the phrase is, and so working in the same direction, are opposed by the third cell. Do they prevail over it?

Try three cells in series leading one way against two cells in series leading the other way.

Join the zinc of one cell to the zinc of a second, and the copper of the first cell to the copper of the second. Connect the two coppers thus joined with the copper of a third cell. Connect the zinc of this third cell with one of the terminals of the galvanoscope, and connect the joined zincs of the first two cells with the other terminal of the galvanoscope. The first and second cells are now working together, but side by side, rather than, as before, end to end. They are now said to be joined in *parallel*, or *in multiple arc*, or *abreast*. They are opposed by the third cell. Do they prevail over it?

Try four or five cells joined abreast against a single cell, and note the result.

The ordinary unit of electromotive force is called the *volt*, in honor of Volta (§ 296). The electromotive force of an ordinary Daniell cell is about 1.1 volts. If one were to test with an electrometer two points near to but on oppo-

site sides of a strong *arc-light*, now a familiar object in city streets, the instrument should be about as much affected as if it were connected with 45 Daniell cells, joined like the cells used in § 303; that is, it should indicate that an electromotive force of about 50 volts was employed in maintaining the current through the lamp in question.

312. EXERCISE XLIII.

ACTION OF CURRENTS ON MAGNETS: GALVANOSCOPE.

[Worthington, Exp. 1, p. 275; Exp. 2, p. 276.]

Apparatus: The galvanoscope and the two-fluid cell[1] used in the preceding experiment. A copper wire, about No. 20, 2 or 3 metres long. A small compass for "tracing lines of force," as in Ex. XL. (The compass of the galvanoscope may be removable from its place on that instrument and so available for this purpose.)

FIRST PART.—Connect the long wire in circuit with the cell, and then bend all the wire of the circuit into a square, with the cell at one of the corners. Place this square so that two sides will run east and west and two sides north and south. Call the side through which the current, on its way from the copper to the zinc, flows east, E, that in which it flows west, W, etc. Place the compass just over the middle of side E and note the general direction in which the north-seeking end of the needle points. Then place it just beneath the middle of E and again note the general direction in which the needle points. Do the same with each of the sides W, N, and S.

When the needle is near the middle of any one side of the square, it is affected somewhat, but not seriously, by the other sides.

The direction of pointing will in every case show the resultant of the earth's action and of the current's action upon the needle. The student should study the indications of the various cases and endeavor to make out one general rule covering the behavior of the needle in all of them.

SECOND PART.—Put the cell in circuit with the 15 turns of the galvanoscope and place the latter so that the plane of its circle

---

[1] [This cell is not always powerful enough to give entirely satisfactory results in this Exercise. Two such cells joined *abreast* (§ 311, Experiment 77) will serve better.]

shall be east and west, and the current shall flow from east to west on the top of the circle. Place the compass 10 cm. south of the centre of the circle and record the direction of pointing of the needle. Move the compass north four stages of 5 cm. each, recording the direction of the needle at each stopping-place. [If the cell used is very powerful, somewhat greater distances than those mentioned in the last two sentences may be used.] Then place the centre of the compass inside the circle, close to the eastern side, and note the direction of the needle. Place the compass in turn north, east, and south of this part of the circle and all the time close to it, recording the direction of the needle in each position. Make similar observations about the western side of the circle.

Reverse the current in the galvanoscope by changing the connections with the cell, and then repeat all the operations described in the preceding paragraph.

The observations in the second part of this exercise can best be recorded in diagrams in the note-book. Nothing of the galvanoscope need appear in this diagram except two lines representing the sections of the east and west sides made by a horizontal plane passing through the centre of the circle. The student should compare these diagrams with those obtained in Ex. XL with two magnets placed parallel.

313. Discussion of Exercise XLIII.—From the teachings of this Exercise the student should fill out the following rule:

To a person looking along a conductor in the direction which the positive current (§ 306) *follows, the lines of force due to the current run around the conductor in a direction the same as (?) opposite to (?) that which the tip of a watch-hand follows around the dial.*

314. Definition and Measurement of Current Strength.—It is customary in the study of pure physics to define the strength of an electric current by reference to the force which a certain length of the conductor transmitting it exerts upon a magnetic pole of a certain strength at a certain distance. A current which exerts n times as great a magnetic force as a second current, the conditions being

the same for both, is said to be n times as strong as the second. On this basis currents are measured by means of the *galvanometer*, an instrument similar to the galvanoscope of the preceding Exercises, but more carefully made.

If several currents, measured in this way, are sent for the same length of time through similar solutions of chemical compounds, each current will produce an amount of chemical change (see §§ 305 and 309) proportional to its strength as measured by the galvanometer. Hence it has come about that currents are frequently measured by the rate at which they effect chemical changes of certain kinds.

An extended discussion of the measurement of electric currents by their magnetic action is beyond the scope of this book, and it seems best for our purpose to define the unit strength of current by reference to chemical action.[1] The ordinary commercial current of unit strength is called the *ampere*, in honor of a French investigator (§ 296), and may be defined as *a current of such strength as to free from its chemical combinations* .0003296 *gram of copper, or* .0003392 *gram of zinc in one second.*

The proper strength of current for an arc-lamp is about 10 amperes.

PROBLEM. Estimate as well as you can the strength of current passing through the Daniell cell during the half-hour test of Exercise XLII.

315. EXERCISE XLIV.

ELECTRICAL RESISTANCE OF WIRES.

Apparatus: A two-fluid cell, like the one used in Ex. XLII, and a galvanoscope. A current reverser, or "commutator."[2] Uncovered

---

[1] See Ayrton's *Practical Electricity*, § 6.

[2] See Fig. 165, p. 286, in Worthington, but make *four* holes, one at each corner of a square, and connect each hole with a separate binding-screw or spring-clamp. Two detached stout copper wires, amalgamated at the ends, are bent so as to connect any hole with either of the two adjoining holes. When the wires from a cell are

German silver wire, 4.2 m. of No. 30, B. & S., and 2.1 m. of No. 28. Covered copper wire, 20.5 m. of No. 30. A rack[1] for holding the wires. An "English" binding-post.

FIRST PART.—Connect by means of a short copper wire the two binding-posts which stand alone on one upright, thus uniting the two pieces of G. S. wire which are attached to these binding-posts. Put this wire in continuous circuit with the galvanic cell and the 15 turns of the galvanoscope, inserting the commutator in such a way that it will be easy to reverse the direction of the current through the galvanoscope [See Fig. XXVIII.] (The porous cup of the cell should be filled with acid several minutes before it is placed in the sulphate of copper in order that the cup may become wet through before the measurements begin. See Ex. XLII.) The current will now be flowing through 2 m. of the G. S. wire. Record the reading of the compass needle; then, by means of the commutator, reverse the current through the galvanoscope

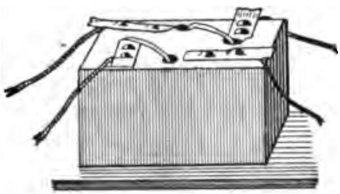


FIG. XXVII.

---

fastened into two diagonally opposite binding-screws and the two wires leading to a galvanoscope are held in the two other binding-screws, the two bent copper connectors may evidently be used to send the current from the battery in either direction at will through the galvanoscope. [See Fig. XXVII.]

[1] Take a pine plank 1 m. long, about 10 cm. wide, and about 4 cm. thick. To each end of one edge fasten a piece of wood about 25 cm. long, 8 cm. wide, and 1 cm. thick, in such a way that these strips will be vertical when the plank lies broadside down on a table. Connect the tops of these uprights by means of a meter-rod screwed to them, with its marks vertical. [See Fig. XXVIII.] In one upright fasten two small binding-posts on the same level and about 1.5 cm. apart. In the other upright at the same level fasten a corresponding pair of binding-posts, and above these, at intervals of 5 cm., place two more pairs. Connect the inner binding-post on that upright which carries only one pair with the corresponding inner binding-post on the other upright by means of a No. 30 German-silver wire, soldered or otherwise fastened to the *base* of each post Connect the two outer binding-posts by means of a similar wire led around the outer edge of

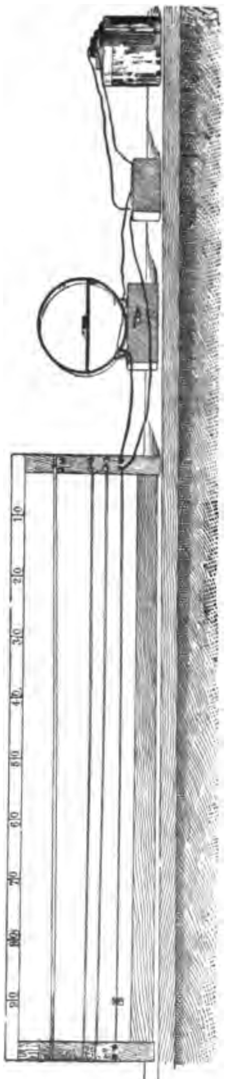


Fig. XXVIII.

and again record the reading. Remove the short copper wire which connects the two pieces of G. S. wire and by means of the "English" binding-post connect the same two wires in such a place that the current will now pass through 180 cm. of G. S. wire. [See Fig. XXVIII.] Read, reverse, and read again as before. Then go through the same operation with 160, 140, 120, 100, 80, and 60 cm. in turn of the G. S. wire. Finally put in 2 m. again and take readings to make sure that the power of the battery has not changed since the beginning of the measurements.

The mean of two deflections, read before and after a reversal, is to be taken as the true deflection. For instance, suppose the north-seeking end of the needle to have come to rest 40° east of north before reversal, and 38° west of

---

each upright. From one binding-post of another pair lead a No. 30 German-silver wire around both uprights to the other binding-post of the same pair. Connect in the same way another pair of binding-posts by means of No. 28 German-silver wire. Finally, beginning at one binding-post of the last pair with No. 30 covered copper wire, lead this wire ten complete circuits about the uprights and end at the other binding-post of the pair. In this winding of the copper wire, and in its subsequent use, it will be necessary to use some care in order to prevent *short circuiting* [leaking across] when the turns of the wire touch each other.

All the wires should be drawn rather tight, and the binding-posts should be made fast in their places."

north after reversal. The mean, 39°, will be called the true deflection. This method makes it unnecessary to determine accurately the position of the needle without current.

These measurements will indicate in a very general way the dependence of resistance upon length of wire; but the exact law cannot be deduced from them, and it should be stated to the student. These measurements are of use, however, in connection with the following parts of the exercise.

SECOND PART.—Take out from the circuit the No. 30 G. S. wire and insert in its place the 2 m. of No. 28 G. S. wire. Read, reverse, and read again.

Compare the deflection with those obtained with the various lengths of No. 30 wire, and estimate what length of the latter would be equivalent in resistance to the 2 m. of No. 28 wire. Then, taking the area of cross-section of No. 28 as 1.59 times[1] that of No. 30, find the relation between cross-section and resistance.

THIRD PART.—Put the two pieces of No. 30 G. S. wire that have been already used, united as before, into the circuit in "multiple arc" with the other 2 m. of [No. 30] G. S. wire, that is, in such a way that one part of the current will go through one 2-meter wire and the other part through the other 2-meter wire. Read, reverse, and read again.

Estimate what length of No. 30 wire, used as in the first part of this exercise, has a resistance equal to that of the two 2-meter pieces of No. 30 wire used here.

FOURTH PART.—Find, by methods already sufficiently indicated, what length of No. 30 G. S. wire is equivalent in resistance to the 20 m. of No. 30 copper wire.

Other parts of the circuit than the coils of the galvanoscope may perceptibly affect the needle if the wires are carelessly placed. They should not be unnecessarily near the galvanoscope, and the wires leading to and from this instrument should lie close together.

If it is found necessary to divide this Exercise between two days, the student should on the second day repeat some of the previous measurements, in order to make sure that the cell has the same power as on the first day.

[[1] It is better to *measure* the wires used with an accurate micrometer gauge and then calculate the ratio of the areas of cross-section.]

316. Unit of Resistance; Resistance Coils.—Electrical resistance has already been discussed, very briefly, in § 299. The ordinary unit of resistance is called the *ohm*, in honor of the investigator Ohm, who first made out clearly the relation between electromotive force, resistance of circuit, and strength of current. The so-called *legal ohm* is a resistance equal to that of a column of pure mercury 106 cm. in length and 1 sq. mm. in area of cross-section, at 0° C. It would take about 45 metres of good copper wire 1 sq, mm. in area of cross-section to furnish a resistance of one ohm.

It is frequently necessary in electrical work to introduce resistance into a circuit, and so-called *resistance-coils* are part of the ordinary equipment of a physical laboratory. They consist of spools upon which are wound pieces of silk-covered or cotton-covered wire, long or short, thick or thin, according to the particular service each coil has to perform. To avoid magnetic action from these coils the wire is wound upon them in such a way that half the turns carry a current in one direction and half in the other direction. This can be easily done if the wire is doubled before it is wound on the spool. German silver is very commonly used in the construction of such coils, on account of its high resistance (among metals) and the fact that its temperature must be raised about 23 degrees Centigrade to increase its resistance one per cent of itself, whereas the resistance of most common metals is increased one per cent of itself by a rise of about three degrees Centigrade.

317. Ohm's Law.—Ohm showed that, other things remaining equal, the current through a given conductor is proportional to the electromotive force which is applied to the conductor. In other words, if E stands for the electromotive force (§ 311) driving a current C (§ 314) through a given conductor, *the ratio* $E \div C$ *is a constant quantity for that conductor, so long as its physical condition remains*

unchanged. This ratio was taken by Ohm, and has been taken generally since his time, as the numerical measure of the conductor's *resistance.* The numerical relations of the three quantities L, C, and R, may be expressed by the equation,

$$R = \frac{E}{C},$$

which, with its other forms,

$$C = \frac{E}{R}, \text{ and } E = R \times C,$$

is called *Ohm's Law.*

Example. If a battery having an electromotive force of 5 volts is placed in a circuit the total resistance of which is 100 ohms, the strength of current will be $5 \div 100 \, (= \frac{1}{20})$ ampere.

318. EXERCISE XLV.

BATTERY RESISTANCE: COMBINATIONS OF BATTERIES.

Apparatus: Two two-fluid cells like those used in the preceding exercises. A strip of sheet zinc, amalgamated, 10 cm. long and 0.5 cm. wide, and a copper strip of similar dimensions, each provided with a copper wire like those on the large zinc and copper. The galvanoscope, the resistance-rack and the commutator already described.

FIRST PART.—Set up one cell with the large zinc and copper and the other cell with the small zinc and copper, making the liquids equally deep in both cells, taking care, as before, that the porous cups are wet through by the solutions before any measurements are made. Put the large-plate cell in circuit with the 5-turn section of the galvanoscope, inserting the commutator as in the preceding exercise. Set the plates as far apart as is practicable in the cell, then read, reverse and read again. (It should be understood hereafter that the direction to read means to take two readings, reversing between them.)

Take the large-plate cell out of the circuit and put in its place the narrow-strip cell. Put the strips as far apart as the plates of the other cell were, then read. Put them as near together as possible and then read.

Keeping the narrow-strip cell, put into the circuit 2 m. of No. 30 G. S. wire and the whole 15 turns of the galvanoscope. Repeat with this arrangement the operations just performed with the same cell. Then replace this cell by the large-plate cell; put, as before, the plates as far apart as possible, then read.

Comparing the various readings which have been made, determine whether the current is, when the external resistance is added, more or less sensitive to changes in size and position of the plates than when this resistance is not in use.

SECOND PART.—Replace the narrow strips of zinc and copper in their cell by full-sized plates. Join together the wires leading from the coppers of the two cells. Join also those leading from the zincs of the two cells. Insert the two cells thus arranged in the circuit as a single cell, and, retaining the 2 meters of G. S. wire and the 15 turns of the galvanoscope, note the deflection obtained. Then join the copper of one cell to the zinc of the other cell by means of their wires, and connect the other two wires in the circuit as the wires from a single cell would be connected. Other things being unchanged, note the deflection produced by this arrangement.

Take out the G. S. resistance, connect again with 5 turns only of the galvanoscope coil, and with this arrangement repeat both the two-cell experiments which have just been described. Study the results of all the tests with two cells with the purpose of determining, in a general way, under what conditions it is better to join the zincs together and the coppers together, and under what conditions it is better to join the zinc of one cell to the copper of the next.

319. Discussion of Exercise XLV.—So many elements are involved in the experiments of this Exercise that considerable care is necessary in drawing conclusions from it. The experiments of § 311 showed that the electromotive force of a battery increases with increase of the number of cells, provided these are joined in *series*, but that the combination of several cells *abreast* has an electromotive force little if any greater than that of a single cell. The fact is that, when tests are made with *open circuit* by means of an electrometer, the electromotive force of n cells in series is found to be just n times as great as that of one cell

alone, while the electromotive force of n cells joined abreast is just equal to that of one cell used alone. The electromotive force of cells *in operation,* that is, sending a current through a closed circuit, is not always the same as that of the same cells in open circuit, but the differences will not affect the results of our reasoning on the observations of this exercise.

Let us see what help Ohm's law (§ 317) will give with these observations, it being assumed that the R of that law is equal to $R_e + R_b$, R_e being the resistance of the circuit *external* to the battery, and R_b the resistance of the battery itself (frequently called the *internal* resistance), so that we may write the law thus,

$$C = \frac{E}{R_e + R_b}.$$

Observe that if R_e is very large in comparison with R_b, one hundred times as large, for instance, doubling or trebling, R_b will make but little difference in the value of C. But if R_e is not large compared with R_b, doubling or trebling the latter will make a great difference in the value of C. This suggestion will be found useful in discussing the *First Part* of the exercise.

For the *Second Part* it should be noted that, according to Ohm's law, the change from two cells abreast to the same number in series should, by doubling E, double C also, if the resistance remains unchanged. One part of the resistance, the R_e, does remain unchanged, and if we find that C is increased less, proportionally, than E, we conclude that R_b must be greater when the two cells are in series than when they are abreast.

Further experiments with more accurate apparatus would show that the resistance of n cells in series is n times as great as that of one cell used alone, and that the

resistance of n cells joined abreast is only $\frac{1}{n}$ as great as that of one cell alone.[1] (Compare this rule with that for *wires* joined in series or joined abreast, as in Exercise XLIV.)

When one has a given number of similar cells to be arranged at will for the purpose of sending the maximum current through a certain known external resistance, the proper rule to follow (of which no proof will be here given) is the following : *Join the cells in such a way as to make the resistance of the battery equal, as nearly as may be, to the resistance of the external part of the circuit.*

This rule will sometimes require all the cells to be arranged in series, which will make the battery resistance a maximum, but it must not be supposed that this arrangement is adopted *for the sake* of making the battery resistance large. It is adopted for the sake of making the *electro-motive force* large and *in spite of* the fact that it makes the battery resistance large.

To illustrate the working of the rule let us suppose that we have to arrange 12 cells, each having an electromotive force of 1 volt and a resistance of 1 ohm, in such a way as to send the strongest current through an external resistance of 3 ohms.

With the cells all in line we have

$$C = \frac{E}{R_e + R_b} = \frac{12}{3 + 12} = \frac{4}{5} \text{ (ampere).}$$

With the cells 2 abreast and 6 in series,

$$C = \frac{6}{3 + 3} = 1 \text{ (ampere).}$$

[1] It is to be noted, however, that the resistance of a cell is somewhat variable, being dependent upon the strength of current and certain other conditions.

With 3 abreast and 4 in series,

$$C = \frac{4}{3 + \frac{4}{3}} = \frac{12}{13} \text{ (ampere).}$$

With 4 abreast and 3 in series,

$$C = \frac{3}{3 + \frac{3}{4}} = \frac{4}{5} \text{ (ampere).}$$

With 6 abreast and 2 in series,

$$C = \frac{2}{3 + \frac{1}{3}} = \frac{3}{5} \text{ (ampere).}$$

With all the cells abreast,

$$C = \frac{1}{3 + \frac{1}{12}} = \frac{12}{37} \text{ (ampere).}$$

320. EXERCISE XLVI.

ELECTROMAGNETISM.

Let the student make and describe carefully a very simple sounder and key by which telegraphic signals may be made by means of the current furnished by one of the cells used in the preceding experiments, no resistance being used except that of the battery and instruments.

Materials suggested: An ordinary carriage-bolt about 7 cm. long and 0.7 cm. thick to be used as the *core* of the electromagnet; a soft iron nut to be attracted by the magnet; sheet spring-brass about 0.5 mm. thick; well covered copper wire, No. 30; a block of wood about 3 cm. thick 15 cm. wide and 20 cm. long; round-headed screws for binding-posts; cement of bees-wax and and rosin, etc.

The student should find out for himself how much wire is necessary for his electromagnet. He should wind the wire upon the core from a spool, and instead of cutting the wire whenever he wishes to make a trial he need merely scrape the covering off for a distance of about 1 cm. Care must be taken to prevent

short-circuiting from such breaks in the covering. It is well to draw the wire through melted paraffine before beginning to wind it.

Questions and Problems on Chapters XV and XVI.

1) The influence of the earth's magnetism upon a magnetic needle is merely *directive*. Show why it is so.

2) How can the intensity of various portions of the magnetic field be roughly estimated from the behavior of the compass-needle?

3) Draw diagrams illustrating the lines of magnetic force in the neighborhood of, 1st, a magnet with north pole pointing north, 2d, a magnet with north pole pointing south; and explain the difference between the diagrams.

4) If the magnets used in Exercise XL were of unequal strength, how could the relative intensity of their fields be approximately ascertained by use of a compass placed between them?

5) What proofs are there that the condition of a wire closing the circuit of a galvanic cell is different from that of the same wire not connected with a cell.

6) Explain why battery "polarization" weakens the current from the battery.

7) What do you, from your own experience, conclude to be the advantage of, 1st, amalgamating the zinc in a galvanic cell? 2d, using two fluids separated by a porous partition?

8) What effect will "short circuiting," that is, connecting the terminals by means of a conductor of very small resistance, produce on the consumption of zinc and of copper in the Daniell cell?

9) How strong a current will be required to cause the copper cylinder in a Daniell cell to gain 5 gm. per hour?

10) Describe carefully and fully some galvanic cell that you have used.

11) Name the ordinary unit of electromotive force. Name the ordinary unit of electric current. Name the ordinary unit of electric resistance. State the relation which exists between electromotive force, current, and resistance.

12) If you had two galvanic cells which in "short circuit," that is, with very small outside resistance, gave equal currents, could you safely assume them to have equal electromotive forces? If not, how could you decide the matter by experiment?

13) If the electromotive force of a certain battery is 2 volts, its resistance 4 ohms, and the external resistance of the circuit 6 ohms, what is the strength of the current? Name the unit in which the current is measured.

14) If a wire offers a resistance of 40 ohms, what current can be sent through it by 6 galvanic cells arranged 2 abreast and 3 in series, the internal resistance of each cell being 3 ohms, and its electromotive force 1.1 volts?

15) Make a diagram of an electric circuit including two cells, a galvanometer, and a resistance wire, with a commutator for reversing the current through the galvanometer, the cells being arranged for a *small* external resistance.

16) Make a diagram of an electric circuit including four cells, a galvanoscope, and a resistance wire, marking the positive and negative pole of each cell and arranging the cells in the best manner for a large external resistance.

APPENDICES.

APPENDIX I.

THE METRIC SYSTEM.

THE *metric system* of weights and measures derives its name from its fundamental unit the *meter*. This was intended to be just one ten-millionth of a quadrant of the earth's meridian, in the longitude of Paris. It is probably not exactly a ten-millionth of the quadrant of that meridian, but is perhaps more nearly a ten-millionth of the quadrant of the meridian of New York City.*

The metric units most employed in physics are the following ones, with their multiples and subdivisions :

The	*meter*,	measure of	length.
"	*square meter*,	measure of	area.
"	*cubic* "	" "	volume.
"	*gram*	" "	weight.

Multiples of these units have been given a series of names by the use of prefixes derived from Greek numerals, as follows :

Deca,	meaning	10.
Hecto,	"	100.
Kilo,	"	1000.
Myria	"	10,000.

---

* See article *Metric System* in Johnson's Cyclopædia.

Subdivisions of units are named by the use of Latin prefixes as follows:

Deci, meaning $\frac{1}{10}$.
Centi, " $\frac{1}{100}$.
Milli, " $\frac{1}{1000}$.

Applying these prefixes to form a table of linear measures derived from the meter, the table would read:

10 meters	= 1 decameter.
10 decameters or 100 meters	= 1 hectometer.
10 hectometers, or 100 decameters, or 1000 meters	= 1 kilometer.
$\frac{1}{10}$ meter	= 1 decimeter.
$\frac{1}{10}$ decimeter or $\frac{1}{100}$ meter	= 1 centimeter.
$\frac{1}{10}$ centimeter, or $\frac{1}{100}$ decimeter, or $\frac{1}{1000}$ meter	= 1 millimeter.

Corresponding tables may be constructed for square meters, for cubic meters, and for grams.

The abbreviations for names of metric units most used in this book are:

m. = meter.
cm. = centimeter.
mm. = millimeter.
gm. = gram.
kgm. = kilogram.

Some of the metric units have more than one name. For instance, the cubic decimeter is frequently called a *liter,* especially when the substance measured is a fluid.

All the other units of the metric system are derived from the linear meter. Of course the square meter is a square, one meter on a side, and the cubic meter a cube, one meter on an edge. The gram was intended to be, and is very

nearly, the weight of a cubic centimeter of water at 4° C.* It is convenient to remember that a cubic decimeter, or *liter*, of water, under standard conditions, weighs a kilogram.

Reduction of English units to the corresponding French ones, and the reverse, can be performed most rapidly by reference to tables of equivalents, such as Craig's Decimal System (Van Nostrand, N. Y.). A few of the more important equivalents are given here:

1 meter = 1.0936 yards.
1 " = 3.2809 feet.
1 " = 39.3705 inches.
1 kilometer = 0.6214 mile.
1 gram = 15.4323 grains = 0.0353 ounce.
1 kilogram = 2.2046 pounds avoirdupois.

1 yard = 0.9144 meter.
1 foot = 0.3048 "
1 inch = 0.2540 "
1 mile = 1.6093 kilometers.
1 pound avoirdupois = 0.4536 kilogram.
1 ounce = 28.35 grams.

The following are approximate equivalents:

1 decimeter = 4 inches.
1 meter = 1.1 yards.
1 kilometer = $\frac{5}{8}$ of a mile.
1 kilogram = $2\frac{1}{5}$ pounds.

---

* 4° C. is approximately the temperature of the maximum density of water. Distilled water is understood, and the weighing is supposed to be made in a vacuum, at sea-level, and in the latitude of Paris.

APPENDIX II.

VALUE IN MILLIMETERS OF BROWN & SHARPE WIRE-GAUGE NUMBERS.

Number.	Diameter mm.	Number.	Diameter mm.
1	7.348	17	1.150
2	6.544	18	1.024
3	5.827	19	0.912
4	5.189	20	0.812
5	4.621	21	0.723
6	4.115	22	0.644
7	3.656	23	0.573
8	3.264	24	0.511
9	2.906	25	0.455
10	2.582	26	0.405
11	2.305	27	0.361
12	2.053	28	0.321
13	1.828	29	0.286
14	1.628	30	0.255
15	1.459	31	0.227
16	1.291	32	0.202

APPENDIX III.

FUNDAMENTAL AND DERIVED UNITS:

THE C. G. S. SYSTEM OF UNITS.

THE C. G. S. system of units, to which occasional reference is made in this book, takes the *centimeter* as the unit of distance, the *gram* as the unit of mass, the *second* as the unit of time, and upon these three bases the definitions of all other units needed for the measurement of purely phys-

ical quantities. The first three are called *fundamental* units, the others *derived* units.

One of the simplest derived units is that of velocity. The unit velocity is that of a body moving at the rate of the unit distance, the centimeter, in the unit time, the second. For this unit there is no one-word name.

A somewhat more complex derived unit is that of force. The unit force, in the C. G. S. system, is that force which, acting for the unit time, the second, upon the unit mass, the gram, imparts to it unit velocity. To this particular unit the name *dyne* is given (see § 112).

It would be possible to state how other units, the volt and the ohm, for instance, are based upon the fundamental units, but the demonstration would in some cases be difficult for the users of this book to follow.

Sound and light, as forms of motion and energy, are to be described completely in terms of the fundamental units, but the *impressions made upon our senses* by sound and light are not to be described or measured in terms of the fundamental units.

Temperature is not commonly measured in terms of the fundamental units, although it is possible so to measure it.

As the fundamental units of the C. G. S. system are quite independent of the attraction of our earth, this system is applicable to the physics of the heavenly bodies, in fact to matter wherever it may be found. Hence it is called the *absolute* system of units. Other systems can be made equally "absolute" by avoiding gravitation units of force (see §§ 4 and 112).

The C. G. S. system, although borrowing two of its three fundamental units from the French, was first adopted, *as a system*, by the British Association for the Advancement of Science.

APPENDIX IV.

TABLE OF PHYSICAL CONSTANTS.

Name.	Breaking-strength, kgm. per sq. cm.	Density, grams per cu. cm.	Mean Coefficient of Cubical Expansion from 0° to 100° C.	Mean Specific Heat between 0° and 100° C.	Melting-point, Centigrade.	Latent Heat of Melting, gm-deg. C.	Boiling-point, Centigrade.	Latent Heat of Evaporation, gm-deg. C.	Electrical Resistance, silver the standard.
Copper........	4100	8.9	.000051	.093	1100	30?			1.07
Gold..........	3000	19.3	.000044	.032	1100				1.36
Iron..........	6100	7.8	.000036	.113	1600?	35?			6.4
Lead..........	100 to 300	11.3	.000088		330	5.6	1500		12.3
Platinum....	3100	21.5	.000027	.032	1900?	27			6.4
Silver.........	3100	10.5	.000058	.056	1000	21			1.0
Tin...........	200 to 410	7.2	.000069	.055	230	14	1500		7.1
Zinc..........	200 to 510	7.0	.000088	.093	420	28	1000		3.55
Brass:									
(cast).......	1200	8.3	.00005+	.094−	900?				6.4 to 3.2
(hard-drawn)	4100 to 9200	8.5	.000057	.09+	"				
Steel:									
(cast).......	8200 to 10200	7.8+	.000034	.12	1400?				
(wire).... ..	10200 to 20400	7.9?	.000037	.12?					
Glass:									
(crown).....	300 to 600		.000025	.19	400				Very large
(flint).......		3.5−	.000025	.19?					Very large

APPENDIX V.

VAPOR-PRESSURES AT VARIOUS TEMPERATURES EXPRESSED IN MEGADYNES* PER SQUARE CENTIMETER.

Temperature Centigrade.	Alcohol.	Bisulphide of Carbon.	Ether.	Mercury.	Water.
0°	.017	.17	.25	.0000	.006
10°	.032	.27	.38	.0000	.012
20°	·059	.40	.58	.0000	.023
30°	.10+	.58	.85	.0000	.042
40°	.18	.82	1.2	.0000	.073
50°	.29	1.14	1.7	.0000	.123
60°	.47	1.6	2.3	.0001	.198
70°	.72	2.1	3.1	·0001	.310
80°	1.08	2.7	4.0	.0002	.472
90°	1.6	3.5	5.2	.0004	.701
100°	2.3	4.4	6.6	.0006	1.014
110°	3.2	5.5	8.3	.0010	1.44
150°	9.8	12.		.0045	4.8
190°				.018	12.6

* The megadyne is 1,000,000 dynes. To find the pressure in centimeters of mercury column multiply the pressure in megadynes per square centimeters by 75.

APPENDIX VI.

INDICES OF REFRACTION OF VARIOUS SUBSTANCES AS COMPARED WITH A VACUUM (SEE § 254).

Agate	1.540	Selenium (crystals)	2.98
Canada balsam	1.53		
Diamond	2.5	Alcohol	1.36
Fluor spar	1.434	Petroleum (heavy)	1.45
Glass (ordinary crown)	1.53	" (light)	1.4+
" (" flint)	1.61*	Water	1.333
Ice	1.31		
Quartz	1.544	Nitrogen	1.000298
Rock salt	1.544	Oxygen	1.000271

* The *dispersive power* (§ 272) of flint glass is nearly twice as great as that of crown glass.

APPENDIX VII.

THE USE OF THE PORTE-LUMIÈRE AND THE SOLAR LANTERN.

WHENEVER a beam of sunlight is to be introduced into the room the porte-lumière is the most convenient means of obtaining such a beam. The use of this instrument when employed alone or as a part of the solar lantern (a modification of the magic lantern, using sunlight for the illumination) is described in several elementary books, one of the best of which is Dolbear's *Art of Projection* (Lee & Shepard, Boston, $1.65).

The form of instrument made by A. P. Gage is shown in Fig. 92, § 231. Directions for making a simple form of porte-lumière will be found in Prof. Dolbear's book above mentioned, or in Mayer and Barnard's *Light* (D. Appleton & Co., N. Y., $1.00). The instrument should be set in a southerly window, if possible; if that cannot be secured, it should be placed where the sun will strike it during the portion of the day when the physics class is at work.

Provision should be made for darkening all of the windows of the room in which the porte-lumière is to be used. This may conveniently be done by the use of black enamelled cloth curtains, hung on any convenient fixtures. Remove a pane from the lower part of the window that is to be used, and put in its place a piece of board, well painted to prevent warping. Fasten to this board a box about 8 or 10 inches square at its junction with the board, and projecting out-of-doors far enough to bring the mirror, when attached, clear of the shadow of the wall of the building, as it surrounds the window. The outer end of the box should be covered with a moderately thick piece of board; the

inner is to be left uncovered. In the outer end of the box cut a hole $7\frac{1}{2}$ inches in diameter, and cut four sockets to receive the ears *A*, *H*, *C*, and *D*, shown in Fig. 62. Screw the larger of the concentric plates to the inside surface of the outer end of the box.

Rotating the mirror by the use of the handles near *A* and *C*, and adjusting its angle of inclination to the plane of the rotating plate will enable the experimenter to maintain a horizontal beam of sunlight in the laboratory at will.

To project the image of a picture on an ordinary lantern-slide or any small object, on a screen or white wall, fasten a slide-holder to the inner of the concentric plates by means of thumbscrews provided for the purpose, and insert the slide or other object. Then stand a mounted, double-convex lens* horizontally in front of the object, between it and the white wall or screen on which the image is to be projected. Shift the position of the lens until a sharp image is obtained.

To show convection-currents in air, hold a lighted candle in the cone of light which extends from the focus of the lens to the screen.

To show the action of dilute acid upon the zinc in a simple voltaic element, nearly fill with the diluted acid a tank formed by clamping together two pieces of thick plate-glass, with a piece of rubber tubing, bent into a U-shape interposed between them.

Insert in the liquid a slender strip of zinc and a piece of stout copper wire, place the tank in or against the plate-holder of the porte-lumèire, focus the lens, as already described, and note the effect of joining and of separating the free ends of the zinc and copper.

---

* If the distance from object to screen is less than 30 feet, a lens of about 8-inch focus is to be used; if more than 30 feet, one of about 12-inch focus. The diameter of the lens should be 5 inches or more.

To show the electrolysis of water, submerge two platinum electrodes in dilute sulphuric acid in the tank and pass a powerful current through them.

Diagrams copied from the graphical record of experiments, the trace of the tuning-fork and the pendulum on smoked glass, as obtained in Exercise XXXI, and many similar things may be projected by means of the solar lantern. Such diagrams may be drawn on ordinary glass with thick india-ink, to which a very little caustic soda has been added, or drawn with a pencil on ground glass, which is afterwards made transparent by varnishing with damar varnish or with Canada balsam thinned with chloroform. Smoked-glass tracings may be rendered permanent by holding the smoked side for a few moments in the vapor which rises from alcohol poured into a heated saucer, and then flowing the smoked surface with thin alcohol-varnish or turpentine-varnish.

A great number of other modes in which the lantern may be made useful for class-room illustration will be found described in Professor Dolbear's *Art of Projection*, above mentioned.

APPENDIX VIII.

FOCAL LENGTH, ETC., OF LENSES AND COMBINATIONS OF LENSES.

It is customary to define the focal length, F, of a single lens as the distance from the focus to the nearest point of the surface of the lens, and in the formula $\frac{1}{F} = \frac{1}{D_0} + \frac{1}{D_i}$ to consider D_0 and D_i as measured from the object and image, respectively, to the nearest point of the lens. With this in-

terpretation of the letters, the formula is not exactly fulfilled by any actual lens. It holds strictly true only for the ideal case of a lens of zero thickness, but it is sufficiently near the truth for common purposes in the case of ordinary lenses. The formula is about equally accurate, for a double convex lens, at least, when all the distances, F, D_0, and D_i, are measured to the *optical centre* of the lens (§ 258).

When a combination of lenses is used, as in a microscope-objective or a photographic camera, a formula similar to that just given can be applied, but the F, D_0, and D_i occurring in it are not now measured either to the nearest point of the combination or to the optical centre. They are measured to certain other points determined by the radii of curvature, thickness, and refractive index of each lens, and the distance between the two lenses. In the ordinary use of such a combination, its magnifying power is substantially equivalent to that of a single ideal thin lens having a focal length equal to what is called the focal length of the combination. The calculation of the focal length of the combination is frequently very laborious.

Dealers in photographic objectives very frequently state as the focal length of a combination of lenses the distance from the principal focus to the nearer surface of the nearest lens. They sometimes call this the "back focal length," or, rather, the "back focus" of the combination. It is a convenient quantity to use in the description of a lens, but is not intended for use in the formula $\frac{1}{F} = \frac{1}{D_0} + \frac{1}{D_i}$.

The term "equivalent focal length," or "equivalent focus," is sometimes applied, in the case of a *symmetrical* combination of two equal lenses, to the distance from the principal focus to a point midway between the two lenses.

APPENDIX IX.

EXTRACT FROM THE HARVARD COLLEGE PAMPHLET CALLED "DESCRIPTIVE LIST OF ELEMENTARY PHYSICAL EXPERIMENTS."

It is well to obtain and compare catalogues and circulars from a number of firms having apparatus and supplies to sell. It is best, whenever it is practicable, to examine apparatus before buying it. Teachers must insist that no unnecessary finish shall be given to apparatus. Otherwise the prices will be unduly high. Most of the articles needed for this course can be obtained from the firms named below:—

Came & Co., Sudbury St., Boston. Second-hand billiard balls, $1 each.

Chatillon & Sons, 89 Cliff St., New York. Spring-balances. See Exs. I and VII.

Educational Supply Company, 6 Hamilton Place, Boston. Materials and general supplies. Apparatus for this course.

Eimer & Amend, 205 Third Ave., New York. Materials and general supplies. Some apparatus. (This firm has furnished to schools the scale-pan balance mentioned in the note to Ex. VII, with weights, for about $2.25.)

Fairbanks & Co., 311 Broadway, New York; 83 Milk St., Boston, etc. Scales and balances. (Scales having a beam about 8 in. long and a side beam with sliding weight reading to 0.1 grm. are furnished, without weights, by this firm for about $5. The firm sells also the Chatillon spring-balances.)

Gage, A. P., 13 Tremont Place, Boston. Apparatus.

Goodnow & Wightman, Sudbury St., Boston. Hardware and tools.

Hall, Thomas, Bromfield St., Boston. Apparatus and materials.

Metric Bureau, Boston. Meter-rods; measuring-vessels.

Queen & Co., 924 Chestnut St., Philadelphia. Optical apparatus. (The spy-glass for Ex. XXX need not cost more than $2.50. Spectacle-lenses can be obtained from wholesale jewellers for a few cents each.)

Ritchie & Sons, Brookline, Mass. Apparatus for this course.

Russell, A. L., 109 Court St., Boston. Electrical apparatus and supplies. [Apparatus for this course.]

The E. S. Greeley & Co., 5 Dey St., New York. Electrical apparatus and supplies. Useful catalogue of electrical supplies.

A. J. Wilkinson & Co., 184 & 188 Washington St., Boston. Hardware and tools.

It is doubtful whether in this course a teacher can, in general, superintend with the best results the laboratory work of more than twelve students at once.

The cost of apparatus and tools, exclusive of fixtures—such as tables, water-pipes and gas-pipes—necessary for a class working in divisions of twelve upon the experiments of this pamphlet [those called "Exercises" in this book] would probably be $350 or $400.* In making this estimate, it is assumed that certain articles, e.g., the scale-pan balance mentioned in the note to Exercise VII and the Fairbanks platform-balances, need not be furnished to each student, but that one instrument will serve for three or four students. One barometer and one air-pump will suffice. [If the suggestion given in the last paragraph of Ex. XL is followed, there should be one small air-pump, without plate, for every three or four students. See Ritchie's Catalogue.] The cost of apparatus will depend to some extent upon the particular exercises which are chosen for performance.

Throughout most of this course twelve students can work simultaneously upon the same experiment at two tables 10 ft. long and 3 ft. wide, made accessible on both sides and at the ends. Each student should have gas at his command on such a table. A convenient arrangement would be to run a gas-pipe, sustained from above, parallel to the length of each table and about 2 ft. above its middle, tapping this pipe to supply each burner. For the experiments in magnetism and electricity it would be well to have the tables movable from the vicinity of the gas-pipes, but this is not necessary. The laboratory should have one or two water-taps and a lead-lined or, better, a soap-stone, sink of generous proportions.

The Educational Supply Company mentioned above issues a price-list of the articles needed for the exercises of the College pamphlet. A. L. Russell & Co. will probably do the same.

---

* I shall be glad to give special regard to buying particular articles to teachers who are not satisfied with prices demanded.—E. H. H.

INDEX.

THE AMERICAN SCIENCE SERIES.

THE principal objects of the series are to supply the lack—in some subjects very great—of authoritative books whose principles are, so far as practicable, illustrated by familiar American facts, and also to supply the other lack that the advance of Science perennially creates, of text-books which at least do not contradict the latest generalizations. The scheme systematically outlines the field of Science, as the term is usually employed with reference to general education, and includes ADVANCED COURSES for maturer college students, BRIEFER COURSES for beginners in school or college, and ELEMENTARY COURSES for the youngest classes. The Briefer Courses are not mere abridgments of the larger works, but, with perhaps a single exception, are much less technical in style and more elementary in method. While somewhat narrower in range of topics, they give equal emphasis to controlling principles. The following books in this series are already published:

THE HUMAN BODY. By H. NEWELL MARTIN, Professor in the Johns Hopkins University.

Advanced Course. 8vo. 655 pp.

Designed to impart the kind and amount of knowledge every educated person should possess of the structure and activities and the conditions of healthy working of the human body. While intelligible to the general reader, it is accurate and sufficiently minute in details to meet the requirements of students who are not making human anatomy and physiology subjects of special advanced study. *The regular editions of the book contain an appendix on Reproduction and Development. Copies without this will be sent when specially ordered.*

From the CHICAGO TRIBUNE: "The reader who follows him through to the end of the book will be better informed on the subject of modern physiology in its general features than most of the medical practitioners who rest on the knowledge gained in comparatively antiquated text-books, and will, if possessed of average good judgment and powers of discrimination, not be in any way confused by statements of dubious questions or conflicting views."

THE HUMAN BODY.—*Continued.*

Briefer Course. 12mo. 364 pp.

Aims to make the study of this branch of Natural Science a source of discipline to the observing and reasoning faculties, and not merely to present a set of facts, useful to know, which the pupil is to learn by heart, like the multiplication-table. With this in view, the author attempts to exhibit, so far as is practicable in an elementary treatise, the ascertained facts of Physiology as illustrations of, or deductions from, the two cardinal principles by which it, as a department of modern science, is controlled,—namely, the doctrine of the "Conservation of Energy" and that of the "Physiological Division of Labor." To the same end he also gives simple, practical directions to assist the teacher in demonstrating to the class the fundamental facts of the science. *The book includes a chapter on the action upon the body of stimulants and narcotics.*

From HENRY SEWALL, *Professor of Physiology, University of Michigan:* "The number of poor books meant to serve the purpose of text-books of physiology for schools is so great that it is well to define clearly the needs of such a work: 1. That it shall contain accurate statements of fact. 2. That its facts shall not be too numerous, but chosen so that the important truths are recognized in their true relations. 3. That the language shall be so lucid as to give no excuse for misunderstanding. 4. That the value of the study as a discipline to the reasoning faculties shall be continually kept in view. I know of no elementary text-book which is the superior, if the equal, of Prof. Martin's, as judged by these conditions."

Elementary Course. 12mo. 261 pp.

A very earnest attempt to present the subject so that children may easily understand it, and, whenever possible, to start with familiar facts and gradually to lead up to less obvious ones. *The action on the body of stimulants and narcotics is fully treated.*

From W. S. PERRY, *Superintendent of Schools, Ann Arbor, Mich.:* "I find in it the same accuracy of statement and scholarly strength that characterize both the larger editions. The large relative space given to hygiene is fully in accord with the latest educational opinion and practice; while the amount of anatomy and physiology comprised in the compact treatment of these divisions is quite enough for the most practical knowledge of the subject. The handling of alcohol and narcotics is, in my opinion, especially good. The most admirable feature of the book is its fine adaptation to the capacity of younger pupils. The diction is simple and pure, the style clear and direct, and the manner of presentation bright and attractive."

ASTRONOMY. By SIMON NEWCOMB, Professor in the Johns Hopkins University, and EDWARD S. HOLDEN, Director of the Lick Observatory.

Advanced Course. 8vo. 512 pp.

To facilitate its use by students of different grades, the subject-matter is divided into two classes, distinguished by the size of the type. The portions in large type form a complete course for the use of those who desire only such a general knowledge of the subject as can be acquired without the application of advanced mathematics. The portions in small type comprise additions for the use of those students who either desire a more detailed and precise knowledge of the subject, or who intend to make astronomy a special study.

From C. A. YOUNG, *Professor in Princeton College:* "I conclude that it is decidedly superior to anything else in the market on the same subject and designed for the same purpose."

Briefer Course. 12mo. 352 pp.

Aims to furnish a tolerably complete outline of the astronomy of to-day, in as elementary a shape as will yield satisfactory returns for the learner's time and labor. It has been abridged from the larger work, not by compressing the same matter into less space, but by omitting the details of practical astronomy, thus giving to the descriptive portions a greater relative prominence.

From THE CRITIC: "The book is in refreshing contrast to the productions of the professional schoolbook-makers, who, having only a superficial knowledge of the matter in hand, gather their material, without sense or discrimination, from all sorts of authorities, and present as the result an *indigesta moles*, a mass of crudities, not unmixed with errors. The student of this book may feel secure as to the correctness of whatever he finds in it. Facts appear as facts, and theories and speculations stand for what they are, and are worth."

From W. B. GRAVES, *Master Scientific Department of Phillips Academy:* "I have used the Briefer Course of Astronomy during the past year. It is up to the times, the points are put in a way to interest the student, and the size of the book makes it easy to go over the subject in the time allotted by our schedule."

From HENRY LEFAVOUR, *late Teacher of Astronomy, Williston Seminary:* "The impression which I formed upon first examination, that it was in very many respects the best elementary text-book on the subject, has been confirmed by my experience with it in the class-room."

ZOOLOGY. By A. S. PACKARD, Professor in Brown University.

Advanced Course. 8vo. 719 pp.

Designed to be used either in the recitation-room or in the laboratory. It will serve as a guide to the student who, with a desire to get at first-hand a general knowledge of the structure of leading types of life, examines living animals, watches their movements and habits, and finally dissects them. He is presented first with the facts, and led to a thorough knowledge of a few typical forms, then taught to compare these with others, and finally led to the principles or inductions growing out of the facts.

From A. E. VERRILL, *Professor of Zoology in Yale College:* "The general treatment of the subject is good, and the descriptions of structure and the definitions of groups are, for the most part, clear, concise, and not so much overburdened by technical terms as in several other manuals of structural zoology now in use."

Briefer Course. 12mo. 334 pp.

The distinctive characteristic of this book is its use of the *object method.* The author would have the pupils first examine and roughly dissect a fish, in order to attain some notion of vertebrate structure as a basis of comparison. Beginning then with the lowest forms, he leads the pupil through the whole animal kingdom until man is reached. As each of its great divisions comes under observation, he gives detailed instructions for dissecting some one animal as a type of the class, and bases the study of other forms on the knowledge thus obtained.

From HERBERT OSBORN, *Professor of Zoology, Iowa Agricultural College:* "I can gladly recommend it to any one desiring a work of such character. While I strongly insist that students should study animals from the animals themselves,—a point strongly urged by Prof. Packard in his preface,—I also recognize the necessity of a reliable text-book as a guide. As such a guide, and covering the ground it does, I know of nothing better than Packard's."

First Lessons in Zoology. 12mo. 290 pp.

In method this book differs considerably from those mentioned above. Since it is meant for young beginners, it describes but few types, mostly those of the higher orders, and discusses their relations to one another and to their surroundings. The aim, however, is the same with that of the others; namely, to make clear the general principles of the science, rather than to fill the pupil's mind with a mass of what may appear to nim unrelated facts.

PSYCHOLOGY—Advanced Course. By WILLIAM JAMES, Professor in Harvard University. 2 vols. 8vo., 689, 704 pp.

From Prof. E. H. GRIFFIN, *John Hopkins University:* "An important contribution to psychological science, discussing its present aspects and problems with admirable breadth, insight, and independence."

From Prof. JOHN DEWEY, *University of Michigan;* "A remarkable union of wide learning, originality of treatment, and, above all, of never-failing suggestions. To me the best treatment of the whole matter of advanced psychology in existence. It does more to put psychology in scientific position both as to the statement of established results and a stimulating to further problems and their treatment, than any other book of which I know."

From Hon. W. T. HARRIS, *National Bureau of Education:* "I have never seen before a work that brings together so fully all of the labors, experimental and analytic, of the school of physiological psychologists."

BOTANY. By CHARLES E. BESSEY, Professor in the University of Nebraska.

Advanced Course. 8vo. 611 pp.

Aims to lead the student to obtain at first-hand his knowledge of the anatomy and physiology of plants. Accordingly, the presentation of matter is such as to fit the book for constant use in the labaratory, the text supplying the outline sketch which the student is to fill in by the aid of scalpel and microscope.

From J. C. Arthur, Editor of *The Botanical Gazette:* "The first botanical text-book issued in America which treats the most important departments of the science with anything like due consideration. This is especially true in reference to the physiology and histology of plants, and also to special morphology. Structural Botany and classification have up to the present time monopolized the field, greatly retarding the diffusion of a more complete knowledge of the science."

Essentials of Botany. 12mo. 292 pp.

A guide to beginners. Its principles are, that the true aim of botanical study is not so much to seek the family and proper names of specimens as to ascertain the laws of plant structure and plant life; that this can be done only by examining and dissecting the plants themselves; and that it is best to confine the attention to a few leading types, and to take up first the simpler and more easily understood forms, and afterwards those whose structure and functions are more complex.

From J. T. ROTHROCK, *Professor in the University of Pennsylvania:* "There is nothing superficial in it, nothing needless introduced, nothing essential left out. The language is lucid; and, as the crowning merit of the book, the author has introduced throughout the volume 'Practical Studies,' which direct the student in his effort to see for himself all that the text-book teaches."

CHEMISTRY. By IRA REMSEN, Professor in the Johns Hopkins University.

Advanced Course. 8vo.

The general plan of this work will be the same with that of the Briefer Course, already published. But the part in which the members of the different families are treated will be considerably enlarged. Some attention will be given to the lines of investigation regarding chemical affinity, dissociation, speed of chemical action, mass action, chemical equilibrium, thermochemistry, etc. The periodic law, and the numerous relations which have been traced between the chemical and physical properties of the elements and their positions in the periodic system will be specially emphasized. Reference will also be made to the subject of the chemical constitution of compounds, and the methods used in determining constitution.

Introduction to the Study of Chemistry. 12mo. 389 pp.

The one comprehensive truth which the author aims to make clear to the student is the essential nature of chemical action. With this in view, he devotes the first 208 pages of the book to a carefully selected and arranged series of simple experiments, in which are gradually developed the main principles of the subject. His method is purely inductive; and, wherever experience has shown it to be practicable, the truths are drawn out by pointed questions, rather than fully stated. Next, when the student is in a position to appreciate it, comes a simple account of the theory of the science. The last 150 pages of the book are given to a survey, fully illustrated by experiments, of the leading families of *inorganic* compounds.

From ARTHUR W. WRIGHT, *Professor in Yale College*:—The student is not merely made acquainted with the phenomena of chemistry, but is constantly led to reason upon them, to draw conclusions from them, and to study their significance with reference to the processes of chemical action—a course which makes the book in a high degree disciplinary as well as instructive.

From THOS. C. VAN NUYS, *Professor of Chemistry in the Indiana University*:—It seems to me that Remsen's "Introduction to the Study of Chemistry" meets every requirement as a text or class book.

From C. LES MEES, *Professor of Chemistry in the Ohio University*:—I unhesitatingly recommend it as the best work as yet published for the use of beginners in the study. Having used it, I feel justified in saying this much.

CHEMISTRY—*Continued.*

Elements of Chemistry. 12mo. **272 pp.**

Utilizes the facts of every-day experience to **show what** chemistry is and how things are studied chemically. The language is untechnical, and the subject is fully illustrated by simple experiments, in which the pupil is led by questions to make his own inferences. The author has written under the belief that "a rational course in chemistry, whether for younger or older pupils, is something more than a lot of statements of facts of more or less importance; a lot of experiments of more or less beauty; or a lot of rules devised for the purpose of enabling the pupil to tell what things are made of. If the course does not to some extent help the pupil to think as well as to see it does not deserve to be called rational."

CHASE PALMER, *Professor in the State Normal School, Salem, Mass.:* —It is the best introduction to chemistry that I know, and I intend to put it into the hands of my pupils next Fall.

A. D. GRAY, *Instructor in Springfield (Mass.) High School*:—Neat, attractive, clear, and accurate, it leaves little to be desired or sought for by one who would find the best book for an elementary course in our High Schools and Academies.

GENERAL BIOLOGY. By WILLIAM T. SEDGWICK, Professor in the Mass. Institute of Technology, and EDMUND B. WILSON, Professor in Bryn Mawr College. *Part I.* 8vo. 193 pp.

This work is intended for college and university students as an introduction to the theoretical and practical study of biology. It is not zoology, botany, or physiology, and is intended not as a substitute, but as a foundation, for these more special studies. In accordance with the present obvious tendency of the best elementary biological teaching, it discusses broadly some of the leading principles of the science on the substantial basis of a thorough examination of a limited number of typical forms, including both plants and animals. Part First, now published, is a general introduction to the subject illustrated by the study of a few types. Part Second will contain a detailed survey of various plants and animals.

W. G. FARLOW, *Professor in Harvard University, Cambridge, Mass.:* —An introduction is always difficult to write, and I know no work in which the general relations of plants and animals and the cell-structure have been so well stated in a condensed form.

POLITICAL ECONOMY. By FRANCIS A. WALKER, President of the Massachusetts Institute of Technology.

Advanced Course. 8vo. 537 pp.

The peculiar merit of this book is its *reality*. The reader is brought to see the application of the laws of political economy to real facts. He learns the extent to which those laws hold good, and the manner in which they are applied. The subject is divided, as usual, into the three great branches of production, exchange, and distribution. An interesting and suggestive "book" on consumption is added, which serves to bring in conveniently the principles of population. The last part of the volume is given to the consideration of various practical applications of economic principles.

From RICHMOND MAYO SMITH, *Professor in Columbia College, N. Y.*:—In my opinion it is the best text-book of political economy that we as yet possess.

From WOODROW WILSON, *Professor in Princeton University, N. J.*:—It serves better than any other book I know of as an introduction to the most modern point of view as to economical questions.

Briefer Course. 12mo. 415 pp.

The demand for a briefer manual by the same author for the use of schools in which only a short time can be given to the subject has led to the publication of the present volume. The work of abridgment has been effected mainly through excision, although some structural changes have been made, notably in the parts relating to distribution and consumption.

From ALEXANDER JOHNSTON, *late Professor in Princeton University, N. J.*:—Using the "Briefer Course" as a text-book, suited to any capacity, I am able at the same time to recommend the "Advanced Course" to those who are better able to use it as a book of reference, or more inclined to carry their work further.

Elementary Course. 12mo. 323 pp.

What has been attempted is a clear arrangement of topics; a simple, direct, and forcible presentation of the questions raised; the avoidance, as far as possible, of certain metaphysical distinctions which the author has found perplexing; a frequent repetition of cardinal doctrines, and especially a liberal use of concrete illustrations, drawn from facts of common experience or observation.

... 2t − 1
... number: ½ t^2
... many one $(2t-1)/2$

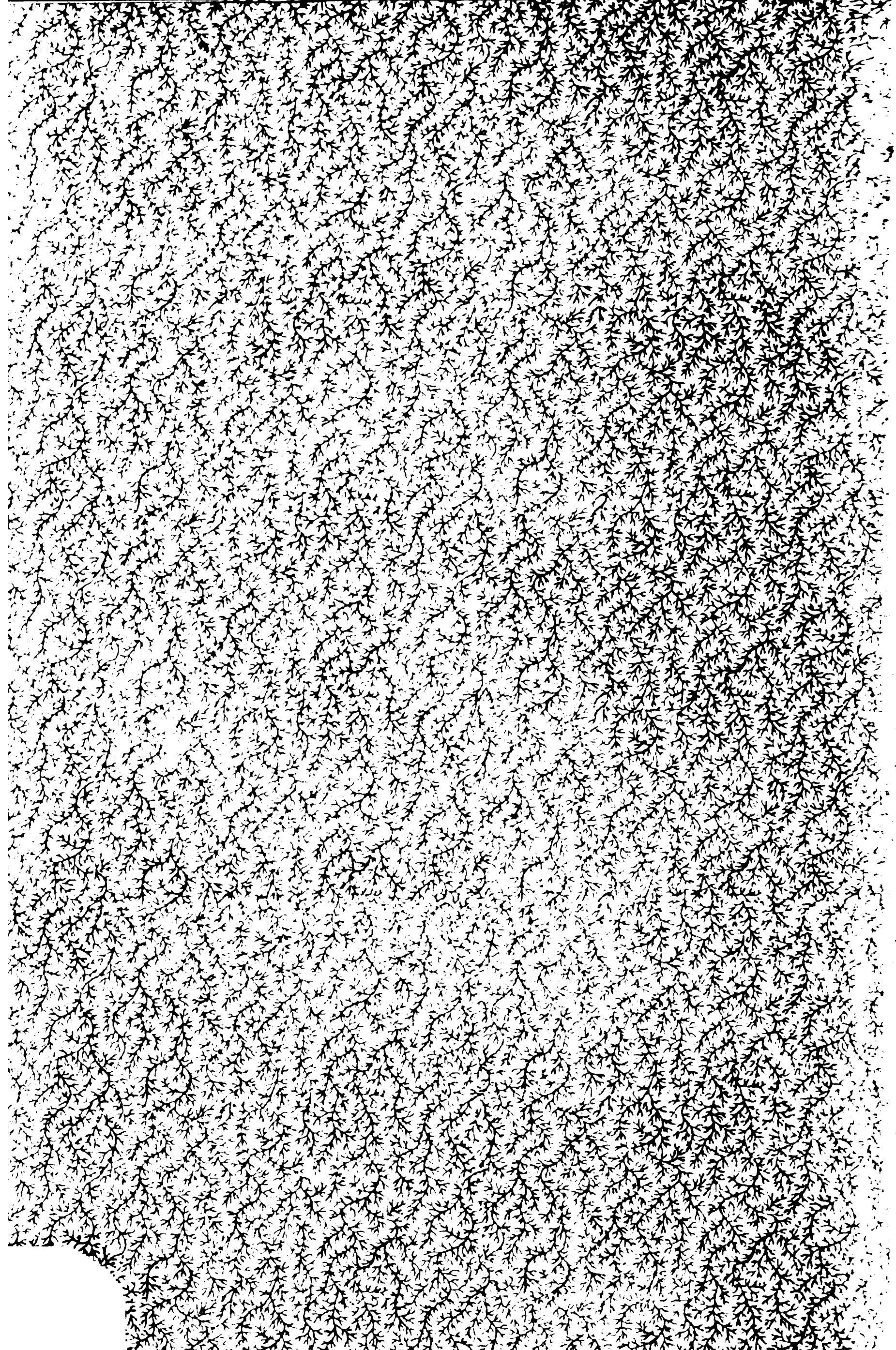
---

1. Pressure of atmosphere = 14.7 lbs per sq in
or = 1033 grams per sq cm
Barometer (av) = 76 cm.

2. Boyle's Law
vol. of a gas varies inversely as the pressure (and directly as the (absolute) temperature)
or, $v : v' = \frac{p'}{T} : \frac{p}{T'}$
To get absolute temperatures add 273 to °C. readings.

---

3. Archimedes' Principle.
a body immersed in a liquid loses the weight of its own volume of the liquid.



JANUARY 25, 1924